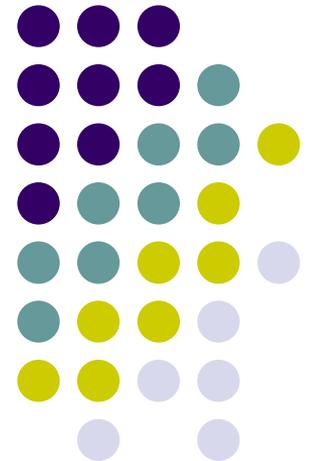


Chapter 34

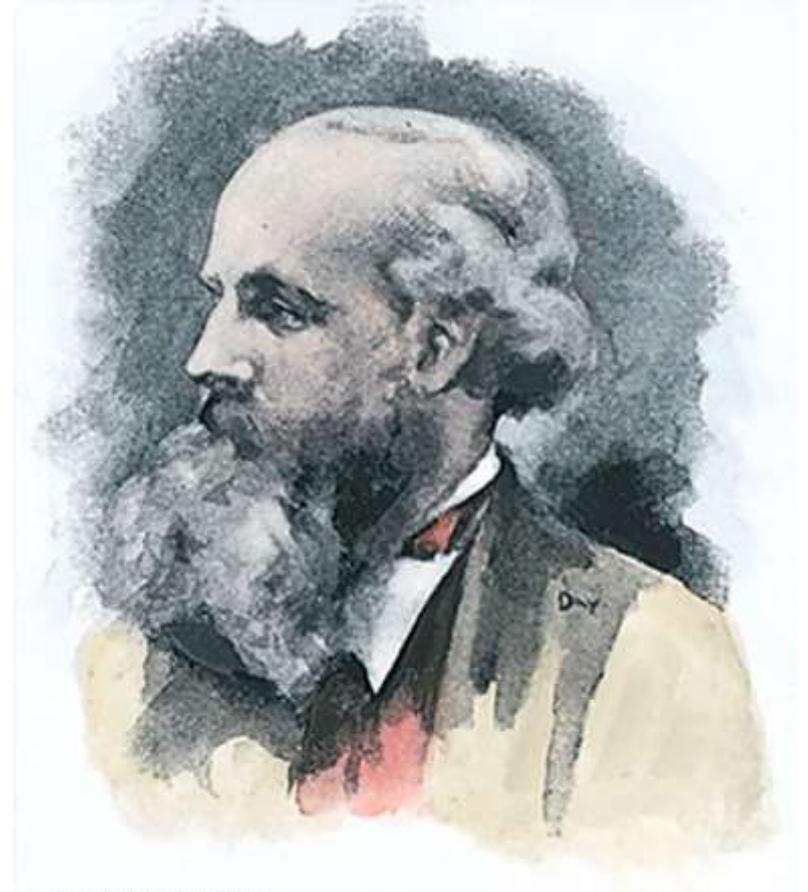
Electromagnetic Waves



James Clerk Maxwell



- 1831 – 1879
- Scottish physicist
- Provided a mathematical theory that showed a close relationship between all electric and magnetic phenomena
- His equations predict the existence of electromagnetic waves that propagate through space
- Also developed and explained
 - Kinetic theory of gases
 - Nature of Saturn's rings
 - Color vision



© 2004 Thomson - Brooks/Cole

Modifications to Ampère's Law

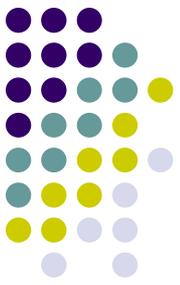


- Ampère's Law is used to analyze magnetic fields created by currents:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_o I$$

- But this form is valid only if any electric fields present are constant in time
- Maxwell modified the equation to include time-varying electric fields
- Maxwell's modification was to add a term

Modifications to Ampère's Law, cont



- The additional term included a factor called the **displacement current**, I_d

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt}$$

- This term was then added to Ampère's Law
 - Now sometimes called Ampère-Maxwell Law
- This showed that magnetic fields are produced both by conduction currents and by time-varying electric fields



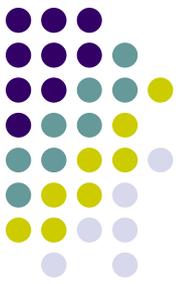
Maxwell's Equations

- In his unified theory of electromagnetism, Maxwell showed that electromagnetic waves are a natural consequence of the fundamental laws expressed in these four equations:

$$\oiint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0} \quad \oiint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

$$\oiint \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = -\frac{d\Phi_B}{dt} \quad \oiint \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell's Equation 1 – Gauss' Law



- The total electric flux through any closed surface equals the net charge inside that surface divided by ϵ_0

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0}$$

- This relates an electric field to the charge distribution that creates it

Maxwell's Equation 2 – Gauss' Law in Magnetism



- The net magnetic flux through a closed surface is zero

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

- The number of magnetic field lines that enter a closed volume must equal the number that leave that volume
- If this wasn't true, there would be magnetic monopoles found in nature
 - There haven't been any found

Maxwell's Equation 3 – Faraday's Law of Induction



- Describes the creation of an electric field by a time-varying magnetic field
- The emf, which is the line integral of the electric field around any closed path, equals the rate of change of the magnetic flux through any surface bounded by that path

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

- One consequence is the current induced in a conducting loop placed in a time-varying magnetic field

