A Numerical Model to Study the Effect of Alumina Tri-Hydrate on Mechanical Properties of Fiber Reinforced Polymers

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ABSTRACT

Fiber-reinforced polymer (FRP) is made of high-strength fibers embedded in resins, which can provide high-strength and durability. In the last two decades, general attention has been significantly increased towards the use of FRP in various structural types in civil engineering. However, the low fire endurance remains a major obstacle. A literature review shows that a fair amount of research has used insulation as an effective way to achieve the required fire performance, which is costly. A more cost-effective way to enhance the fire resistance of FRP is to add fire-retardant filler, such as alumina tri-hydrate (ATH), to the resin. Extensive research has been conducted in this area, including the author’s research group, mainly focused on experimental investigations. The objective of this paper is to develop a numerical model to predict mechanical properties of FRP with different ATH ratios based on a composite micromechanical model. The accuracy of this model is validated against existing experimental data. Discussions and recommendations for further research are provided.

INTRODUCTION

Fiber-reinforced Polymers (FRP) made from fibers, such as carbon, aramid and glass and epoxy are being used in engineering applications because of their enhanced strength and relatively low cost compared to steel. Glass FRP (GFRP) is widely used in the construction area due to its low cost compared to carbon FRP (CFRP). However, it is not fire resistant and emits toxic gases when it burns. Therefore, fire endurance should be provided in order to meet International Building Code (IBC) requirements.

One way to meet the requirements is to provide insulation for structural elements. Among others, Bisby et al. (2005) tested fire endurance of FRP-confined circular columns. They concluded that FRP-wrapped concrete columns could achieve over 5 hours’ fire endurance by using two insulation layers. López et al. (2013) conducted experimental and numerical studies on the fire effect of reinforced concrete (RC) slabs strengthened with externally bonded CFRP. Fire protection was provided at the bottom of the slab using calcium silicate (CS) boards or vermiculite/perlite (VP) cement based mortar. They concluded that 20 mm thick CS insulation could provide 90 minutes protection in normal zones of the slabs. Similar research on beams was done by Kodur et al. (2007) and other researchers including Firmo et al. (2014); Kodur and Ahmed (2013); Palmieri et al. (2013); Sayin (2014); Yu and Kodur (2014); etc.

Although the aforementioned studies can effectively increase the fire resistance, the insulation is costly. A more cost-effective way to meet the requirements of the fire endurance is to modify properties of the FRP. For example, fillers can be added to FRP to enhance its specific properties such as fire resistance or strength. Among others, alumina tri-hydrate (ATH) was found to be an effective filler (Petersen et al. 2015). Although the effect of FRP-filled ATH on
mechanical properties of FRP was addressed experimentally, limited work was devoted to analytical modeling, which is the objective of this study.

Numerous research has been done to predict the stiffness and strength of FRP. However, only limited research is available to predict the stiffness and strength of FRP with fillers. Chang and Weng (1979) developed an analytical model to predict the stiffness of a sheet molding compound (SMC) filled with calcium carbonate. SMC is considered as a form of FRP where the chopped-fibers are randomly dispersed in a filled or unfilled resin. The model was divided into two sections: matrix consisted of the resin and filler, and the composite consisted of the matrix and fiber. The analytical model was then validated against experimental results.

However, until now, no research was done to predict the strength of conventional FRP with filler. The idea of dividing the analytical model into two sections is very promising, since it follows the same procedure of manufacturing the conventional FRP. Therefore, this paper adopts the same idea to develop an analytical model that can predict stiffness and strength for FRP with ATH filler. The analytical model is then compared to experimental results, and conclusions and further recommendations are provided.

