Effective Width of Insulated Sandwich Panels with Flexible FRP Shear Connectors Considering Partial Degree of Composite Action

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ABSTRACT

Insulated sandwich panels consist of two layers of wythe separated by a foam insulation. Many different types of connectors have been used in the past to connect the two wythes, including steel ties, wire trusses, bent wires, truss-shaped connectors, and solid concrete zones. In recent year, fiber-reinforced polymer (FRP) materials, in both truss and mesh configurations, have begun to be incorporated as shear connectors, as they have a thermal conductivity about 14% that of steel, and can significantly reduce thermal bridging. Until now, there is no effective guidelines for the design of these panels. Generally, they are treated as rectangular beams, which is not reasonable since the longitudinal stress over the wythe section is non-uniform due to the in-plane shear flexibility of the wythe, which is called shear lag effect.

Effective flange width has been used to describe the shear lag effect for a deck-on-girder composite beam system, reducing a three dimensional behavior of the composite beam system to the analysis of a T-beam section with a reduced width of deck. This paper extends the concept of effective flange width to insulated concrete sandwich panels. A shear lag model is firstly developed to study the sandwich panel system, where partial Degree of Composite Action (DCA) due to flexible FRP shear connector is considered. The analytical model is then verified through close correlations between finite element and analytical results for a concrete sandwich panel with FRP shear connectors. Next, a parametric study is conducted using the analytical model to study the effects of deck stiffness and aspect ratio on effective flange width. Finally, a simplified method is proposed to calculate effective width.

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INTRODUCTION

Insulated sandwich panels are widely used in buildings as building envelopes. As shown in Figure 1, they are typically composed of two wythes connected by shear connectors, which transfer shear force in between the two wythes to ensure that the wythes can work compositely. Various types of shear connectors have been used, including steel ties, wire trusses, bent wires, truss-shaped connectors, and solid zones [1]–[4]. Although these connectors can establish effective connections, they cause thermal bridging between the wythes, which impairs the advantage of the sandwich panel as an insulating structural element. In recent years, FRP materials, in both truss and mesh configurations, have begun to be incorporated as shear connectors, as they have a thermal conductivity about 14% that of steel and can significantly reduce thermal bridging.

Until now, there is no effective guidelines for the design of these panels. Generally, they are treated as rectangular beams, which is not reasonable since the longitudinal stress over the wythe section is non-uniform due to the in-plane shear flexibility of the wythe, which is called shear lag effect, as shown in Figure 2.

Figure 1 – Sandwich Panels with FRP connectors
(a) Tested Panel with FRP sheargrid (Frankl et al. 2011)
(b) Cross section of test panel (Hopkins et al. 2014)

Figure 2 – Effective width

Figure 3 - FE model
Reissner introduced the concept of the shear-lag effect [5]. Since then, extensive studies have been carried out on this topic mainly for deck-on-girder beam system, including Reissner [6]; Newmark et al. [7]; Heins and Fan [8]; Cheung and Chan [9]; Kemmochi et al. [10]; etc. In all these studies, effective flange width was used to describe the shear lag effect, reducing a three dimensional behavior of the composite beam system to the analysis of a T-beam section with a reduced width of deck. This paper extends the concept of effective flange width to insulated concrete sandwich panels. As shown in Figure 2, a uniform stress can be assumed along the width $b_{eff}$. The area of this stress block is equivalent to the area of the actual stress distributed along width $b$. This reduced width $b_{eff}$ is called effective width.

Headed steel studs are typically used to connect the deck and girder for a deck-on-girder composite beam system. These connections are rigid and the slip between the deck and girder is minimum. This slip is usually neglected in the calculation of effective flange width. Unlike rigid headed steel studs, FRP connectors are flexible. Therefore, the slip between the two wythes cannot be neglected for sandwich panels with FRP shear connectors. Degree of Composite Action (DCA) can be used to interpret this slip.

DCA depends on the shear force transferred through shear connectors between the two wythes. As shown in Figure 4, Lorenz and Stockwell [11] pointed out that 100% and zero shear forces are transferred for full-composite and non-composite cases, respectively. Partial DCA can be obtained at limited amount of slip due to the inadequacy of the shear connectors to maintain strain compatibility.

