# Abstract

## Hossam E. Shawky

### Lie-Group Method for Unsteady Flows in a Semi-infinite Expanding Contracting Pipe with Injection Suction through a Porous Wall

The unsteady incompressible laminar flow in a semi-infinite porous circular pipe with injection suction through the pipe wall whose radius varies with time is considered. The present analysis simulates the flow field by the burning of inner surface of cylindrical grain in a solid rocket motor, in which the burning surface regresses with time. We apply Lie-group method for determining symmetry reductions of partial differential equations. Lie-group method starts out with a general infinitesimal group of transformations under which given partial differential equations are invariant, then, the determining equations are derived [Ibragimov, Elementary Lie Group Analysis and Ordinary Differential Equations, Wiley, New York, 1999 Hydon, Symmetry Methods for Differential Equations, Cambridge University Press, Cambridge, 2000 Olver, Applications of Lie Groups to Differential Equations, Springer, New York, 1986 Seshadri, Na, Group invariance in engineering boundary value problems, Springer, New York, 1985 Yi, Fengxiang, Lie symmetries of mechanical systems with unilateral holonomic constraints, Chinese Sci. Bull. 45 (2000) 1354–1358 Moritz, Schwalm, Uherka, Finding Lie groups that reduce the order of discrete dynamical systems, J. Phys. A: Math. 31 (1998) 7379–7402 Nucci, Clarkson, The nonclassical method is more general than the direct method for symmetry reductions. An example of the Fitzhugh–Nagumo equation, Phys. Lett. A 164 (1992) 49–56 Basarab, Lahno, Group classification of nonlinear partial differential equations: a new approach to resolving the problem, Proceedings of Institute of Mathematics of NAS of Ukraine, vol. 43, 2002, pp. 86–92 Burde, Expanded Lie group transformations and similarity reductions of differential equations, Proceedings of Institute of Mathematics of NAS of Ukraine, vol. 43, 2002, pp. 93–101 Gandarias, Bruzon, Classical and nonclassical symmetries of a generalized Boussinesq equation, J. Nonlinear Math. Phys. 5 (1998) 8–12 Hill, Solution of Differential Equations by Means of One-Parameter Groups, Pitman Publishing Co., 1982]. The determining equations are a set of linear differential equations, the solution of which gives the transformation function the infinitesimals of the dependent and independent variables. After the group has been determined, a solution to the given partial differential equation may be found from the invariant surface condition such that its solution leads to similarity variables that reduce the number of independent variables in the system. Effect of the cross-flow Reynolds number Re and the dimensionless wall expansion ratio ? on velocity, flow streamlines, axial and radial pressure , and wall shear stress has been studied both analytically and numerically and the results are plotted.