**STIFFNESS: MATRIX (RESIN/FILLER)**

Many theories has been developed to predict the stiffness and strength of the heterogeneous material with inclusion. In terms of stiffness, Paul (1959) predicted the Young’s modulus of a filled system. Hashin (1962) developed a model which was considered as a keystone to predict bulk and shear moduli of heterogenous materials. Nielsen (1970) modified Halpin-Tsai (Halpin, 1969) equation for better prediction of shear moduli. Chang and Weng (1979) expanded Paul and Hashin’s expressions to evaluate elastic, shear and shear moduli of a filled matrix. Although these papers discussed the stiffness of a filled matrix, little research studied the stiffness of matrix filled with ATH.

Wainwright (1991) conducted tensile and fracture tests on epoxy filled with ATH to predict both stiffness and strength. Experimental results were used to evaluate the aforementioned analytical models. It was concluded that Nielsen model (Nielsen 1970) could successfully predict the stiffness of an ATH filled matrix. Nie and Basaran (2005) formulated equations for Young’s modulus and Possion’s ratio that could consider imperfect interfacial bonds of composites. The model was validated against experimental data of ATH reinforced poly methyl methacrylate (PMMA) composite. However, the model was considered more complicated compared to Nielsen model. Stapountzi et al. (2009) conducted an experimental program to predict the stiffness of the ATH filled PMMA for different filler volume fractions and temperatures. Previous analytical models were compared with the experimental results. Only four models were found in agreement with test results, including Halpin-Tsai model (H-T) (Halpin, 1969), Generalized self consistent model(GSC) (Kerner, 1956; Hermans 1967), Nielsen model (Lewis and Nielsen 1970), and Lieliens model (Lielens et al. 1998). Lieliens model was concluded to provide the most accurate prediction of the stiffness in terms of different volumetric fractions.

In this paper, Nielsen model is adopted to predict the stiffness of the matrix filled with ATH, since it has been validated by Wainwright (1991).

**STIFFNESS: COMPOSITE (MATRIX/FIBER)**

Numerous models were developed to predict the stiffness of the composites with randomly oriented fibers. Tsai and Pagano (1968) were the first to develop approximate equations to predict elastic and shear moduli. More complicated models were later developed by others,
including Manera (1977), Christensen and Waals (1972), Chang and Weng (1979) and Weng and Sun (1979) to get more enhanced results. However, evaluation using these complicated models requires fracture properties which are not always available for the FRP filled with ATH. Therefore, Tsai and Pagano (1968) expressions are used in this study to predict the stiffness of the FRP composite.

**STRENGTH: MATRIX (RESIN/FILLER)**

The ultimate strength of the matrix depends on the weakest path. There are several factors that affect the strength of the matrix. The most important factor is stress concentration, other factors include material elastic properties, size of flaws, reinforcing effect which can act as barrier to crack growth (Fu et al. 2008; Wainwright 1991). Although there are numerous models that discuss the strength of the matrix with fillers, there is no unified strength model. Most strength models provide more conservative values due to the negligible microscopic flaws, which are usually formed due to air voids, dust or filler, etc.

Danusso and Tieghi (1986) assumed that the strength of the matrix depended on the strength of the resin, which essentially neglected the effect of the filler. Different models were developed to achieve more accurate results including Jancar et al. (1992); Nicolais and Narkis (1971); Nicolais and Nicodemo (2006); Nielsen (1966). More details can be found from Fu et al. (2008).

Wainwright (1991) verified strength expressions developed by Nicolais and Mashelkar (1976), Nicolais and Nicodemo (1973), and Schrager (1978) with experimental results of the epoxy filled with different ATH volume fractions and other experimental data from the literature. However, the model can only be used to provide an approximate value of the strength and over-predicted the strength.

Fu et al. (2008) reviewed different strength models for different types of the matrix. They concluded that the strength model depended on several factors such as particle size, interface adhesion, and particle loading. It was also concluded that there was no single equation that could apply to all types of the matrix. However, these models could predict the trend of the strength against other parameters such as filler volume fraction.

In this paper, Nicolais and Mashelkar (1976) model is used to predict the strength of the epoxy filled with ATH, since this model has been validated by Wainwright (1991) for the same type of filler (ATH).

