Dispersion Compensation in Transmission Using AFBG Chirped with Two Beam Interference Fringe Spacing

Mohamed Hany, Moustafa H. Aly and M. Nassr

1Arab Academy for Science, Technology and Maritime Transport, Alexandria, Egypt.
2Faculty of Engineering, University of Tanta, Egypt.
3Member of the Optical Society of America (OSA).

Abstract: The dispersion characteristics of linearly chirped apodized fiber Bragg gratings (AFBGs) have been modelled and investigated. The chirping is made using the two beam interference fringe spacing method and its potential as a dispersion compensator in transmission. Eight different apodization profiles are studied including their effects on the performance of the compensator. The positive hyperbolic tangent profile results in an overall superior performance, as it provides a maximum bandwidth and a minimum transmission penalty.

Key words: Apodized chirped fiber Bragg grating, dispersion and dispersion slope, interferometer, photosensitive fiber.

INTRODUCTION

Apodized chirped fiber Bragg gratings have become one of the preferred approaches to dispersion compensation in optical systems. Alternative approaches for using fiber Bragg grating (FBGs) for dispersion compensation have also been researched including the use of unchirped ramped gratings operated in reflection and in transmission[1].

The use of AFBGs in transmission for dispersion compensation has several advantages over using them in reflection. To operate the grating in reflection requires the use of a circulator which introduces losses into the system. The circulator also increases the complexity and the cost of the dispersion compensation unit. In contrast, if an AFBG is used in transmission, the device can be spliced directly into the transmission link. This will avoid the losses due to bulk optical devices such as the circulator [3]. Another problem with using AFBGs in reflection is that the signal optical field must interact strongly with the grating. That is, it must reflect from the grating. This means that any imperfection in the fabrication of the AFBG will have a detrimental impact on its properties as a dispersion compensator. In particular, delay ripple (due to noise in the UV writing process) and polarization mode dispersion introduced into the signal by the grating can severely degrade system performance [3]. In contrast, if the grating is operated in transmission, the interaction between the signal optical field and the grating is much weaker, and hence imperfections in the grating do not become impressed upon the signal field.

Chirped gratings may be fabricated using all the techniques referred to thus far for inscribing conventional Bragg gratings. The first report of chirped grating fabrications was by A. Othonos et al. [3] who used a conventional two beam UV interferometer to produce a uniform period fringe pattern in a tapered photosensitive fiber. In this method, the chirp is achieved by the approximately linear variation of the fiber effective index along the tapered section. In another method with the same uniform period exposure arrangement, chirp can be introduced by bending the fiber with respect to the interference fringes. This, in effect, results in a fringe separation that varies continuously along the exposed fiber length. This method is capable of producing more than 99% reflectivity over a 7.5 nm reflection bandwidth in hydrogen loaded, high germania fiber without incurring short wavelength loss [4].

This paper discusses the use of AFBGs, operated in transmission, as dispersion compensators using a far more flexible and controllable approach to chirped grating fabrication relying on two beam interference and is based on the use of dissimilar curvatures in the interfering wave fronts. This is accomplished by placing two lenses in the two arms of an interferometer. Different apodization profiles and design parameters are studied showing their effects on the performance of the compensator. The best performance is characterized by a short grating (less than 0.5 m)
that provides, simultaneously, a maximum bandwidth and a minimum eye closure penalty.

**THEORY:** The chirping effect on the grating performance as a dispersion compensator for different profiles is analyzed by Kerry Hinton\(^1\). In order to model the operation of an AFBG in transmission, the "effective medium method" is used. The refractive index, at any distance, z, of the AFBG along its length, L, is

\[
n(z) = n_0 + \sigma(z) + 2h(z)\cos\left(\frac{2\pi}{\Lambda} z + 2\phi(z)\right),
\]

(1)

where, \(n_0\) is the background refractive index, L is the nominal Bragg grating pitch, \(s\) is the variation in the average refractive index, \(h(z)\) is the grating apodization profile, \(h\) is the modulation amplitude (> 0) and \(f\) is the grating phase.

Using Eq. (1) in Maxwell's equations and applying perturbation techniques, the grating can be represented as an "effective medium" with an effective refractive index, \(n_{\text{eff}}\), an effective dielectric permittivity, \(\varepsilon\), an effective magnetic permeability, \(\mu\), and an effective local impedance, \(Z\).