Several studies have studied DCA for sandwich panels. Benayoune et al. [12] calculated the DCA for sandwich panel based on the stress distribution as follows:

$$ DCA = \frac{I_e}{I_g}, \quad I_e = \frac{M \times h}{\sigma_b - \sigma_t} $$

(1)
where $\sigma_b$, $\sigma_t$ are the stresses at the bottom and top face of the panel, respectively, $M$ is the applied bending moment, $h$ is the depth of the panel, and $I_g$ is the moment inertia of the sandwich panel.

Frankl et al. [1] conducted experimental study and proposed a displacement method to calculate DCA as:

$$DCA = \frac{\Delta_{\text{noncomposite}} - \Delta_{\text{experimental}}}{\Delta_{\text{noncomposite}} - \Delta_{\text{composite}}} (100\%)$$  \hspace{1cm} (2)

where, $\Delta_{\text{experimental}}$, $\Delta_{\text{composite}}$ and $\Delta_{\text{noncomposite}}$ are displacement measured at selected load level, displacements corresponding to full- and non-composite, respectively. Hopkins et al. [13] adopted the displacement method and concluded that DCA has a significant effect on both stiffness and flexural strength of the panel.

Chen et al. [14] performed experimental tests on sandwich panels with different shear connectors. DCA of the test panels was calculated based on:

$$DCA = \frac{\left(\frac{1}{EI}\right)_{0\%} - \left(\frac{1}{EI}\right)_{\text{Actual}}}{\left(\frac{1}{EI}\right)_{0\%} - \left(\frac{1}{EI}\right)_{100\%}} (100\%)$$  \hspace{1cm} (3)

where $\left(\frac{1}{EI}\right)_{0\%}$, $\left(\frac{1}{EI}\right)_{\text{Actual}}$ and $\left(\frac{1}{EI}\right)_{100\%}$ represent the values for panels with non-composite; partial DCA; and full-composite, respectively.

However, there is still no study available on effective width of sandwich panel considering partial DCA, which is the objective of this study. To this end, a shear lag model is firstly developed, where partial DCA due to flexible FRP shear connector is considered. The analytical model is then verified through close correlations among Finite Element (FE) and analytical results for a concrete sandwich panel with FRP shear connectors. Next, a parametric study is conducted using the analytical model to study the effects of deck stiffness and aspect ratio on effective flange width. A simplified method is then proposed to calculate effective width.

**ANALYTICAL MODEL**

Applying the Classical Lamination Theory (CLT), the constitutive relationship for an orthotropic wythe can be described as:

$$\begin{bmatrix} \{N\} \\ \{M\} \end{bmatrix} = \begin{bmatrix} [A] & 0 \\ 0 & [D] \end{bmatrix} \begin{bmatrix} \{\varepsilon\} \\ \{\kappa\} \end{bmatrix}$$  \hspace{1cm} (4)
where \( \{N\} \) and \( \{M\} \) are in-plane load and moment vectors, respectively; \( [A] \) and \\
\( [D] \) are extensional stiffness and bending stiffness matrices, respectively; and \( \{\varepsilon\} \) and \\
\( \{\kappa\} \) are in-plane strain and curvature vectors, respectively. \( [A] \) and \\
\( [D] \) can be calculated as:

\[
[A] = h_1 [Q] \tag{5}
\]

\[
[D] = \frac{h_1^3}{12} [Q] \tag{6}
\]

where \( h_1 \) is the thickness of the wythe. For an orthotropic material, we have

\[
[Q] = \begin{bmatrix}
E_x & \nu_{xy} E_x & 0 \\
\frac{1-\nu_{xy}}{E_x} & \frac{1-\nu_{xy}}{E_y} & 0 \\
\nu_{yx} E_y & \frac{1-\nu_{yx}}{E_y} & 0 \\
0 & 0 & G
\end{bmatrix}
\]

where \( E_x, \nu_{xy} \), and \( E_y, \nu_{yx} \) are Young’s modular and Poisson’s ratios in x and y directions, respectively. Equation (4) can be re-written as follows by performing appropriate matrix inversions:

\[
\begin{bmatrix}
\{\varepsilon\} \\
\{\kappa\}
\end{bmatrix} = \begin{bmatrix}
\alpha & 0 \\
0 & \delta
\end{bmatrix} \begin{bmatrix}
\{N\} \\
\{M\}
\end{bmatrix} \tag{8}
\]

where the compliance matrices are given by \( \alpha = [A]^{-1} \) and \( \delta = [D]^{-1} \).

