THERMAL MICROBENDING IN GRADED-INDEX MULTIMODE OPTICAL FIBERS

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ABSTRACT

Optical fibers exhibit losses due to thermal microbending that occurs to the fiber axis during any significant temperature drop. Thermal microbending loss coefficient is found to depend on the fiber parameters as well as the number of propagating modes.

INTRODUCTION

Multimode optical fibers are still important for special applications such as local area networks. Research and development on these fibers is now devoted to decreasing operational losses exhibited by the fiber during optical power transmission. Of these mechanisms, is the microbending loss [1,2] and the thermal microbending loss which results from any significant drop in the operating temperature.

For stable performance, optical fibers are coated to be protected from external influences. However, the coatings may have different thermal expansion coefficients than the fiber core. These differences yield unlikely length contractions and, consequently, a series of random bends along the fiber axis.

Thermal microbending is calculated through a loss coefficient which is a function of both the fiber parameters and the operating conditions. We have shown, in an earlier work [3], how to control the value of this coefficient for single-mode optical fibers. The object of the present work is to generalize the previous work for the multimode type of fibers. We present an analytic study to indicate how the parameters affecting the thermal microbending coefficient can be compromised in the case of graded index multimode optical fibers.

MODEL AND ANALYSIS

Under the compressive forces at low temperature, optical fibers suffer a buckling deformation. However, fibers will not buckle unless the cooling strain exceeds the mechanical limit set by the Young's modulus of elasticity.

A. Cooling Strain

Using the rule of mixtures, the different thermal expansion coefficients of the fiber coatings can be lumped together into an effective expansion coefficient, \( \alpha_e \), under the form [4]:

\[
\alpha_e = \frac{\sum \alpha_i A_i E_i}{\sum A_i E_i}
\]

(1)

where \( \alpha_i \), \( A_i \) and \( E_i \) are, respectively, the thermal expansion coefficient, cross-sectional area and Young's modulus of elasticity of the ith coating layer.

Due to the thermal expansion coefficients difference, a drop in the temperature by an amount \( \Delta T \), results in an axial contraction strain, \( \varepsilon_t \), which is defined by:

\[
\varepsilon_t = \int_{\Delta T} (\alpha_e - \alpha_i) dT,
\]

(2)

where \( \alpha_i \) denotes the fiber thermal expansion coefficient. In general, \( \alpha_e \) is temperature dependent through the dependence of Young's modulus on temperature. This dependence must be cleared to perform the integration of equation (2) numerically.

B. Loss Increase Due to Buckling

In the case under investigation, a compressive force, \( F \), occurs during the temperature drop. To explain the loss increase, a bending model related to the buckling effect was proposed [5]. In this model, it is assumed that the buckling deformation follows a helical path to which the fiber bending radius is related by:
\[ \rho_b = \frac{R}{\cos \phi} = \frac{R}{2 \varepsilon_d}, \]  

where \( R \) is the spiral radius, \( \phi \) is the angle between the fiber and the normal to the central axis and \( \varepsilon_d \) is the strain difference represented by:

\[ \varepsilon_d = \varepsilon_t - \varepsilon_m, \]  

where \( \varepsilon_m \) is the mechanical strain.

Applying the theory of elastic stability, the equilibrium equation for the forces at low temperature is given by [6]:

\[ E_t \frac{d^4 y}{dz^4} + F \frac{d^2 y}{dz^2} + \kappa y = 0, \]  

where \( E_t \) is the fiber Young's modulus of elasticity, \( I \) is the geometrical moment of inertia and \( \kappa \) is the spring constant determined, under the assumption that other coatings rather than the primary are rigid, from [7]:

\[ \kappa = \frac{4 \pi E_p (1 - \nu_p) (3 - 4 \nu_p)}{(1 + \nu_p) [(3 - 4 \nu_p)^2 \ln \left( \frac{r_p}{r_f} \right) - \left( \frac{r_p - r_f}{r_p + r_f} \right)^2]}, \]  

where \( r_p, E_p \) and \( \nu_p \) are, respectively, the primary radius, Young's modulus of elasticity and Poisson's ratio, and \( r_f \) is the fiber radius.

The periodical solution that satisfies the buckling conditions can be written as:

\[ y = y_{\text{max}} \sin \left( \frac{2 \pi z}{P} \right), \]  

where \( P \) is the spiral pitch. When the solution is submitted to equation (5), one can get the following expression for the minimum force, \( F_{\text{min}} \), that can result in a buckling to the fiber axis:

\[ F_{\text{min}} = \tau_f \frac{4 \pi E_t \kappa}{\rho_b}, \]  

to which the corresponding spiral pitch is:

\[ P_{\text{min}} = \pi r_f \left( \frac{4 \pi E_t \kappa}{\rho_b \rho} \right)^{1/4}. \]  

But, the fiber bending radius, \( \rho_b \), is related to the spiral pitch, \( P \), by the relation:

\[ \rho_b = \frac{P}{2 \pi y/2 \varepsilon_d}. \]  

Therefore, using equation (10), the bending radius, \( \rho_b \), can be rewritten as:

\[ \rho_b = \frac{r_f \pi E_t \kappa^{1/4}}{2 \varepsilon_d^{3/2}}. \]  

The obtained thermal deformation is used to estimate the thermal microbending loss coefficient through the Marcuse formula [8]:

\[ \alpha_m = \frac{\sqrt{\pi} \zeta^2 \exp \left[ -2 \rho_b \gamma^3 / 3 \beta^2 \right]}{e_m \gamma^{15} \sqrt{3 \pi} \rho_b K_{m-1}(\gamma r_c) K_{m+1}(\gamma r_f)}, \]  

where \( \beta \) is the axial propagation constant, \( \zeta \) and \( \gamma \) are, respectively, the transverse propagation constants of the core and the cladding, \( \nu \) is the normalized frequency, \( r_c \) is the core radius and \( K_0(x) \) is the modified Bessel function of the 0th order, with:

\[ e_{\nu} = 2, m = 0 \quad e_{\nu} = 1, m \neq 0 \]