Maxwell's Equation 4 – Ampère-Maxwell Law



- Describes the creation of a magnetic field by a changing electric field and by electric current
- The line integral of the magnetic field around any closed path is the sum of μ_0 times the net current through that path and $\epsilon_0\mu_0$ times the rate of change of electric flux through any surface bounded by that path

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$



Lorentz Force Law

- Once the electric and magnetic fields are known at some point in space, the force acting on a particle of charge q can be found

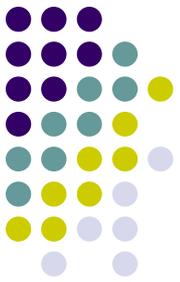
$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

- Maxwell's equations with the Lorentz Force Law completely describe all classical electromagnetic interactions

Speed of Electromagnetic Waves

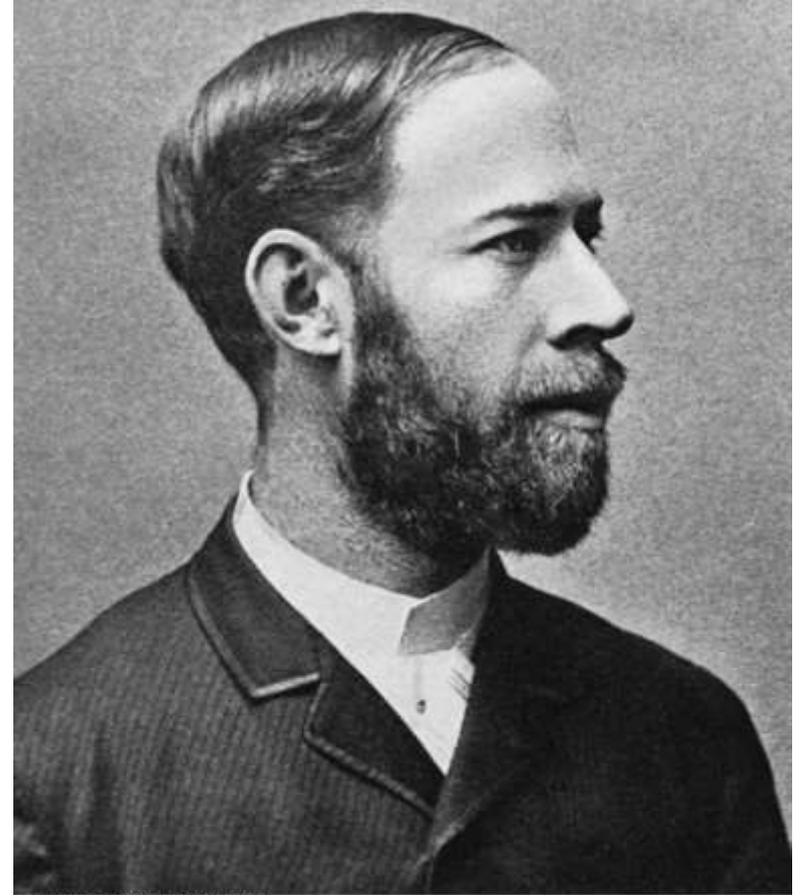


- In empty space, $q = 0$ and $I = 0$
- The last two equations can be solved to show that the speed at which electromagnetic waves travel is the speed of light
- This result led Maxwell to predict that light waves were a form of electromagnetic radiation

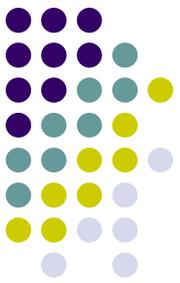


Heinrich Rudolf Hertz

- 1857 – 1894
- German physicist
- First to generate and detect electromagnetic waves in a laboratory setting
- The most important discoveries were in 1887
- He also showed other wave aspects of light

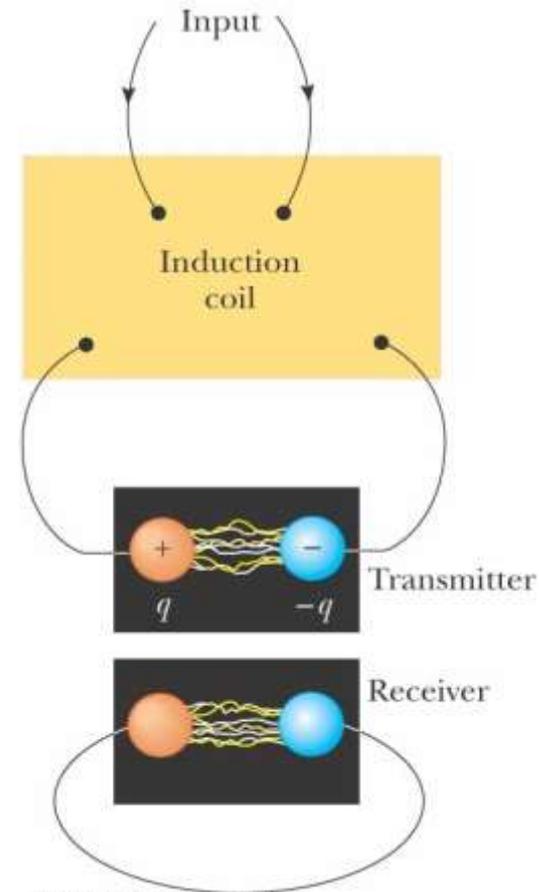


© 2004 Thomson - Brooks/Cole



Hertz's Experiment

- An induction coil is connected to a transmitter
- The transmitter consists of two spherical electrodes separated by a narrow gap

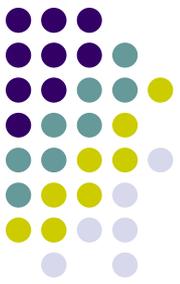




Hertz's Experiment, cont.