**STRENGTH: COMPOSITE (MATRIX/FIBER)**

Much research in the last era was focused on developing the strength of the composite. Cox (1952) calculated the stiffness and strength of the fiber mat based on the strength of the fiber only and neglected the matrix contribution. Nielsen and Chen (1968) and Chon and Sun (1980) developed new models for off-axis short fiber composites.

Baxter (1992) developed a strength model based on averaging Tsai-Hill expression (Tsai 1968) for off-axis strength. Lees (1968) calculated the strength based on the maximum stress criterion by averaging the strength over all angles. Chen (1971) followed Lees’ procedure and included a strength efficiency factor to take into account discontinues fibers.

Hahn (1975) developed an empirical formula to calculate the strength of randomly oriented fibers based on unidirectional composite properties. However, the formula required fracture properties to calculate the in-plane shear and transverse tensile strength, which are not always available.

In conclusion, there are two general approaches to calculate the strength of randomly
oriented FRP composites. The first approach, as pointed out by Barbero (2011), uses the strength of fibers and matrix. However, fracture properties such as strain energy rate \( (G_I \text{ and } G_{II}) \) are needed to calculate the strength. The second approach was discussed by Lees (1968), which is simpler than the first approach and does not require fracture properties. In this paper, Lees (1968) expressions is used to predict the strength of randomly oriented discontinues FRP filled with ATH.

**ANALYTICAL MODEL**

FRP filled with ATH consists of fibers, resin, and ATH filler. Although this paper mainly uses ATH filler, the analytical model can be generalized to include other types of fillers. Assuming the volumetric fraction of the fiber \( (v_f) \) is a constant and the matrix consists of resin only, the weight of the matrix can be determined as

\[
W_r = \frac{W_f - \rho_f W_f}{\rho_f} = \frac{W_f}{\rho_r} - \frac{W_f}{\rho_f}
\]

where \( W \) and \( \rho \) are weight and density, respectively; and \( r \) and \( f \) donate resin and fiber, respectively.

The weight of the matrix can be then divided into the weight of the resin and filler weights. For example, if 75% of the weight of the matrix is resin, the other 25% is ATH filler. It can be described as part per hundred (pphr), where the resin will be 75 pphr and the ATH will be 25 pphr.

Given the density of the resin and filler, the volumetric fraction of the filler \( (V_{ATH}) \) can be expressed as

\[
V_{ATH} = \frac{W_{ATH}}{\frac{1}{100*\rho_{ATH}} + \frac{W_{ATH}}{\rho_r + 100*\rho_{ATH}}}
\]

where \( W_{ATH} \) is the percentage of the filler per weight given in pphr (part per hundred), and subscript ATH donates ATH filler.

**ANALYTICAL MODEL: STIFFNESS**

Using Nielsen (1970) expression to predict the stiffness of the matrix, we get

\[
E_m = E_r \left( \frac{1 + A B V_{ATH}}{1 - B \psi V_{ATH}} \right)
\]

where, \( A = k_E - 1; B = \frac{E_r}{E_{ATH} + A}; \psi = 1 + \left( \frac{1 - \phi_m}{\phi_m^2} \right) \); \( k_E \) is known as Einstein’s coefficient, which is \( k_E = 2.5 \) when the Poisson’s ratio of the matrix equals to 0.5 (Einstein 1905), and \( E \) is Young’s modulus.

The stiffness of randomly discontinuous FRP can be calculated by averaging the stiffnesses of unidirectional composites. Unidirectional fiber properties can be calculated using the rule of mixtures as follows:
where \( E_{11} \) is the stiffness of the composite in the direction of the fibers and \( V_f \) is the volumetric fraction of the fiber.