Barbero et al. [15] found out that when a plate is subjected to out-of-plane forces, the axial force \( N_y \), moment \( M_y \) in y direction and twisting moment \( M_{xy} \) can be assumed to be equal to zero. If a wythe is considered to be connected by two shear connectors at both edges, then only edge shear transaction \( N_{xy} \) and axial force \( N_x \) exist as shown in Figure 5. Based on this assumptions, Equation (8) can be reduced to:
Based on equilibrium equation of an infinitesimal section of the plate, we have

\[ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \]  \hspace{1cm} (10)

The compatibility equation for an orthotropic plate is given as:

\[ \frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} \]  \hspace{1cm} (11)

Since the deck is restrained in y direction by two shear connectors at two sides, \( \varepsilon_y \) is insignificant compared to \( \varepsilon_x \), which can be neglected. Equation (11) becomes:

\[ \frac{\partial^2 \varepsilon_x}{\partial y^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} \]  \hspace{1cm} (12)

Substituting Equation (9) into (12), and combining with Equation (10), we get:
\[ \alpha_{11} \frac{\partial^2 N_x}{\partial y^2} + \alpha_{66} \frac{\partial^2 N_x}{\partial x^2} = 0 \]  

(13)

Equation (13) can be reduced to an ordinary differential equation by a harmonic analysis. Assume the wythe is simply supported at \( x=0 \) and \( a \), the axial force can be obtained as:

\[ N_x(x, y) = \sum_{j=1}^{\infty} N_j(y) \sin\left(\frac{j\pi x}{a}\right) \]  

(14)

where \( N_j(y) \) is an amplitude function. Substituting Equation (14) into (13), we get:

\[ \frac{\partial^2 N_j}{\partial y^2} = \xi_j^2 N_j \]  

(15)

where

\[ \xi_j = \frac{j\pi}{a} \sqrt{\frac{\alpha_{66}}{\alpha_{11}}} \]  

(16)

Salim and Davalos [16] obtained a general solution for Equation (15), which is:

\[ N_j(y) = C_{1j} \cosh(\xi_j y) + C_{2j} \sinh(\xi_j y) \]  

(17)

where \( C_{1j} \) and \( C_{2j} \) are constants that need to be determined. Based on symmetric conditions about the \( x \)-axis, we have \( C_{2j} = 0 \). Therefore, Equation (17) can be reduced to:

\[ N_j(y) = C_{1j} \cosh(\xi_j y) \]  

(18)

and \( N_x \) can be expressed as:

\[ N_x(x, y) = \sum_{j=1}^{\infty} C_{1j} \cosh(\xi_j y) \sin\left(\frac{j\pi x}{a}\right) \]  

(19)

Substituting Equation (18) into (10), we get:
\[
\frac{\partial N_{xy}(x, y)}{\partial y} = -\frac{\partial N_{x}(x, y)}{\partial x} = -\sum_{j=1}^{\infty} \frac{j\pi}{a} C_{1j} \cosh(\xi_j, y) \cos\left(\frac{j\pi x}{a}\right) \tag{20}
\]

Therefore,

\[
\frac{\partial N_{xy}(x, y)}{\partial y} = -\sum_{j=1}^{\infty} \frac{j\pi}{a} C_{1j} \sinh(\xi_j, y) \cos\left(\frac{j\pi x}{a}\right) \tag{21}
\]

Based on symmetric condition, at \( y = b/2 \), we have

\[
N_{xy}(x, \frac{b}{2}) = \frac{1}{2} \frac{dF(x)}{dx} \tag{22}
\]

where \( F(x) \) is the force transferred through the shear connection. Combining Equations (21) and (22), we have

\[
\frac{dF(x)}{dx} = -2N_{xy}(x, \frac{b}{2}) = -2 \sum_{j=1}^{\infty} \frac{j\pi}{a} C_{1j} \frac{\xi_j b}{2} \sinh\left(\frac{\xi_j b}{2}\right) \cos\left(\frac{j\pi x}{a}\right) \tag{23}
\]

Therefore, \( F(x) \) can be expressed as:

\[
F(x) = 2 \sum_{j=1}^{\infty} C_{1j} \frac{\xi_j b}{2} \sinh\left(\frac{\xi_j b}{2}\right) \sin\left(\frac{j\pi x}{a}\right) \tag{24}
\]