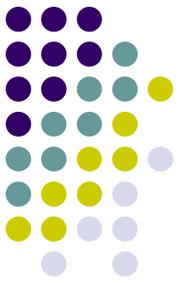
- The coil provides short voltage surges to the electrodes
- As the air in the gap is ionized, it becomes a better conductor
- The discharge between the electrodes exhibits an oscillatory behavior at a very high frequency
- From a circuit viewpoint, this is equivalent to an LC circuit

Hertz's Experiment, final

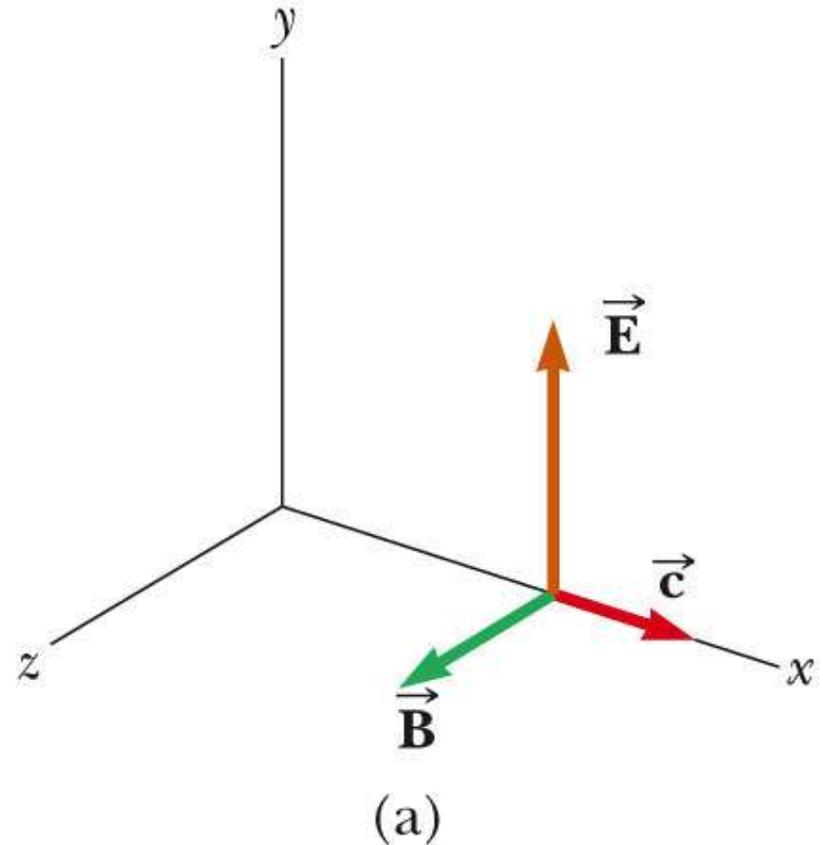


- Sparks were induced across the gap of the receiving electrodes when the frequency of the receiver was adjusted to match that of the transmitter
- In a series of other experiments, Hertz also showed that the radiation generated by this equipment exhibited wave properties
 - Interference, diffraction, reflection, refraction and polarization
- He also measured the speed of the radiation

Plane em Waves



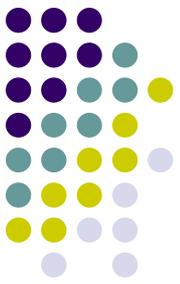
- We will assume that the vectors for the electric and magnetic fields in an em wave have a specific space-time behavior that is consistent with Maxwell's equations
- Assume an em wave that travels in the x direction with $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ as shown



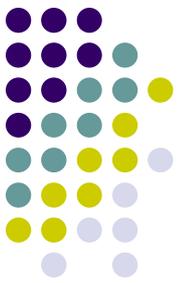
© Thomson Higher Education

PLAY
ACTIVE FIGURE

Plane em Waves, cont.



- The x -direction is the *direction of propagation*
- The electric field is assumed to be in the y direction and the magnetic field in the z direction
- Waves in which the electric and magnetic fields are restricted to being parallel to a pair of perpendicular axes are said to be *linearly polarized* waves
- We also assume that at any point in space, the magnitudes E and B of the fields depend upon x and t only



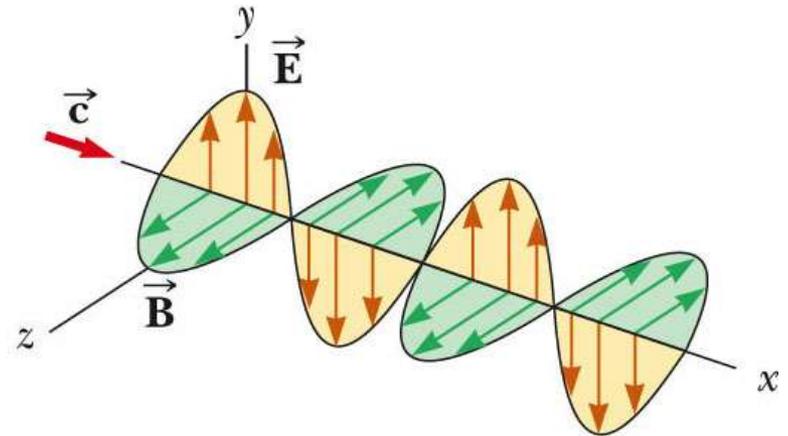
Rays

- A **ray** is a line along which the wave travels
- All the rays for the type of linearly polarized waves that have been discussed are parallel
- The collection of waves is called a **plane wave**
- A surface connecting points of equal phase on all waves, called the **wave front**, is a geometric plane