The stiffness of the composite in the transverse direction of the fibers (\( E_{22} \)) can be obtained using Halpin-Tsai expression (Halpin and Tsai 1969) as follows:

\[
E_{22} = E_m \left( \frac{1 + \zeta \cdot \eta \cdot V_f}{1 + \eta \cdot V_f} \right) 
\]

\[
\eta = \frac{(E_f / E_m) - 1}{(E_f / E_m) - \zeta} 
\]

Barbero (2011) pointed out that the empirical parameter \( \zeta \) can be assumed to be 2 for both square and circular fibers.

Using the rule of the mixtures approach, in-plane and out-of-plane Poisson’s ratio can be calculated as:

\[
u_{12} = V_f \nu_f + (1-V_f) \nu_m
\]

\[
u_{21} = \frac{E_{22}}{E_{11}} \nu_{12}
\]

where \( \nu \) denotes Poisson’s ratio. Tsai and Pagano (1968) developed simplified expressions to calculate the elastic modulus and shear modulus of randomly oriented FRP as:

\[
E = \frac{3}{8} E_{11} + \frac{5}{8} E_{22}
\]

\[
G = \frac{1}{8} E_{11} + \frac{1}{4} E_{22}
\]

Since the randomly oriented FRP can be treated as isotropic material, Poisson’s ratio can be calculated as:

\[
\nu = \frac{E}{2G} - 1
\]

**ANALYTICAL MODEL: STRENGTH**

According to Nicolais and Nicodemo (1973), the strength of the matrix (\( S_m \)) is based on both the resin’s strength (\( S_r \)) and filler’s volume fraction (\( V_{ATH} \)), which is:

\[
S_m = S_r (1 - f \cdot V_{ATH}^{2/3})
\]

where \( f = 1.1 \) for dense hexagonal packing of filler particles.

Lees (1968) proposed that the strength of the composite (\( S \)) is:

\[
S = \frac{2S_{LT}}{\pi} (1 + \frac{S_T}{S_{mf1}} + \ln \frac{S_T S_{mf1}}{S_{LT}^2})
\]

where, \( S_{LT} \) is the in-plane shear strength, \( S_T \) is the transverse strength, and \( S_{mf1} \) is the matrix stress corresponding to fiber failure strain.

Gibson (2011) pointed out that the transverse strength (\( S_T \)) can be expressed as:

\[
S_T = \frac{E_{22} \cdot S_m}{E_m \cdot F}
\]
\[ F = \frac{1}{d \left( \frac{E_m}{E_f} - 1 \right) + 1} \]  

(14)

where \( d \) is the fiber diameter, and \( s \) is the fiber spacing which can be calculated as:

\[ s = \frac{\sqrt{\pi d^2}}{4 \gamma_f} \]  

(15)

Using the same approach, the in-plane shear strength (\( S_{LT} \)) can be calculated by replacing the tensile stiffness in Equations (13) and (14) with corresponding shear stiffness as follows:

\[ S_{LT} = \frac{G \cdot S_m}{G_m \cdot F_s} \]  

(16)

\[ F_s = \frac{1}{d \left( \frac{G_m}{G_f} - 1 \right) + 1} \]  

(17)

where \( G \) represents the shear stiffness. Assuming that randomly oriented discontinues FRP can be treated as an isotropic material, the shear stiffness (\( G \)) is:

\[ G = \frac{E}{2(1 - \nu)} \]  

(18)

The matrix longitudinal stress (\( S_{mf1} \)) can be expressed as:

\[ S_{mf1} = E_m \cdot e_f \]  

(19)

where \( e_f \) is the maximum strain of the fiber, which is:

\[ e_f = \frac{S_f}{E_f} \]  

(20)

where \( S_f \) is the strength of the fibers.

5. VERIFICATION

For verification purpose, the derived model is verified with previous experimental results. Although this model can be applied to FRP filled with any type of fillers, ATH filler is chosen for verification purpose. As discussed before, this model is divided into two sections: matrix and composite sections. The matrix section is based on Equations (3) and (11), which predict the stiffness and strength of the matrix composed of epoxy and ATH filler. These equations were from Wainwright (1991) and verified against experimental results.

