Optimization of Self-Trapping and Thermal Effects in W-Shaped Optical Fibers

El-Sayed A. El-Badawy

Higher Institute of Engineering, Thebes Academy, Cairo 11434, Egypt.
Emer. Prof., Elec. Eng. Department, Faculty of Engineering, Alexandria University, Alexandria 21544, Egypt.
Senior Member IEEE, Member of the Optical Society of America (OSA), s0badw@yahoo.com

Farag Z. El-Halafawy* Abd El-Naser A. Mohammed

* Member of the Optical Society of America (OSA) and the Intern. Society of Optical Engineering (SPIE)

Moustafa H. Aly

Electronics and Communication Eng. Dept., College of Engineering, Arab Academy for Science and Technology & Maritime Transport, Alexandria, Egypt,
Member of the Optical Society of America (OSA), E-mail: mousaly@hotmail.com.

ABSTRACT

In the present paper, both self-trapping (paraxial propagation) and thermal effects (caustic zones and maximum axial temperature) in W-shaped refractive index fibers are parametrically investigated in order to minimize the thermal effects and to maximize the self-trapping. Ray optics is employed with a closed form solution for the ray trajectory. Conditions for maximum self-trapping are parametrically investigated. A figure of merit is designed and tested for good performance.

I. INTRODUCTION

Single mode (SM) optical fibers are most suitable for high data-rate transmission systems because of the absence of modal dispersion. However, some drawbacks arise due to difficulty in excitation and splicing because of the small core radius. Single-mode graded-core W-fibers are proposed to overcome these problems [1] and the structure of the phase devices [2]. According to the rapid development in recent all-optical networks or all-optical signal processing technology [3], optical fibers or waveguide based optical devices [4] have been investigated extensively.

El-Halafawy et al. [5,6 and 7] have investigated the soliton transmission in nonlinear inhomogeneous biquadratic graded (W-shaped) index fibers. They have tailored the W-shaped refractive index as (Fig. 1):

\[ n^2 = n_0^2 \left( 1 - \alpha \rho^2 + \beta \rho^4 \right), \]  

where \( n_0 \) is the axial refractive index, both \( \alpha \) and \( \beta \) are tailoring parameters and \( \rho \) is the radial position (\( \rho = r/R \)). Both \( \alpha \) and \( \beta \) control the position and the value of the minimum refractive index in the W-shaped structure.

In the present paper, two basic problems are investigated; namely, the control of both self-trapping and thermal effects in the W-shaped optical fiber carrying high power as in optical amplifiers, sensors, and medical applications. Section I is a concise review, Section II handles the basic model and analysis, while Section III processes the obtained results with a general discussion and Section IV summarizes the major conclusions.

II. BASIC MODEL AND ANALYSIS

The object of the above investigations starts by solving the ray equation [8,9]:

\[ \frac{\partial^2 \rho}{\partial \eta^2} = \frac{1}{2n_0^2 N_e} \frac{\partial n^2}{\partial \rho}, \]  

where \( n = 2h/a \) is the normalized axial distance and \( N_e \) is the directional cosine of the incident ray.

The use of \( \rho_n = \rho \sqrt{2B/\alpha} \) and \( \eta_n = \eta \sqrt{\alpha} \) in Eq.(2) yields:

\[ \frac{\partial^2 \rho_n}{\partial \eta_n^2} + \rho_n - \rho_n^2 = 0. \]  

El-Halafawy et. al. [5] and El-Badawy et al. [9] employed the power series technique to solve Eq.(3) while El-Halafawy et. al.[5] processed the phenomena under a closed form. The launch conditions (initial conditions) are:

\[ \rho_n = \rho_n \left|_{\eta_n = 0} \right. = A_n = \frac{r_s}{a} \sqrt{\frac{2B}{\alpha}} = \rho_n \sqrt{\frac{2B}{\alpha}} \]  

and

\[ \frac{\partial \rho_n}{\partial \eta_n} \bigg|_{\eta_n = 0} = A_n = \frac{dr}{dz} \left|_{\eta_n = 0} \right. \frac{N_e}{\alpha} \sqrt{\frac{2B}{\alpha}} = \rho_n N_e \sqrt{\frac{2B}{\alpha}} \]  

