Tunable Multi-Wavelength Erbium Doped Fiber Laser Using Cascaded Apodized Fiber Bragg Gratings

Islam A. Ashry*, Mohamed M. Keshk#, Moustafa H. Aly# and Ali M. Okaz*

# Faculty of Engineering, University of Alexandria, Alexandria, Egypt.
* College of Engineering and Technology, Arab Academy for Science & Technology & Maritime Transport, Alexandria, Egypt and Member of OSA.

Abstract _ A novel tunable multi-wavelength laser using an erbium doped fiber (EDF) as a gain medium is proposed theoretically. The laser source is terminated by a chirped fiber Bragg grating (CFBG) on one side and cascaded apodized FBGs on the other side. Three and five-wavelength tunable EDF laser (EDFL) are demonstrated using applied strains and temperature variations to tune the lasing wavelengths.

Index Terms _ Apodization, erbium doped fiber laser, fiber Bragg grating, strain and temperature.

I-INTRODUCTION

Narrow-line width, wavelength tunable multi-wavelength fiber lasers operating in the 1550-nm band have important applications in high-capacity communication systems, especially in wavelength division multiplexing (WDM) systems. Various techniques have been proposed to realize tunable multi-wavelength oscillations in erbium doped fiber lasers (EDFLs) [1, 2].

Fiber Bragg gratings (FBGs) are ideal wavelength selection components for fiber lasers due to the unique advantages such as fiber compatibility, ease of use and low cost. Different types of FBGs and different topologies have been used in order to perform simple or more elaborated filtering functions. These include cascaded fiber Bragg grating [3], a FBG written in a high birefringent fiber [4] and a sampled FBG [5]. The Bragg resonance wavelength is altered by changes in applied strain and ambient temperature. This phenomenon can be used to tune the lasing wavelengths of the EDFL.

In this paper, a simple design to a three and five-wavelength tunable EDFL is proposed using a chirped fiber Bragg grating (CFBG), cascaded apodized fiber Bragg gratings and EDF.

II- THEORY

II.1. Reflectivity of FBGs

Consider a uniform Bragg grating formed within the core of an optical fiber with an average refractive index \( \bar{n} \). The refractive index profile can be expressed as [6]

\[
n = \bar{n} + \Delta n \cos \left( \frac{2\pi}{\Lambda} z \right),
\]

where \( \Delta n \) is the amplitude of the induced refractive index perturbation (typical values are \( 10^{-3} \) to \( 10^{-4} \)), and \( z \) is the distance along the fiber longitudinal axis [6]. Using the coupled-mode theory [6–8] that describes the reflection properties of a Bragg grating, the reflectivity of a grating with constant modulation amplitude and period is given by [6]

\[
R(L, \lambda) = \frac{k^2 \sinh^2(sL)}{s^2 \cosh^2(sL) + \delta^2 \sinh^2(sL)},
\]

where \( R(L, \lambda) \) is the reflectivity function of the grating length \( L \) and wavelength \( \lambda, k \) is the absolute value of the coupling coefficient, \( \delta (= \eta \omega / c - \pi \Lambda) \) represents the detuning from the Bragg grating resonance wavelength \( \lambda_{re} (= 2n_{eff} \Lambda), n_{eff} \) is the effective refractive index of the fiber core at the free space center wavelength, \( \Lambda \) is the grating spacing and \( s^2 = k^2 - \delta^2 \). The coupling coefficient, \( k \), for sinusoidal variation of index perturbation along the fiber axis is given by [8]

\[
k(z) = \frac{\eta \pi \Delta n(z)}{\lambda_{re}},
\]

with \( \arg k(z) = \theta(z) + \frac{\pi}{2} \),

\[
arg k(z) = \theta(z) + \frac{\pi}{2},
\]

where \( \eta \) is the fraction of the fiber mode power contained by the fiber core. On the basis that the grating is uniformly written through the core, \( \eta \) can be approximated by \( 1 - V^{-2} \) [6] where \( V \) is the normalized frequency of the fiber. In the uniform gratings, \( k(z) = k = \text{constant} \) because \( \Delta n(z) = \Delta n = \text{constant} \) and \( \theta(z) = 0 \).