Wave Propagation, Example



- The figure represents a sinusoidal em wave moving in the x direction with a speed c
- Use the active figure to observe the motion
- Take a “snapshot” of the wave and investigate the fields



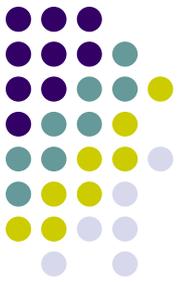
(b)

**PLAY
ACTIVE FIGURE**



Waves – A Terminology Note

- The word *wave* represents both
 - The emission from a single point
 - The collection of waves from all points on the source
- The meaning should be clear from the context



Properties of em Waves

- The solutions of Maxwell's third and fourth equations are wave-like, with both E and B satisfying a wave equation
- Electromagnetic waves travel at the speed of light:

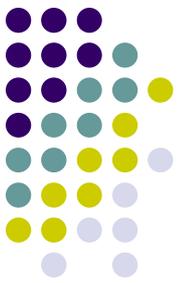
$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

- This comes from the solution of Maxwell's equations

Properties of em Waves, 2



- The components of the electric and magnetic fields of plane electromagnetic waves are perpendicular to each other and perpendicular to the direction of propagation
 - This can be summarized by saying that electromagnetic waves are transverse waves



Properties of em Waves, 3

- The magnitudes of the electric and magnetic fields in empty space are related by the expression

$$c = \frac{E}{B}$$

- This comes from the solution of the partial differentials obtained from Maxwell's equations
- Electromagnetic waves obey the superposition principle

Derivation of Speed – Some Details



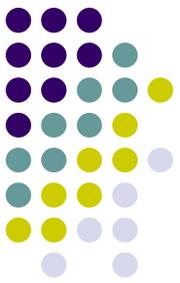
- From Maxwell's equations applied to empty space, the following partial derivatives can be found:

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \quad \text{and} \quad \frac{\partial^2 B}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}$$

- These are in the form of a general wave equation, with

$$v = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

- Substituting the values for μ_0 and ϵ_0 gives $c = 2.99792 \times 10^8$ m/s



E to B Ratio – Some Details

- The simplest solution to the partial differential equations is a sinusoidal wave:
 - $E = E_{\max} \cos(kx - \omega t)$
 - $B = B_{\max} \cos(kx - \omega t)$
- The angular wave number is $k = 2\pi/\lambda$
 - λ is the wavelength
- The angular frequency is $\omega = 2\pi f$
 - f is the wave frequency



E to B Ratio – Details, cont.

- The speed of the electromagnetic wave is

$$\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = \lambda f = c$$

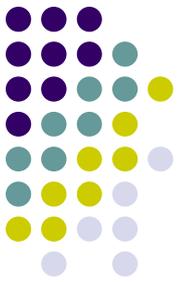
- Taking partial derivations also gives

$$\frac{E_{\max}}{B_{\max}} = \frac{\omega}{k} = \frac{E}{B} = c$$



Poynting Vector

- Electromagnetic waves carry energy
- As they propagate through space, they can transfer that energy to objects in their path
- The rate of flow of energy in an em wave is described by a vector, $\vec{\mathbf{S}}$, called the **Poynting vector**

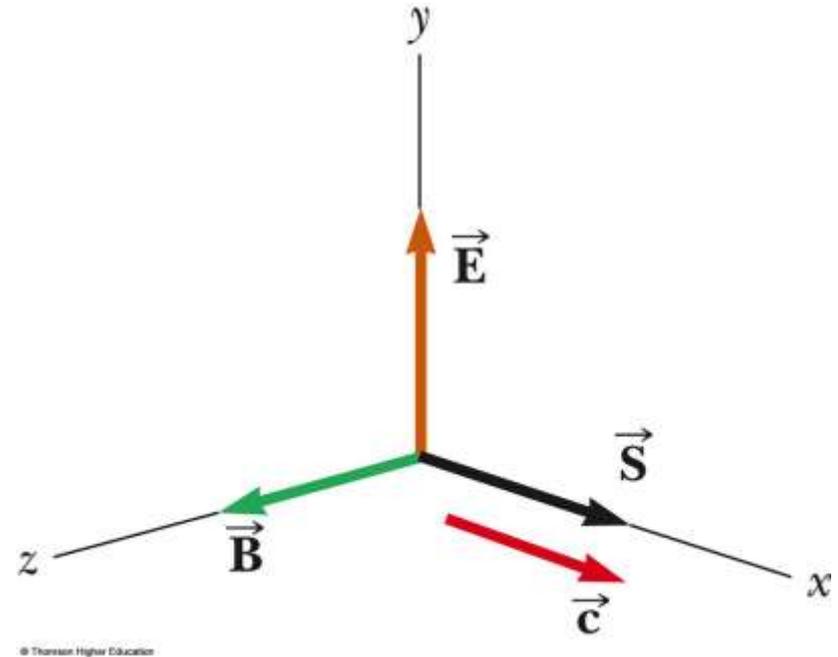


Poynting Vector, cont.