A local detuning of a grating, \(d(z, D)\), is defined as \(^1\)

\[
d(z, D) = \Delta + \frac{\pi}{\Lambda} \sigma(z) - \frac{d}{dz} \phi(z),
\]

(2)

where

\[
\Delta(k) = k_n - \frac{\pi}{\Lambda}.
\]

(3)

with \(k(=2\pi/\Lambda)\) the free space wave number of the incident light.

Both medium permittivity and permeability as functions of \(z\) are obtained as

\[
\sigma(z, \Delta) = \delta(z, \Delta) + \kappa(z),
\]

(4)

and

\[
\mu(z, \Delta) = \delta(z, \Delta) - \kappa(z),
\]

(5)

where

\[
\kappa(z) = h(z) \frac{\pi}{\Lambda}.
\]

(6)

Therefore, the effective refractive index, \(n(z, \Delta)\), is

\[
n_{\text{eff}}(z, \Delta) = \sqrt{\varepsilon \mu} = \sqrt{\delta^2(z, \Delta) - \kappa^2(z)},
\]

(7)

and the effective local impedance, \(Z(z, \Delta)\), is

\[
Z(z, \Delta) = \frac{\mu}{\varepsilon} = \frac{\delta(z, \Delta) - \kappa(z)}{\delta(z, \Delta) + \kappa(z)}.
\]

(8)

The dispersion, \(d(k)\), and the dispersion slope, \(d'(k)\), of the grating are given, respectively, by \(^1\)

\[
d(k) = -\frac{2\pi n^2}{\lambda^2 c} \frac{d^2}{d\Delta^2} \arg[t(\Delta)] \text{ ps/nm.}
\]

(9)

and

\[
d'(k) = \left(\frac{2\pi n}{\lambda^2 c}\right)^2 \frac{d}{d\Delta} \arg[K(\Delta)] \text{ ps/nm}^2.
\]

(10)

where the transmission coefficient has an amplitude, \(t(D)\), given by

\[
|t(\Delta)| = |\frac{Z(L, \Delta)}{Z(0, \Delta)}|^2 - e^{-\pi [Z(0, \Delta) - 1]/Z(L, \Delta) - 1)]
\]

(11)

and a phase, \(\varphi(z)\), given by

\[
\varphi(z) = \int_0^z \frac{d}{dz} \left[n_{\text{eff}}(z, \Delta)\right] dz.
\]

(12)

When AFBGs are used, the grating profile, \(h(z)\), grows and decays continuously from and to zero. Therefore, \(K(0) = K(L) = 0\). Hence

\[
Z(0, \Delta) = Z(L, \Delta) = 1.
\]

(13)

Therefore, the transmission coefficient can be rewritten in the form

\[
t(\Delta) = e^{i\varphi}.
\]

(14)

Well away from the reflection band edges, the argument of the transmission coefficient, \(\arg[t]\), can be simplified to

\[
\arg[t(\Delta)] = \text{sgn}(\Delta) \varphi(\Delta),
\]

(15)

at its lowest order, because, \(n_{\text{eff}}\) is real, where

\[
\text{sgn}(D) = +1, \quad \text{for} \quad D > 0, \quad \text{(16-a)}
\]

\[
\text{sgn}(D) = -1, \quad \text{for} \quad D < 0. \quad \text{(16-b)}
\]

This result is precise for AFBGs (i.e. \(h(0) = h(L) = 0\) and is continuous for all \(z\)). The effect of apodization on the grating leads to express the dispersion and the dispersion slope of the compensator, respectively, as \(^1\)
One of the most interesting FBG structures with immediate applications in telecommunications is the chirped fiber Bragg grating (CFBG). This grating has a monotonically varying period.\(^1\)

A CFBG is considered with constant variations in the average refractive index for the length of the grating \(s(z) = 0\). The grating phase, \(\phi(z)\), is assumed to change linearly with the grating length as:

\[
\phi(z) = \phi_0 + az,
\]

where \(\phi_0\) is the starting phase and \(a\) is the linear change in the grating phase. The change of phase along the grating will result in "a".

\[
\frac{d}{dz} \phi(z) = a.
\]