As pointed out by Newmark et al. [7] and Bradford [17], the relative movement, or slip, between the slab and girder is given by

\[
\gamma = \frac{q}{K} \tag{25}
\]

where \( \gamma \) is the slip, \( q \) and \( K \) are the horizontal shear and stiffness at the interface of the two layers, respectively. \( q \) is equal to the change of force \( F \), which can be expressed as:

\[
q = \frac{dF(x)}{dx} \tag{26}
\]

Since the slip \( \gamma \) is the difference between the lower wythe and upper wythe as shown in Figure 6, which can be expressed as:
\[ \gamma = u_2 - u_1 \]  

(27)

\[ \varepsilon_2 - \varepsilon_1 = \frac{d\gamma}{dx} \]  

(28)

where \( \varepsilon_1 \) and \( \varepsilon_2 \) are strains calculated at neutral axis of the sandwich panel as shown in Figure 8. Substituting Equations (25) and (26) into (28) results in

\[ \varepsilon_2 - \varepsilon_1 = \frac{d^2F(x)}{K \cdot dx^2} \]  

(29)

Based on the assumption that plane remains plane after deformation, the strain follows a linear distribution along the depth of both layers. Therefore,
\[
\varepsilon_1 = -\alpha_{11}^{(1)} N_*(x, \frac{b}{2}) + \frac{M_1(x) C_1}{D_{11}^{(1)}}
\]  

(30)

and

\[
\varepsilon_2 = \alpha_{11}^{(2)} N_*(x, \frac{b}{2}) - \frac{M_2(x) C_2}{D_{11}^{(2)}}
\]  

(31)

where \( M_i(x) \), \( C_i \), \( b \) and \( D_{11}^{(i)} \) are moment, distance between mid-depth of the wythe to the neutral axis of sandwich panel, width of the sandwich panel, and flexural rigidity, respectively; and \( i = 1 \) and \( 2 \) represent the upper and lower wythes, respectively. The assumption that the upper and lower layer deflect equal amount of moment at all points along the length indicates that the angle changes along the length be equal, which can be expressed as:

\[
\frac{M_1(x)}{b D_{11}^{(1)}} = \frac{M_2(x)}{b D_{11}^{(2)}} = \frac{M_1(x) + M_2(x)}{b D_{11}^{(1)} + b D_{11}^{(2)}}
\]  

(32)

Based on equilibrium of the moment, we have

\[
M_1(x) + M_2(x) + F(x) (C') = M(x)
\]  

(33)

where \( C' \) is the distance between the neutral axis of the upper wythe and the neutral axis of the lower wythe. Substituting Equation (33) into (32), we have

\[
\frac{M_1(x)}{b D_{11}^{(1)}} = \frac{M_2(x)}{b D_{11}^{(2)}} = \frac{M(x) + F(x) (C')}{b D_{11}^{(1)} + b D_{11}^{(2)}}
\]  

(34)

Combining Equations (30), (31) and (34), we have

\[
\varepsilon_2 - \varepsilon_1 = (\alpha_{11}^{(1)} + \alpha_{11}^{(2)}) N_*(x, \frac{b}{2}) - \frac{[M(x) - F(x) (C')] (C_1 + C_2)}{b [D_{11}^{(1)} + D_{11}^{(2)}]}
\]  

(35)

Substituting Equation (29) into (35), we have

\[
\frac{d^2 F(x)}{K \cdot dx^2} = (\alpha_{11}^{(1)} + \alpha_{11}^{(2)}) N_*(x, \frac{b}{2}) - \frac{[M(x) - F(x) (C')] (C')}{b [D_{11}^{(1)} + D_{11}^{(2)}]}
\]  

(36)
In order to use Fourier series to solve Equation (36), the moment can be expressed as:

\[ M(x) = \sum_{j=1}^{\infty} Q_j \sin\left(\frac{j\pi x}{a}\right) \]  

(37)

where

\[ Q_j = \frac{2}{b} \int_{b}^{h} M(x) \sin\left(\frac{j\pi x}{a}\right) dx \]  

(38)

Using Equations (19), (24), and (37) into (36), the solution of the partial differential Equation (36) can be obtained using Fourier series. Thus, through some simple mathematical transformations, we obtain

\[ \sum_{j=1}^{\infty} C_{1j} A_{j} \sin\left(\frac{j\pi x}{a}\right) = \sum_{j=1}^{\infty} Q_j \frac{(C')}{b[D_{11}^{(1)} + D_{11}^{(2)}]} \sin\left(\frac{j\pi x}{a}\right) \]  