Integrating Eq.(3), we get:
where

\[ C = A_3^2 + A_1^2 - 0.5A_4^2, \]

and finally

\[ \frac{\text{dp}_a}{\text{d}n_a} = \sqrt{0.5p_a^2 - p_a^2 + C}. \]

This integration yields the elliptic functions \[10], where:

\[ A^2 + B^2 = 2, \]

and

\[ A^2 B^2 = 2C. \]

Using \( p_a = B u_a \), in Eq \((7)\), one can get:

\[ \int_i \sqrt{B^2 u_n^2 - A^2(B^2 u_n^2 - B^2)} = \int_i \frac{1}{\sqrt{2}} \text{d}n_a, \]

or

\[ \frac{B du_a}{\sqrt{(1 - u_n^2)(1 - m^2)}} = \omega \eta_a, \]

where \( m = B^2 / A^2 < 1.0 \) and \( m = A / \sqrt{2} \).

This is the standard form of the "Jacobian" elliptic periodic sine \((m)\) of period \( T_r \), i.e., we get:

\[ \text{sn} - u_n = \text{sn}^{-1} u_n = \omega \eta_n, \]

or finally:

\[ \rho_n = B \text{sn}(\eta_n + 0). \]

The period of this kind of function is \( T_r \), where:

\[ T_r = \frac{\pi}{\lambda} \left[ 1 + \left( \frac{1}{2} \right)^2 \lambda + \left( \frac{1}{2} \right)^4 \lambda^2 + \left( \frac{1}{2} \right)^6 \lambda^3 + \ldots \right] \]

(11)

The amplitude of this elliptic sine \((m)\) the maximum distance away-of-the-axis \( \rho_n \) is given by:

\[ \rho_n = \frac{B}{\sqrt{1 - (1 - 2C)}}. \]

(12)

\[ \frac{r_n}{a} = \frac{B}{\sqrt{2}} \frac{\alpha}{2 \beta}. \]

(13)

The radial position of minimum refractive index is:

\[ \rho_{\text{mn}} = \frac{\alpha}{2 \beta}. \]

(14)

Thus, good confinement \((\text{paraxial propagation})\) requires:

\[ r_{\text{mn}} \leq 1.0, \quad \text{or} \]

\[ B \leq 1.0. \]

(15-a)

(15-b)

i.e.,

\[ \sqrt{1 - 2C} \leq 0.0. \]

(15-c)

Thermal effects in fibers doped with rare earth or other absorbers play an increasingly important role as in fiber amplifiers, laser sources, attenuators, and all optical fiber switches \[11\].

A theoretical analysis of the temperature rise under two pumping regimes \(\text{(short pump pulse or continuous wave pumping)}\) has been presented \[11\], but we will employ the simple \((\text{but accurate})\) relation \[12\]:

\[ T_n = T_o + 3.54(P_p + P_{\text{RF}}) - T_o + 3.54P_1; \]

(16)

where \( T_n \) is the caustic temperature, \( T_o \) is the ambient temperature, and \( P_p \) is the Raman injected power in watts \((\text{in Raman amplifiers)}\) or the injected power in industrial applications, and \( P_{\text{RF}} \) is the total power of the multiplexed signals in the fiber. \( T_n \) is caused at different periodic lengths \( L_n \) is given by:

\[ L_n = \frac{0.5T_n}{\alpha} = \frac{0.5 \sqrt{2} T_n}{\alpha} = \frac{0.5 \sqrt{2} T_n}{\alpha}. \]

(17)

Reduction of thermal effects requires the optimization of both \( T_n \) and \( L_n \), i.e., the minimization of \( T_n \), and then the maximization of \( L_n \), as well as satisfying the condition of self-trapping given by Eq \((15)\).