The grating pitch, \(L(z)\), is \(^3\):

\[
\frac{d_1 \cos(\theta_1) + z}{\sqrt{d_1^2 + 2d_1 z \cos(\theta_1) + z^2}} + \frac{d_0 \cos(\theta_0) - z}{\sqrt{d_0^2 - 2d_0 z \cos(\theta_0) + z^2}} = \lambda_B.
\]

where the parameters \(d_1\) and \(d_0\) are the distances from the lens focal point to point \(z = 0\), and \(\theta_1, \theta_0\) are the angles that the respective beams make with the optical fiber. The fringe spacing \(L(z)\) along the fiber axis may be obtained from the geometry of the interfering beams. The Bragg wavelength of the grating is clearly a function of the position \(z\), given by \(^1\):

\[
\lambda_B = 2n_{\text{eff}} \lambda(z).
\]

In this paper, a single cylindrical lens is used in just one arm of the interferometer resulting in a large bandwidth, linearly chirped gratings, where \(d_1 \to \infty\), which corresponds to having no lens in the second arm of the interferometer. It clearly indicates an almost linear variation of the reflected wavelength with distance along the grating. The bandwidth of the chirped grating may be selected by the appropriate combination of lens focal length and position \(^1\). Hence, the grating period, \(L\), in Eqs. (2) and (6) is to be replaced by \(L(z)\).

The effect of changing the apodization profiles with the quadratic dispersion parameter for off resonance grating is investigated using eight different profiles: positive tanh, Cauchy, sin, Gauss, Hamming, sine, raised sine and Blackman. These are respectively displayed in Fig. 1. The apodization functions correspond to well known window functions employed in filter design to suppress side lobes in the rejected band \(^1\). The values of the parameter values which provide an efficient way to control the characteristics of the functions are chosen such that all the profiles have similar characteristics. This is composed with a flat region at the grating center and a constant slope decaying characteristics towards the grating edges. The displayed results show that positive tanh profile is the most successful profile. It has \(h(0) = h(L) = 0\), with \(h(z)\) all continuous and attaining a maximum of \(\kappa(z) = 640 \text{ m}^{-1}\) in a 50 cm grating. It also satisfies the previous terms for selection of apodization profile with a flat region at the grating center which is the highest among the different used profiles and a constant slope decaying characteristics towards the grating edges.

The compensator performance can be measured using its maximum bandwidth, \(D_l\), defined as \(^1\):

\[
\Delta \lambda = \frac{d(k)}{d'(k)}.
\]

Another measure for the performance of the compensator is its eye closure penalty, given, in terms of the transmission link length \(l\), by \(^1\):

\[
R(dB) = 10 \log \left( \frac{1}{1 - g e^2} \right),
\]

where \(g\) is directly proportional to the level of intersymbol interference and is given, as a function of the fiber dispersion, \(D\), and the bit rate, \(B\), by \(^1\).
The dispersion, $d$, of the dispersion compensator is set to negate the impact of fiber dispersion. That is

$$d(k) = -D\ell. \quad (26)$$

Using Eq. (26) together with Eqs. (17) and (25) in Eq. (24), one can get

$$P(dB) = 10 \log \left( \frac{1}{1 - \left(\frac{\pi}{4}\right)^2 \left(\frac{A^2 B^4}{c^2}\right) d^2(k)} \right). \quad (27)$$

This equation gives the eye closure penalty for a chirped AFBG compensator to cancel the impact of fiber dispersion.

**RESULTS AND DISCUSSION**

A computer simulation is performed for the described model for the different apodization profiles versus grating length. For large bandwidth compensation, using a 100 Gbps bit rate, a linearly chirped grating is designed using a single cylindrical lens in just one interferometer arm. This situation is modeled by Eq. (21) with $d_i = \infty$, and a grating designed for a center wavelength at 1550 nm. The chirp rate and grating length and hence the bandwidth may be selected by the appropriate combination of lens focal length and position. The key parameters that determine the limitations of the profile as a dispersion compensator are: the strength of the grating (maximum value of $k(z)$), the grating length, $L$, and the degree of apodization, $F_{rc}$, which is the fraction of the grating length used to increase $k(z)$ from zero to its maximum value.

The effect of the beam 1 angle parameter, $\theta_1$, on the different grating profiles is investigated. When the beam distance, $d_i$, is kept constant (e.g. $d_i = 100$ mm), the obtained values of $k(z)$ showed an alternating variation (like sinusoidal variation) with the beam angle, $\theta_1$, for all apodization profiles. This is in a fair agreement with the published results in Ref. [1].