(39)

where

\[ A_j = \frac{2}{K \xi_j} \left(\frac{j\pi}{a}\right)^2 \sinh\left(\frac{\xi_j b}{2}\right) + \frac{2}{\xi_j} \frac{(C')^2}{b[D_{11}^{(1)} + D_{11}^{(2)}]} \sinh\left(\frac{\xi_j b}{2}\right) + (\alpha_{11}^{(1)} + \alpha_{11}^{(2)}) \cosh\left(\frac{\xi_j b}{2}\right) \]  

(40)

Solving for Equation (39), we have

\[ C_{1j} = \frac{Q_j}{A_j} \frac{(C')}{b[D_{11}^{(1)} + D_{11}^{(2)}]} \]  

(41)

where \( Q_j \) and \( A_j \) are defined in Equations (38) and (40), respectively. Effective flange width \( b_{eff} \) can then be calculated as:

\[ b_{eff} = \frac{\int_{b}^{0} N_s(x,y) dx}{N_s\left(x,\frac{b}{2}\right)} = \frac{F(x)}{N_s\left(x,\frac{b}{2}\right)} \]  

(42)

DEGREE OF COMPOSITE ACTION
The DCA defined by Lorenz and Stockwell [11] was used in the American Institute of Steel Construction (AISC) [18], as shown in Figure 4. We can extend this concept to the sandwich panel, as shown in Figure 8. Thus, DCA can be expressed as:

\[
DCA = 1 - \frac{x}{x_{\text{MAX}}} \quad (43)
\]

where \(x\) indicates the amount of the horizontal slip. Based on strain distributions shown in Figure 8, Equation (43) can be re-written as:

\[
DCA = 1 - \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_{\text{MAX}}} \quad (44)
\]

Substituting Equation (43) into Equation (35) we have

\[
(1 - DCA) \times \varepsilon_{\text{MAX}} = (\alpha_{11}^{(1)} + \alpha_{11}^{(2)}) N_x (x, \frac{b}{2}) \times \frac{[M(x) - F(x)(C')] [C']}{b[D_{11}^{(1)} + D_{11}^{(2)}]} \quad (45)
\]

where \(\varepsilon_{\text{MAX}}\) can be calculated based on 0% DCA, as shown in Figure 8. \(\varepsilon_{\text{MAX}}\) can be expressed as:

\[
\varepsilon_{\text{MAX}} = \frac{[M(x)] (C_1 + C_2)}{b[D_{11}^{(1)} + D_{11}^{(2)}]} \quad (46)
\]

Substituting Equation (46) into Equation (45), DCA can be expressed as:

\[
DCA = \frac{(\alpha_{11}^{(1)} + \alpha_{11}^{(2)}) N_x (x, \frac{b}{2}) - \frac{F(x)(C')^2}{b[D_{11}^{(1)} + D_{11}^{(2)}]}}{\varepsilon_{\text{MAX}}} \quad (47)
\]

**VERIFICATION**

For verification purpose, the derived closed-form solution and FE method were used to study a concrete sandwich panel, which was tested by Hopkins et al. [13], as shown in Figure 1(b). The wythes were 1,219 mm (48 in) wide, 7,049 mm (277.5 in) long, and 76.2 mm (3 in) thick. They were separated by a 102 mm (4 in) thick foam core. A linear elastic FE model using ABAQUS [19] was created, as shown in Figure 3. The Young’s modulus and Poisson’s ratio for concrete were 29,322 MPa (4,253 ksi) and 0.15, respectively. A 93,857 N (21,100 lb) point load was applied at the mid-span. Shell element (S4R) was used to model the concrete wythes. The mesh size was set to 122
mm (4.8 in). The two concrete wythes are connected using CONN3D element, where all stiffnesses were set to be rigid except the stiffness in y-direction, which was changed to represent different DCAs. The boundary conditions were set to be pin on one side and roller at the other side. The other two sides of the sandwich panel were also restrained.

The stress and deflection results at mid-span from the FE and analytical model for different DCAs are shown in Figure 9 and Figure 10, where good correlations can be seen, which proves the accuracy of the model. Figure 11 shows the relationship between Effective Width Ratio (EWR) and DCA, where again good correlations can be observed. EWR is defined as the ratio between the effective width and width of the panel.