The composite section is compared with experimental results from Petersen et al. (2015), where mechanical tests were performed to predict the strength and stiffness of FRP filled with 0%, 25% and 50% ATH filler by weight. The tension samples were composed of four layers of GFRP filled with ATH. Each layer consisted of 102 g/m² (3 oz./yd²) of chopped strand fibers and certain amount of resin (epoxy) mixed with ATH. The density of the fiber, resin and ATH were 2600 kg/m³, 1100 kg/m³, and 561 kg/m³, respectively. Using a volumetric fraction of the fiber (\( v_f \)) of 0.1877 and density of the materials, the required weight of both resin and ATH could be calculated using Equations (1) and (2). Table 1 shows the material properties of the fibers, resin and ATH. Fiber and resin data are adopted from Petersen et al. (2015), and verified with Chen and Davalos (2010). Since Petersen et al. (2015) did not list the material properties of the ATH, the properties of the ATH were mainly adopted from Wainwright (1991) and verified with other...
literature such as Nie and Basaran (2005) and Stapountzi et al. (2009).

Table 1: Material properties of the resin, ATH and fibers

<table>
<thead>
<tr>
<th></th>
<th>Tensile Strength (MPa)</th>
<th>Tensile Modulus (GPa)</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resin</td>
<td>50</td>
<td>3.7</td>
<td>0.38</td>
</tr>
<tr>
<td>ATH</td>
<td>--</td>
<td>70</td>
<td>0.24</td>
</tr>
<tr>
<td>E-Glass Fiber</td>
<td>2400</td>
<td>73</td>
<td>0.22</td>
</tr>
</tbody>
</table>

The derived stiffness model is compared with Peterson’s results (2015). According to Figure 1, the analytical model shows that the stiffness increases with the increase of the volumetric fraction of the filler ($V_{ATH}$), although Peterson’s experimental results show that the stiffness is almost constant for different $V_{ATH}$. Stapountzi (2008) and Wainwright (1991) have proven that the matrix stiffness increases with $V_{ATH}$, which raise a concern about whether the stiffness can be used as a valid parameter to predict the material behavior.

![Figure 1 – Comparison between analytical model and experimental data for stiffness vs. ATH volume fraction](image)

The tensile strengths of the resin and fiber are given in Table 1, where the strength of the matrix is calculated based on Nicolais and Nicodemo (1973) equation, as shown in Equation (11). Figure 2 shows that the matrix strength decreases with the increase of $V_{ATH}$.

The strength of the composite is calculated based on Equation (12), where the fiber diameter is assumed to be $9.144 \times 10^{-3}$ mm (Barbero 2011). Figure 3 shows the strength of the analytical model compared with the experimental results for different filler volume fractions. Composite strength in both model is decreasing with the increase of the filler content. However, the strength of the experimental results at 50% ATH ($V_{ATH}=0.313$) are 48% lower than the strength at 0% ATH while strength model only decreases around 18% from 0% ATH to 50% ATH.
DISCUSSIONS

The difference between the analytical model and the experimental results is due to several factors. The first factor is that simple models such as Nicolais and Nicodemo (1973) are used to predict the strength. Other complicated models can be used if fracture properties are available from experimental data. The second factor is the uncertainty of the prediction model against the experimental results. In order to remove any uncertainty about the effect of different types of epoxy and filler, mechanical properties of the matrix need to be tested before adding fiber.
CONCLUSIONS

This study develops an analytical model to predict the stiffness and strength of FRP filled with filler, specifically, ATH filler. From this study, the following conclusions can be drawn:

1. The stiffness model can predict existing test results for matrix. However, the analytical model does not correlate well with the only available experimental results for composite (Petersen et al. 2015).
2. The strength model can predict the same decreasing trend as that of the experimental. However, the strength from experimental results almost double that predicted by the analytical model.
3. Further study is needed to include fracture properties for both matrix and composite for better prediction of the strength model.

REFERENCES


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