Abstract

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Solution of the Rayleigh problem for a power law non-Newtonian conducting fluid via group method

An investigation is made of the magnetic Rayleigh problem where a semi-infinite plate is given an impulsive motion and thereafter moves with constant velocity in a non-Newtonian power law fluid of infinite extent. We will study the non-stationary flow of an electrically conducting non-Newtonian fluid of infinite extent in a transverse external magnetic field. The rheological model of this fluid is given by the well-known expression for a power law fluid [Ing. Arch. 41 (1972) 381] where \( \tau \) is the shear stress, \( p \) is the pressure, \( \delta \) is the Kronecker symbol, \( k \) the coefficient of consistency, \( I_2 \) the second strain rate invariant, \( e_{ij} \) the strain rate tensor and \( n \) is a parameter characteristic of the non-Newtonian behavior of the fluid. For \( n=1 \), the behavior of the fluid is Newtonian, for \( n>1 \), the behavior is dilatant and for \( 0<n<1 \), the behavior is pseudo-plastic. The equation of motion of the semi-infinite flat plate in the infinite power law non-Newtonian fluid after an impulsive end loading and maintaining constant velocity thereafter is

\[
\frac{1}{1-n} \frac{\partial}{\partial t} \left( \frac{\partial u}{\partial y} \right) + V \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial p}{\partial n} - \frac{k}{\eta} \frac{\partial^2 u}{\partial y^2} + \frac{\mu}{\mu_0} H \frac{\partial^2 u}{\partial y^2} + \frac{\mu}{\mu_0} H \frac{\partial u}{\partial t}
\]

where \( u(y,t) \) is the velocity of the fluid flow in the horizontal direction, \( V \) is the steady state velocity of the plate, \( t \) is the time, \( y \) is the coordinate normal to the plate, \( n \) is constant, \( \mu_0 \) is constant, \( k \) is the coefficient of consistency, \( \eta \) is the density of the fluid, \( M = \frac{\mu}{\mu_0} \) is constant, \( \mu \) is the magnetic conductivity, \( \mu_0 \) is the magnetic permeability and \( H \) is the magnetic field strength and is function of time \( H(t) \). The solution of this highly non-linear problem is obtained by means of the transformation group theoretic approach. The one-parameter group transformation reduces the number of independent variables by one and the governing partial differential equation with the boundary conditions reduce to an ordinary differential equation with the appropriate boundary conditions. Effect of the parameters \( M \), \( w \) \( (=V_n^{-1}) \), \( n \) and time \( t \) on the velocity \( u(y,t) \) has been studied and the results are plotted.