TABLE I – MATERIAL PROPERTIES ADOPTED IN THE PARAMETERIC STUDY
(CHEN AND DAVALOS 2014)

<table>
<thead>
<tr>
<th>$E_x$ (GPa)</th>
<th>$E_y$ (GPa)</th>
<th>$G$ (GPa)</th>
<th>$\nu_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1475</td>
<td>2747</td>
<td>741</td>
<td>0.321</td>
</tr>
</tbody>
</table>

APPLICATION AND DISCUSSION

A parametric study was performed to study effective width of the same model as described above, but with orthotropic FRP material instead of concrete for the wythes. The material properties of FRP were taken from Chen and Davalos [21] and listed in Table I.

Figure 12 shows that the core thickness has negligible effect on EWR for DCAs up to 75%. For 100% DCA, the effective width is about 10% lower than that of 75% DCA. Therefore, 100% DCA provides the least value for effective width, which can be used as a conservative value representing all DCAs. This finding will be used to derive simplified method, as will be discussed next.

The aspect ratio has a significant effect on EWR, as can be seen from Figure 13. The EWR with an aspect ratio of 2 is about 30% lower than that with an aspect ratio 8. Therefore, it can be concluded that the derived equation can only be used for one-way panel.
Figure 9 – Stress Distribution at mid-span

Figure 10 - Mid-span Deflection vs. DCA

Figure 11 - EWR vs. DCA

Figure 12 - EWR vs. Core Thicknesses

Figure 13 - EWR vs. Aspect Ratios
Figure 14 and Figure 15 show that panel thickness ratio and stiffness have negligible effects on EWR up to 75% DCA. When reaching 100% DCA, EWR drops about 10% for the same stiffness. EWR increases as the panel thickness ratio increases.

SIMPLIFIED METHOD TO CALCULATE THE DEFLECTION AND STRAIN DISTRIBUTION

From Equation (42), $N_x(x, b/2)$ can be expressed as:

\[ N(x, \frac{b}{2}) = \frac{F(x)}{b_{\text{eff}}} \]  

(48)

Based on the findings from the parametric study, $b_{\text{eff}}$ for 100% DCA can be conservatively used to represent effective widths for other DCAs. Therefore, we can assume $b_{\text{eff}}$ to be a constant. Substituting Equation (48) into (47), we have

\[ DCA = \frac{F(x) \times \left[ (\alpha_{11}^{(1)} + \alpha_{11}^{(2)}) - \frac{(C')^2}{b[D_{11}^{(3)} + D_{11}^{(2)}]} \right]}{\varepsilon_{\text{max}}} \]  

(49)

Therefore, DCA is in linear relationship to $F(x)$, i.e.,

\[ F(x) = DCA \times F_c(x) \]  

(50)

where $F_c(x)$ is the maximum interfacial shear force transferred through the shear connection for a composite beam with 100% DCA. Based on this finding, the deflections and stresses of composite beams with partial DCA can be calculated from those with 100% and 0% DCAs as follows.
From Figure 7, we have

\[ \kappa(x) = \frac{M(x)}{b \times D_{11}} = \frac{M_1(x)}{b \times D_{11}^{(1)}} = \frac{M_2(x)}{b \times D_{11}^{(2)}} = \frac{M_1(x) + M_2(x)}{b[D_{11}^{(1)} + D_{11}^{(2)}]} \]  

(51)

where \( \kappa \) is curvature, \( D_{11} \) is the flexural rigidity, and \( M(x), M_1(x), \) and \( M_2(x) \) are moments acting on the sandwich panel, upper layer, and lower layer, respectively. Applying 100% DCA to Equation (51), we have

\[ \kappa_c(x) = \frac{M(x)}{b \times D_{11}} - \frac{M_{1c}(x)}{b \times D_{11}^{(1)}} = \frac{M_{2c}(x)}{b \times D_{11}^{(2)}} = \frac{M_{1c}(x) + M_{2c}(x)}{b[D_{11}^{(1)} + D_{11}^{(2)}]} \]  

(52)

The subscript \( c \) denotes a 100% DCA composite action. For 0% DCA, we have

\[ \kappa_0(x) = \frac{M(x)}{b \times D_{11}} = \frac{M_{10}(x)}{b \times D_{11}^{(1)}} = \frac{M_{20}(x)}{b \times D_{11}^{(2)}} = \frac{M_{10}(x) + M_{20}(x)}{b[D_{11}^{(1)} + D_{11}^{(2)}]} \]  

(53)

where all symbols have the same meaning as those in Equation (52), but subscript \( 0 \) denotes a 0% DCA noncomposite action.