- The Poynting vector is defined as

$$\vec{\mathbf{S}} \equiv \frac{1}{\mu_0} \vec{\mathbf{E}} \times \vec{\mathbf{B}}$$

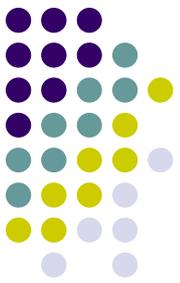
- Its direction is the direction of propagation
- This is time dependent
 - Its magnitude varies in time
 - Its magnitude reaches a maximum at the same instant as $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$





Poynting Vector, final

- The magnitude of $\vec{\mathbf{S}}$ represents the rate at which energy flows through a unit surface area perpendicular to the direction of the wave propagation
 - This is the *power per unit area*
- The SI units of the Poynting vector are $\text{J}/(\text{s}\cdot\text{m}^2) = \text{W}/\text{m}^2$



Intensity

- The wave *intensity*, I , is the time average of S (the Poynting vector) over one or more cycles
- When the average is taken, the time average of $\cos^2(kx - \omega t) = \frac{1}{2}$ is involved

$$I = S_{\text{avg}} = \frac{E_{\text{max}} B_{\text{max}}}{2\mu_0} = \frac{E_{\text{max}}^2}{2\mu_0 c} = \frac{c B_{\text{max}}^2}{2\mu_0}$$



Energy Density

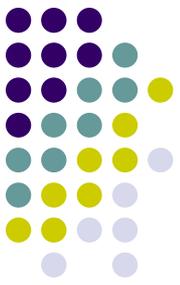
- The energy density, u , is the energy per unit volume
- For the electric field, $u_E = \frac{1}{2} \epsilon_0 E^2$
- For the magnetic field, $u_B = \frac{1}{2} \mu_0 B^2$
- Since $B = E/c$ and $c = 1/\sqrt{\mu_0 \epsilon_0}$

$$u_B = u_E = \frac{1}{2} \epsilon_0 E^2 = \frac{B^2}{2\mu_0}$$



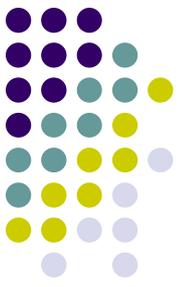
Energy Density, cont.

- The instantaneous energy density associated with the magnetic field of an em wave equals the instantaneous energy density associated with the electric field
 - In a given volume, the energy is shared equally by the two fields



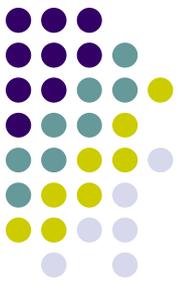
Energy Density, final

- The **total instantaneous energy density** is the sum of the energy densities associated with each field
 - $u = u_E + u_B = \epsilon_0 E^2 = B^2 / \mu_0$
- When this is averaged over one or more cycles, the total average becomes
 - $u_{\text{avg}} = \epsilon_0 (E^2)_{\text{avg}} = \frac{1}{2} \epsilon_0 E_{\text{max}}^2 = B_{\text{max}}^2 / 2\mu_0$
- In terms of I , $I = S_{\text{avg}} = cu_{\text{avg}}$
 - The intensity of an em wave equals the average energy density multiplied by the speed of light



Momentum

- Electromagnetic waves transport momentum as well as energy
- As this momentum is absorbed by some surface, pressure is exerted on the surface
- Assuming the wave transports a total energy T_{ER} to the surface in a time interval Δt , the total momentum is $p = T_{ER} / c$ for complete absorption



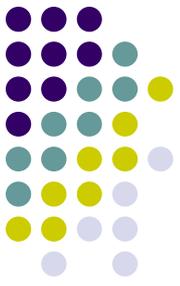
Pressure and Momentum

- Pressure, P , is defined as the force per unit area

$$P = \frac{F}{A} = \frac{1}{A} \frac{dp}{dt} = \frac{1}{c} \frac{(dT_{ER}/dt)}{A}$$

- But the magnitude of the Poynting vector is $(dT_{ER}/dt)/A$ and so $P = S/c$
 - For a perfectly absorbing surface

Pressure and Momentum, cont.



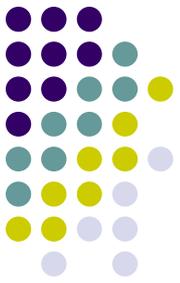
- For a perfectly reflecting surface,
 $\rho = 2T_{ER}/c$ and $P = 2S/c$
- For a surface with a reflectivity somewhere between a perfect reflector and a perfect absorber, the pressure delivered to the surface will be somewhere in between S/c and $2S/c$
- For direct sunlight, the radiation pressure is about $5 \times 10^{-6} \text{ N/m}^2$