The impact of thermal effects arises in different applications in both fields of industry and communications. The local caustic zone results in a temperature rise which increases the heat transfer coefficient \[11\] and consequently faster the cooling in the caustic zones yielding different stresses and stains along the fiber \[13\].

The temperature dependence of erbium doped fiber amplifiers \((\text{EDFA’s})\) characteristics is of great importance as the temperature rise reduces the amplifier gain \[14, 15\], as systems evolve towards more wavelengths \((\text{WDM and UWWDM)}\), higher bit-rate and greater distances, such temperature dependence are no longer acceptable.

Pumping-induced thermal effects in doped fibers due to nonradiative processes are detrimental in most doped fiber devices including high power lasers and amplifiers.

Also, the fiber dispersion characteristics and the spectral losses \[16, 17, 18\] undergo severe changes due to the temperature rise which increases both losses and dispersion, thus reducing the transmitted bit-rate, and the repeater spacing.

It was found early \[19\] that the temperature dependence of the transient time delay shift, \( \tau \), thermal coefficient of the fiber stress \( \sigma \), and the fiber strain \( e \) are functions of the temperature rise \((40.0 < T^\circ C < 60.0)\) and are phenomenologically derived, respectively, as:

\[ \tau = \tau_c(T_n - 1) = 10.9(T_n - 1) \text{ ps km}; \]

\[ \sigma = \sigma_c(1 + \sigma_c(T_n - 1) + \sigma_c(T_n - 1)^2); \]

\[ \sigma - \sigma_c = 493(T_n - 1) - 1009(T_n - 1)^2 \text{ MPa}; \]

\[ e = e_c(1 + e_c(T_n - 1)); \]

\[ e - e_c = 36.25 \times 10^{-3} (T_n - 1) \]

where \( T_c = T_n / T_0 \) and \( e_c, \sigma_c, \sigma_c, \sigma, \tau, e_c, \) and \( e_c \) are constants.
In the following section, the investigated items (self-trapping and thermal effects) one parametrically processed.

III. RESULTS AND DISCUSSION

The software especially designed to parametrically process the present problem investigates the following optimized items:

i) the self-trapping, and

ii) the thermal effects.

The set of causes is \( \{ P, r, r', \alpha, \beta \} \) while the set of effects is \( \{ T_n, A, B, I_p \} \).

Two important features of W-shaped optical fibers must be processed. The clad refractive index \( n_2 \) and the location of minimum radial refractive index \( n_{\text{min}} \) at \( \rho_{\text{min}} \), where:

\[
\begin{align*}
n_2 & = n_0 \left( 1 - 0.5\alpha + 0.5\beta \right), \\
\rho_{\text{min}} & = \frac{\alpha}{2\beta}, \\
n_{\text{min}} & = n_0 \left( 1 - \frac{\alpha^2}{8\beta} \right).
\end{align*}
\]  

We will constraint both \( n_2 \) and \( n_{\text{min}} \) at the following numerical values:

\[
\begin{align*}
n_2 & = 0.95n_0, \\
n_{\text{min}} & = 0.90n_0.
\end{align*}
\]  

Thus, we have:

\[
\begin{align*}
0.5(\alpha - \beta) & = 0.05 \quad \text{or} \quad \alpha - \beta = 0.1, \\
\frac{1}{8\beta} & = 0.1 \quad \text{or} \quad \alpha^2 = 0.8\beta.
\end{align*}
\]

which yields:

\[
\alpha = 0.6828 \text{ or } 0.1172 \quad \text{and} \quad \beta = 0.5828 \text{ or } 0.0172.
\]