The suffix \( m \) stands for the modal number. Hence, equation (12) calculates the respective loss for each mode. All losses for different modes are then added to each other to give \( \alpha_m \), the overall microbending loss coefficient:

\[ \alpha_{\text{m}} = \sum_{m+1}^{M} \alpha_m. \]  

C. Analysis of Biquadratic-Index Fibers

Thermal microbending loss coefficient of an optical fiber, operating at a certain wavelength \( \lambda \), is controlled by its refractive index profile and its propagation constants. Practically, for a biquadratic-index profile, the core refractive index is defined through two tailoring parameters, \( A \) and \( B \), by:

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\[ n(\rho) = n_0 \left(1 - 2A\rho^3 + 2AB\rho^4\right)^{1/2}. \]  

(14)

where \( \rho \) is the normalized radial position and \( n_0 \) is the refractive index at the core axis. The index mean value, \( n_1 \), is determined from the normalization condition and the cladding index, \( n_2 \), is obtained through \( n_1 \) and the relative refractive index difference, \( \Delta \), between the fiber core and cladding.

The axial propagation constant \( \beta \) is calculated through the perturbation theory, to the third order, as [9]:

\[ \beta^2 = k^2 + \frac{2k_0^2/2A}{\tau_c} + \frac{2B}{\tau^2} \frac{9B^2}{2\tau^2 k_0^2/2A} \frac{9B^2}{8\tau^2 k_0^2}. \]  

(15)

where \( k_0 \) is the magnitude of the wavevector in vacuum.

The transverse components, \( \zeta \) and \( \gamma \), of the propagation constant are defined through [10]:

\[ \zeta = k_0 (n_1 - n_2)^{1/2}, \]  

(16-a)

and

\[ \gamma = \zeta n_2/n_1. \]  

(16-b)

Finally, the normalized frequency, \( V \), is defined by [11]:

\[ V = k_0 \frac{r_c}{r_c (n_1^2 - n_2^2)^{1/2}}. \]  

(17)

The parameters obtained from equations (14-17) are used in equations (12 and 13) to calculate the thermal microbending loss coefficient of the multimode fiber.

RESULTS AND DISCUSSION

In view of our interest in the multimode graded index optical fibers, the value of the normalized frequency, equation (17), must be adjusted within the range \( 2.4048 < V < 5.1356 \), which is sufficient to accommodate the propagation of twelve modes [10]. We present our results for a double coated silica fiber whose primary and secondary coatings are made, respectively, of silicon and nylon. All calculations are performed at a wavelength of 10.6 \( \mu \)m with an operating temperature of 50°C.

The thermal deformation of the fiber represented by \( \rho_0 \), equation (11), is firstly calculated as a function of the temperature drop at certain values of radii of the fiber and its coatings. The obtained results are displayed in Figure (1), where it is clear that the bending radius decreases (i.e. the thermal deformation increases) with the temperature drop. The results displayed in Figure (1) can be explained as follows: when the temperature starts to decrease, the fiber length contracts internally (i.e. no external deformation can be observed). However, after a certain drop in temperature, the strain difference between the fiber and its coatings reaches the point at which the bending start to appear.

The calculated bending radius is used in equation (12) to calculate the thermal microbending loss coefficient, \( \alpha_m \), for each propagating mode at different values of fiber parameters with a temperature drop of 60°C. Figure (2) is a plot for \( \alpha_m \) with the core radius, where it is noted that the microbending loss is concentrated on the fundamental mode with a sharp decrease for higher order modes. This is expected because the transmitted optical power is mainly carried by the low order modes, especially the fundamental one.

The overall microbending loss, \( \alpha_{\text{total}} \), is obtained by adding the losses of the different modes, equation (13).
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The effect of the total number of the propagating modes, \( M \), on the microbending loss is given in Figure (3), where it is clear that a negligible loss is added to the overall loss for \( M > 0 \).

**Figure 2.** Variation of the microbending loss coefficient, \( a_m \), for a propagating mode, \( m \), with the core radius, \( r_c \).

**Figure 4.** Variation of the overall microbending loss coefficient, \( a_M \), with the core radius, \( r_c \). (Effect of fiber radius, \( r_f \)).

**Figure 3.** Variation of the overall microbending loss coefficient, \( a_M \), with the core radius, \( r_c \). (Effect of number of modes, \( M \)).

**Figure 5.** Variation of the overall microbending loss coefficient, \( a_M \), with the core radius, \( r_c \). (Effect of primary coating radius, \( r_p \)).
Figures (4, 5 and 6) confirm the importance of the appropriate design of multimode optical fibers. The core radius and the primary and secondary coatings radii must be chosen to maintain the overall microbending loss as low as possible. To achieve this, thinner fibers and thicker coatings are recommended.

![Graph showing variation of overall microbending loss coefficient with core radius and secondary coating radius.](image)

**Figure 6.** Variation of the overall microbending loss coefficient, $\alpha_m$, with the core radius, $r_c$. (Effect of secondary coating radius, $r_s$).

**REFERENCES**


**CONCLUSION**

Thermal microbending loss was studied for multimode biquadratic-index optical fibers. It was shown that the thermal deformation, and hence the microbending loss, increase with the temperature drop. Geometrical dimensions of the fiber and its coatings have to be adjusted, using the obtained curves, to maintain the microbending loss within the permissible level.