Production of em Waves by an Antenna

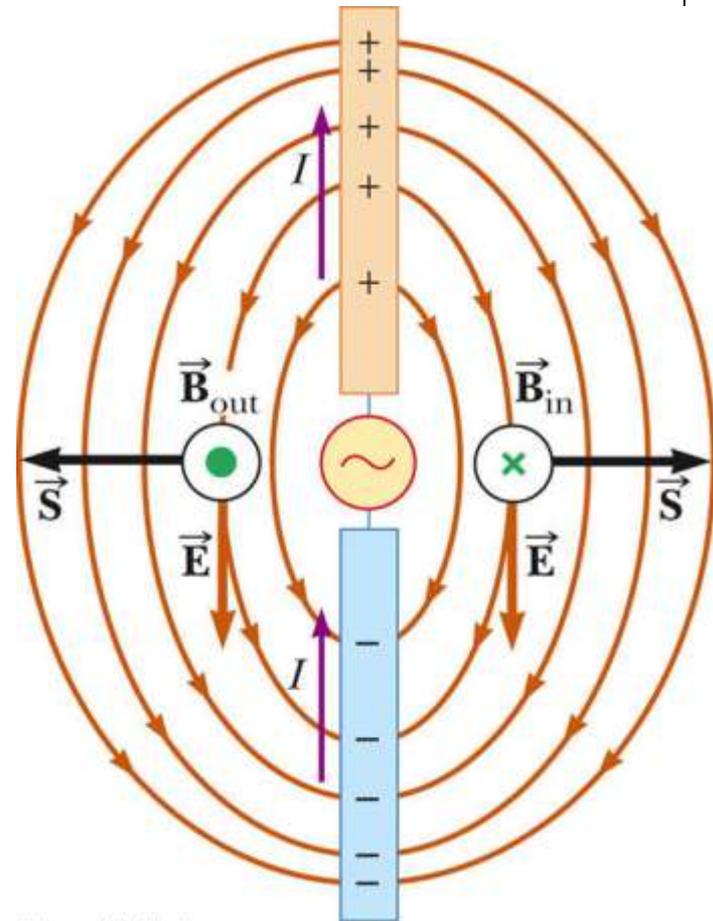


- Neither stationary charges nor steady currents can produce electromagnetic waves
- The fundamental mechanism responsible for this radiation is the acceleration of a charged particle
- Whenever a charged particle accelerates, it radiates energy

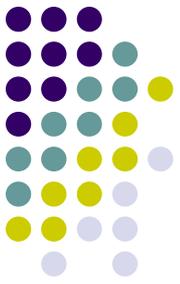
Production of em Waves by an Antenna, 2



- This is a *half-wave* antenna
- Two conducting rods are connected to a source of alternating voltage
- The length of each rod is one-quarter of the wavelength of the radiation to be emitted

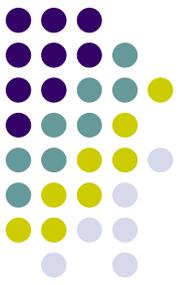


Production of em Waves by an Antenna, 3



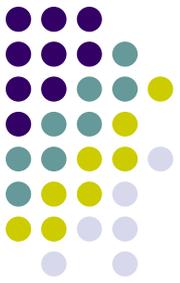
- The oscillator forces the charges to accelerate between the two rods
- The antenna can be approximated by an oscillating electric dipole
- The magnetic field lines form concentric circles around the antenna and are perpendicular to the electric field lines at all points
- The electric and magnetic fields are 90° out of phase at all times
- This dipole energy dies out quickly as you move away from the antenna

Production of em Waves by an Antenna, final

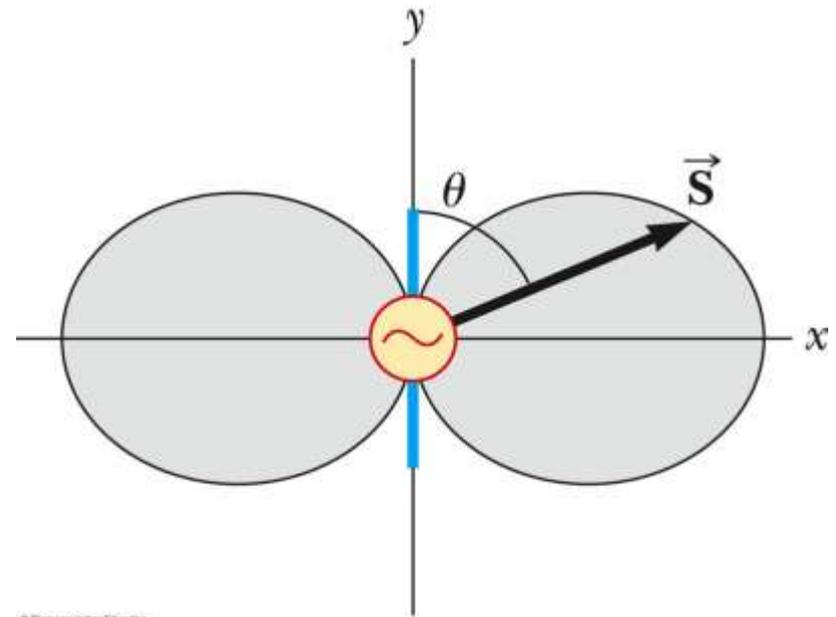


- The source of the radiation found far from the antenna is the continuous induction of an electric field by the time-varying magnetic field and the induction of a magnetic field by a time-varying electric field
- The electric and magnetic field produced in this manner are in phase with each other and vary as $1/r$
- The result is the outward flow of energy at all times