In general, the design assumptions:

\[
\begin{align*}
n_2 & = (1 - \Delta)n_0, \\
n_{\text{min}} & = (1 - 2\Delta)n_0
\end{align*}
\]  

yield:

\[
\begin{align*}
\alpha & = 13.6568\Delta \quad \text{or} \quad 2.3431\Delta, \\
\beta & = 11.6568\Delta \quad \text{or} \quad 0.3431\Delta
\end{align*}
\]

The second result is not reasonable and hence we employ only:

\[
\begin{align*}
\alpha & = 13.6568\Delta \quad \text{and} \\
\beta & = 11.6568\Delta
\end{align*}
\]

Thus, whatever the value of \( \Delta \), we have:

\[
\rho_{\text{min}} = 0.7654
\]

Thus, for self-trapping, \( \rho_s \) must be less than \( \rho_{\text{min}} \), i.e., \( \rho_s < 0.7654 \).

Samples of the variations of the amplitude \( \rho_{\text{max}} \) of the "sn" function, where \( \rho_{\text{max}} = B\sqrt{\alpha/2\beta} = 0.5858B \) and the length of the critical zone \( L_p \) against the variations of \( \Delta, \rho_s, \) and \( \rho_s' \) are displayed in Figs. 2-4, while the thermal effects are displayed in Figs. 5-8. These figures assure the following facts:

i) The best set of launch conditions for good self-trapping, is the following:

a) small \( \rho_s \)

b) small \( \rho_s' \)

\[
\begin{align*}
\rho_{\text{max}} & \rightarrow \text{large } \Delta \quad \text{Thus, we design an figure of merit, } FM \text{ under the form:} \\
F_m & = \frac{\rho_s\rho_s'}{\Delta} \quad (27)
\end{align*}
\]

We feel that \( \rho_{\text{max}} \) and \( \rho_s \) possess positive correlation as in Fig. 9, also \( L_p \) and \( F_m \) are in positive correlation, Fig. 10. In fact as \( F_m \) decreases, \( \rho_{\text{max}} \) decreases. This effect is required for good self trapping. Also, as \( \rho_s \) decreases, \( L_p \) increases yielding less number of caustic zones and consequently decreases the harmful thermal effects.

ii) In general, the increase of the launch power yields higher temperature rise and consequently higher stresses, strains, and delay time.

IV. CONCLUSIONS

The refractive index of W-shaped single mode optical fibers is tailored with deep unambiguity to parametrically investigate two basic problems; namely, the self-trapping and the thermal effects (both are of interest in optical communication systems and industrial applications) to optimize these phenomena. The ray trajectory is derived in a closed form of "Jacobian " elliptic function. The designed dimensionless figure of merit is a good criterion to numerically measure both the self-trapping and the thermal effects phenomena.

REFERENCES


Fig. 1. Variations of the ratio \( n_{min}/n_2 \) against variations of radial position, \( \rho \) for W-shaped refractive index SM fiber.

Fig. 2. Variations of \( \rho_{max} \) against variations of \( \rho_0 \) at the assumed set of parameters.
Fig. 3. Variations of $L_p$ against variations of $\rho_p$ at the assumed set of parameters.

Fig. 4. Variations of $C$ against variations of $\rho_p$ at the assumed set of parameters.

Fig. 5. Variations of $T_n$ against variations of $P_n$ at the assumed set of parameters.

Fig. 6. Variations of $\tau$ against variations of $P$, at the assumed set of parameters.
Fig. 7. Variations of $\sigma - \sigma_o$ against variations of $P_i$ at the assumed set of parameters.

Fig. 8. Variations of $\varepsilon$ against variations of $P_i$ at the assumed set of parameters.

$p = 0.2 \rightarrow 0.65, \quad \rho^2 = 0.1 \rightarrow 0.25 \quad \Delta = 0.001 \rightarrow 0.009 \quad N_e = 0.2$

Fig. 9. Variations of $\rho_{\text{max}}$ against variations of $f_m$ at the assumed set of parameters.

Fig. 10. Variations of $L_{\text{pe}}$ against variations of $f_m$ at the assumed set of parameters.