The values of $\theta_1$ and $d_i$ that give linear detuning, Eq. (2), are used to get the compensator bandwidth, Eq. (23), and eye closure penalty, Eq. (24), for all apodization profiles. As explained earlier, our interest is to find short gratings that provide the widest bandwidth as well as zero eye closure penalty. The bandwidth and eye closure penalty obtained for different profiles are displayed in Figs. 2 through 16.

For the sin profile, Figs. 2 and 3, show a maximum bandwidth of 3.25 nm and exact zero dB eye closure penalty at a grating length $L = 0.03$ m.

![Fig. 2: Dispersion compensator bandwidth against grating length of the sin profile.](image1)

![Fig. 3: Eye closure penalty along the grating of the sin profile.](image2)

Figures 4 and 5 show that more than one grating can be obtained from the raised sine profile at a grating length $L = 0.0275$, 0.0285 and 0.0295, 0.03, 0.031 m with a wide maximum bandwidth of 3.55 nm and zero dB eye closure penalty. The shortest grating length (0.0275 m) is recommended.
Similarly, grating lengths of 0.023, 0.024 and 0.025 m are obtained for the sinc profile with a maximum bandwidth of 3.191 nm at zero dB eye closure penalty Figs. 6 and 7.

The same procedure is repeated for the positive tanh profile and the obtained results are shown in Figs. 8 and 9. It is clear that there is a maximum bandwidth has been obtained as 4.1 nm at zero dB eye closure penalty with a very short grating at grating length $L = 0.005$ m. Longer gratings at the same bandwidth and zero dB eye closure penalty are obtained as 0.006, 0.007, 0.008, 0.009, 0.025, 0.03, 0.035, 0.04 and 0.0425 m.

From Figs. 12 and 13, a maximum bandwidth of 3.74 nm (at and zero dB eye closure penalty) is obtained allowing the design of more than one grating using Gauss profile at grating lengths 0.0275, 0.0285, 0.0295, 0.03, 0.0315 and 0.0325 m.

Fig. 4: Dispersion compensator bandwidth against grating length of the raised sine profile.

Fig. 5: Eye closure penalty along the grating of the raised sine profile.

Fig. 6: Dispersion compensator bandwidth against grating length of the sinc profile.

Fig. 7: Eye closure penalty along the grating of the sinc profile.

Fig. 8: Dispersion compensator bandwidth against grating length of the positive tanh profile.
Results displayed in Figs. 10 and 11 show that very limited grating lengths can be obtained using the Blackman profile as a maximum bandwidth of 1.875 nm at exact zero dB eye closure penalty are obtained only at a grating length \( L = 0.025 \) m, which is smaller than that of all the profiles.

Figures 14 and 15 shows a maximum bandwidth of 3.5 nm (with zero dB eye closure penalty) using the Hamming profile at a grating length \( L = 0.03 \) m. This is wider than that of the sin profile having the same grating length.

Figures 16 and 17 show a narrow maximum bandwidth of 4 nm produced by the Cauchy profile at zero dB eye closure penalty allowing a grating length \( L = 0.029, 0.03 \) m. The Cauchy profile is the only one that approaches the wide bandwidth of the positive tanh profile but without the same flexibility of using more than one grating length.
Fig. 14: Dispersion compensator bandwidth against grating length of the Hamming profile.

Fig. 15: Eye closure penalty along the grating of the Hamming profile.

Fig. 16: Dispersion compensator bandwidth against grating length of the Cauchy profile.

Fig. 17: Eye closure penalty along the grating of the Cauchy profile.

Conclusion: The characteristics of apodized linearly chirped fiber gratings written through the two beam fringe spacing technique have been studied and investigated. It is shown that best apodization profiles have a flat center region and apodized edges with continuously decreasing slopes. The positive hyperbolic tangent profile results in an overall superior performance, as it provides highly linear time delay characteristics with minimum reduction in linear dispersion (as compared with the other apodization cases). This results in compensated fiber links of a maximum bandwidth and a minimum transmission penalty. It has the maximum bandwidth of 4.1 nm and exactly zero dB eye closure penalty at a grating length \( L = 0.005 \) m which is very reasonable practical value for the grating length.

REFERENCES