Based on Equation (33) we have

\[ F(x)(C') = M(x) - [M_1(x) + M_2(x)] \]  

(54)

where \( F(x) \) is the interfacial shear force transferred through the shear connection. Combining Equations (50), (51) and (54), for any DCA, we have

\[ \kappa(x) = \frac{M(x) - F(x)(C')}{b[D_{11}^{(1)} + D_{11}^{(2)}]} = \frac{M(x) - DCA \times F_c(x)(C')}{b[D_{11}^{(1)} + D_{11}^{(2)}]} \]  

(55)

Equation (55) can also be written as:

\[ \kappa(x) = \frac{M(x)}{b[D_{11}^{(1)} + D_{11}^{(2)}]} - DCA \times \frac{F_c(x)(C')}{b[D_{11}^{(1)} + D_{11}^{(2)}]} \]  

(56)

Based on Equation (54), we have

\[ \kappa_0(x) - \kappa_c(x) = \frac{F_c(x)(C')}{b[D_{11}^{(1)} + D_{11}^{(2)}]} = \frac{M(x)}{b[D_{11}^{(1)} + D_{11}^{(2)}]} - \frac{M_1(x) + M_2(x)}{b[D_{11}^{(1)} + D_{11}^{(2)}]} \]  

(57)

Using Equation (51) and (57) into (56), we have
\[
\kappa(x) = \kappa_0(x) - DCA \times [\kappa_0(x) - \kappa_c(x)] = (1 - DCA) \kappa_0(x) + DCA \times \kappa_c(x)
\]  
(58)

The deflection \( \Delta(x) \) can be calculated by double integration of \( \kappa(x) \) considering proper boundary conditions. For a simply supported sandwich panel considered in this study, we have

\[
\Delta(x) = (1 - DCA) \Delta_0(x) + DCA \times \Delta_c(x)
\]  
(59)

The stress can be calculated based on \( \kappa(x) \) for any layer as:

\[
\sigma_z(x) = A_{11} [\kappa(x) \times z - \frac{\alpha_{11} \times DCA \times F_c(x)}{b_{eff}}]
\]  
(60)

Substituting Equation (57) into (60), we have

\[
\sigma_z(x) = A_{11} \left[ (1 - DCA) \kappa_0(x) + DCA \times \kappa_c(x) \right] \times z - \frac{\alpha_{11} \times DCA \times F_c(x)}{b_{eff}}
\]  
(61)

From Equation (60), we have

\[
\sigma_{z0}(x) = A_{11} \kappa(x) \times z
\]  
(62)

\[
\sigma_{zc}(x) = A_{11} \left[ \kappa_c(x) \times z - \frac{\alpha_{11} \times F_c(x)}{b_{eff}} \right]
\]  
(63)
Comparing Equations (61) through (63), we can express Equation (60) in a similar expression to Equation (59) as

\[ \sigma_z(x) = (1 - DCA)\sigma_{z0}(x) + DCA \times \sigma_{zc}(x) \]  \hspace{1cm} (64)

where again the subscripts \(0\) and \(c\) denote composite beams with 0% and 100% DCAs, respectively.

**DISCUSSIONS**

To evaluate the accuracy of the proposed simplified method, Equations (59) and (64) are used to calculate the deflection and stresses at the mid-height and mid-span of either wythe, with the results shown in Figures 16 and 17, respectively. It can be observed that simplified method and analytical results are close to each other except for the deflection at 75% DCA, which has a difference of 31%. Therefore, the simplified equations can be used to provide conservative results for deflections and stresses, which is applicable for practical design purposes.

**CONCLUSIONS**

This study develops an analytical model for calculating effective width of sandwich panel based on shear lag model and considering partial DCA. It can be concluded that:

1. The analytical model can accurately predict the behavior of the sandwich panels in terms of stress/strain and deflection, taking into account the partial DCA.
2. DCA can significantly affect the stress and strain distributions.
3. The parametric study results show that the least value of effective width can be achieved with 100% DCA.
4. Aspect ratio is an important factor that affects the effective width. The derived closed-form equation is only valid for one-way panel.
5. Core thickness and panel stiffness have negligible effects on the effective width.

It is noted that this analytical model is based on elastic behavior. Further study is recommended to consider plastic behavior of sandwich panels and multiple shear connectors.

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**REFERENCES**