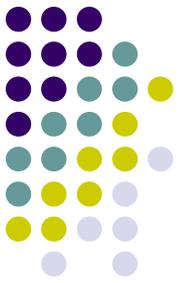
Angular Dependence of Intensity



- This shows the angular dependence of the radiation intensity produced by a dipole antenna
- The intensity and power radiated are a maximum in a plane that is perpendicular to the antenna and passing through its midpoint
- The intensity varies as $(\sin^2 \theta) / r^2$



The Spectrum of EM Waves

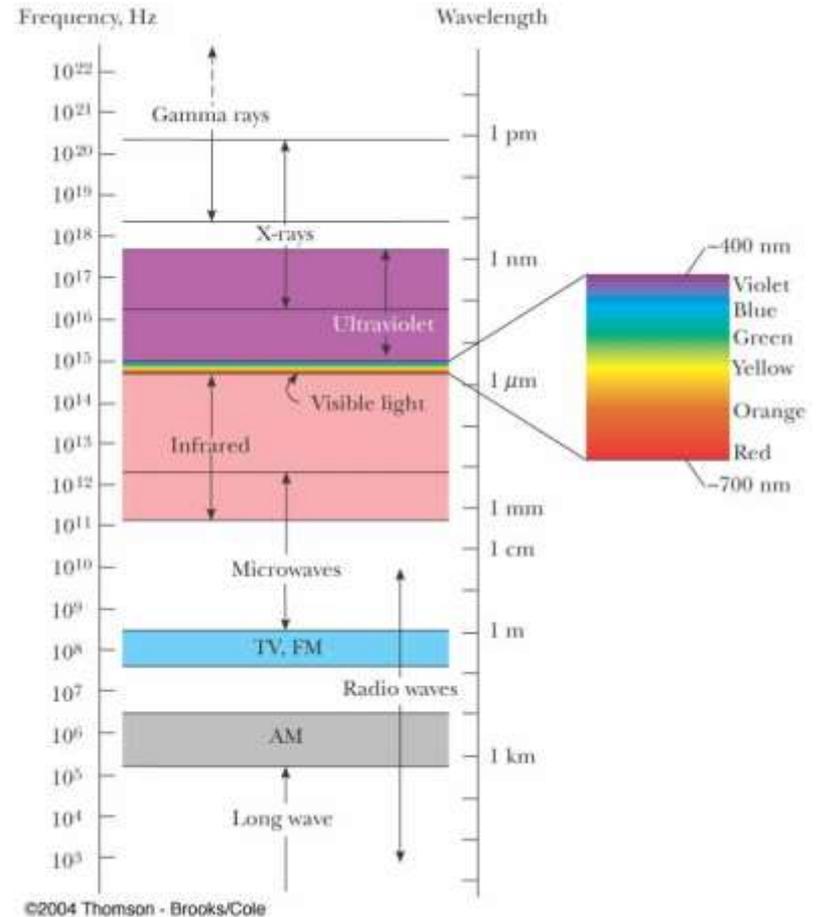


- Various types of electromagnetic waves make up the em spectrum
- There is no sharp division between one kind of em wave and the next
- All forms of the various types of radiation are produced by the same phenomenon – accelerating charges

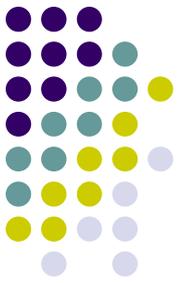


The EM Spectrum

- Note the overlap between types of waves
- Visible light is a small portion of the spectrum
- Types are distinguished by frequency or wavelength



Notes on the EM Spectrum



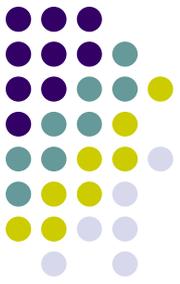
- Radio Waves

- Wavelengths of more than 10^4 m to about 0.1 m
- Used in radio and television communication systems

- Microwaves

- Wavelengths from about 0.3 m to 10^{-4} m
- Well suited for radar systems
- Microwave ovens are an application

Notes on the EM Spectrum, 2

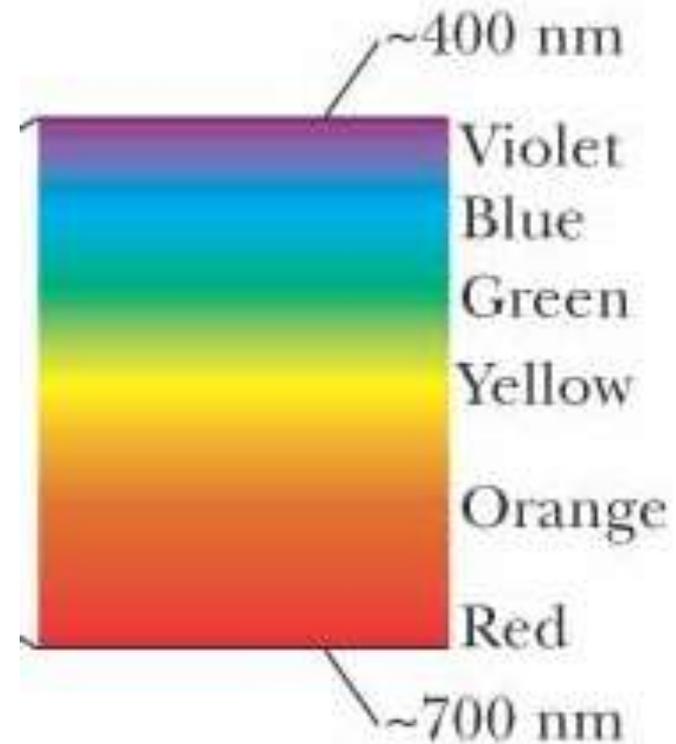


- Infrared waves
 - Wavelengths of about 10^{-3} m to 7×10^{-7} m
 - Incorrectly called “heat waves”
 - Produced by hot objects and molecules
 - Readily absorbed by most materials
- Visible light
 - Part of the spectrum detected by the human eye
 - Most sensitive at about 5.5×10^{-7} m (yellow-green)



More About Visible Light

- Different wavelengths correspond to different colors
- The range is from red ($\lambda \sim 7 \times 10^{-7} \text{ m}$) to violet ($\lambda \sim 4 \times 10^{-7} \text{ m}$)



Visible Light, cont

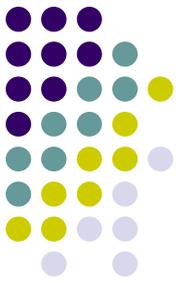


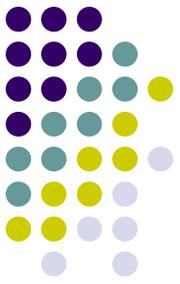
TABLE 34.1

Approximate Correspondence Between Wavelengths of Visible Light and Color

Wavelength Range (nm)	Color Description
400–430	Violet
430–485	Blue
485–560	Green
560–590	Yellow
590–625	Orange
625–700	Red

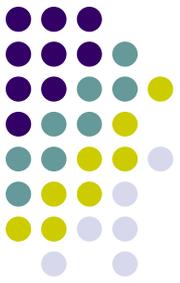
Note: The wavelength ranges here are approximate. Different people will describe colors differently.

Notes on the EM Spectrum, 3



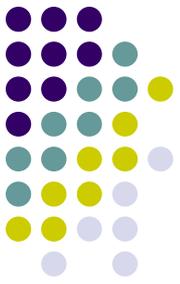
- Ultraviolet light
 - Covers about 4×10^{-7} m to 6×10^{-10} m
 - Sun is an important source of uv light
 - Most uv light from the sun is absorbed in the stratosphere by ozone
- X-rays
 - Wavelengths of about 10^{-8} m to 10^{-12} m
 - Most common source is acceleration of high-energy electrons striking a metal target
 - Used as a diagnostic tool in medicine

Notes on the EM Spectrum, final

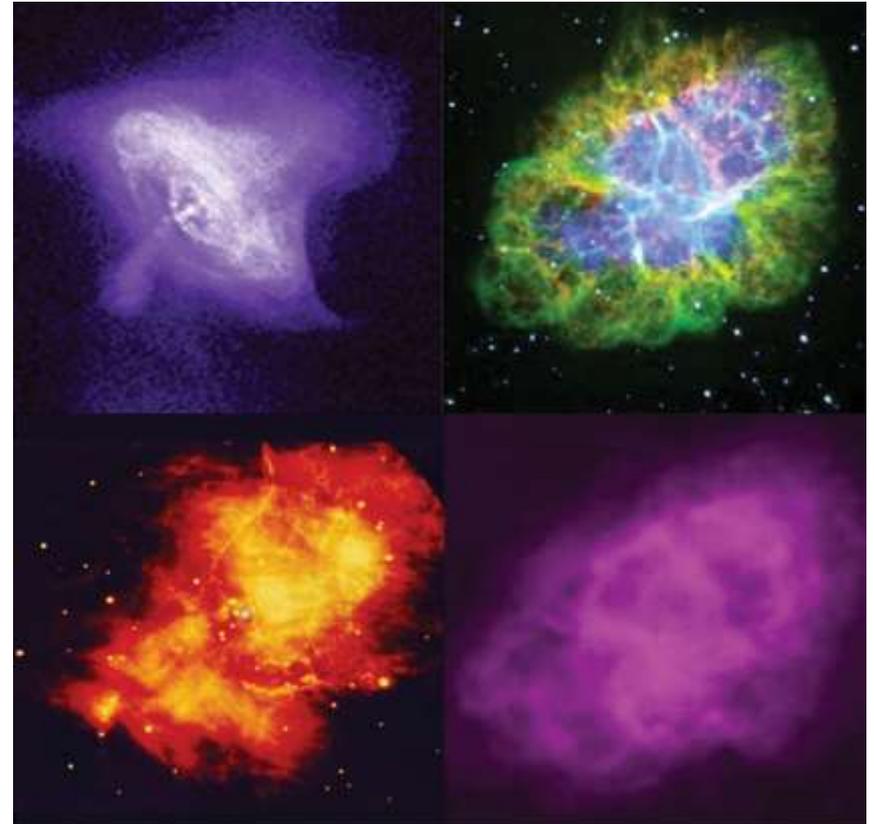


- Gamma rays
 - Wavelengths of about 10^{-10} m to 10^{-14} m
 - Emitted by radioactive nuclei
 - Highly penetrating and cause serious damage when absorbed by living tissue
- Looking at objects in different portions of the spectrum can produce different information

Wavelengths and Information



- These are images of the Crab Nebula
- They are (clockwise from upper left) taken with
 - x-rays
 - visible light
 - radio waves
 - infrared waves



© 2004 Thomson - Brooks/Cole