There is a variety of methods to compute the reflection and transmission spectra for nonuniform (chirped and apodized) gratings [6, 8]. Here, the most extensively used method called transfer matrix method is presented. The grating is divided into a sufficient number \( N \) of sections so that each section can be approximately treated as uniform. Let the section length be \( \Delta = L / N \). Solving the coupled-mode equations, one gets the transfer matrix relation between the fields at \( z \) and at \( z + \Delta \) as

\[
\begin{bmatrix}
u(z + \Delta) \\
v(z + \Delta)
\end{bmatrix} = M_T \begin{bmatrix} u(z) \\ v(z) \end{bmatrix},
\]

where
\[ M_T = \begin{bmatrix} \cosh(s\Delta) + i\sinh(s\Delta) & \frac{k}{s} - \frac{\sinh(s\Delta)}{s} \\ \frac{k^*}{s} - \frac{\sinh(s\Delta)}{s} & \cosh(s\Delta) - i\sinh(s\Delta) \end{bmatrix} \]

Hence, one can connect the fields at the two ends of the grating through

\[
\begin{bmatrix} u(L) \\ v(L) \end{bmatrix} = T \begin{bmatrix} u(0) \\ v(0) \end{bmatrix}, \quad (6)
\]

where \( T = T_{N,1}, T_{N,2}, \ldots, T_{1,1}, T_{1,2} \) is the overall transfer matrix and \( T_j \) is the transfer matrix written in (5) with \( k = k(j\Delta) \) the coupling coefficient of the \( j \)-th section. As a result, \( T \) is a 2x2 matrix of the form

\[
T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}, \quad (7)
\]

Once the matrix \( T \) is determined, the reflection and transmission coefficients are calculated by the relations

\[
r(L, \lambda) = -\frac{T_{21}}{T_{22}},
\]

and

\[
t(L, \lambda) = \frac{1}{T_{22}}. \quad (8)
\]

**II.2. Grating response**

The Bragg resonance wavelength is altered by changes in applied strain and ambient temperature. The contents in this section are based on Refs. [6, 9].

The shift in Bragg wavelength resulting from a change in these parameters is given by [6]

\[
\Delta \lambda_{B} = 2\left( \Lambda \frac{\partial n_{\text{eff}}}{\partial L} + n_{\text{eff}} \frac{\partial \Lambda}{\partial L} \right) \Delta L, \quad (9)
\]

where \( \Delta L \) is the change in grating length due to applied longitudinal strain and \( \Delta \Lambda \) is the temperature change.

Equation (9) can be separated into two parts, the strain and the temperature sensitivity, and both parts can be separately analyzed.

**II.2.1. Strain Sensitivity**

The effects of an applied strain on the value of the Bragg resonance wavelength are twofold. First, strain variations shift the Bragg wavelength by expanding or compressing the grating, and thus changing the grating period, \( \Lambda \). Second, the applied strain causes a change in the effective index, \( n_{\text{eff}} \), the phenomenon which is commonly known as the strain-optic effect. Therefore, the change in the Bragg wavelength resulting from an applied strain is given by:

\[
\Delta \lambda_{B} = 2\left( \Lambda \frac{\partial n_{\text{eff}}}{\partial L} + n_{\text{eff}} \frac{\partial \Lambda}{\partial L} \right) \Delta L, \quad (10)
\]

The above strain effect term may be expressed as [6]

\[
\Delta \lambda_{B} = \lambda_{B} (1 - p_{\text{eff}}) \varepsilon, \quad (11)
\]

where \( \varepsilon \) is the axial strain and \( p_{\text{eff}} \) is an effective strain-optic constant defined as [6]

\[
p_{\text{eff}} = \frac{n_{\text{eff}}^2}{2} \left[ p_{12} - \nu(p_{11} + p_{12}) \right], \quad (12)
\]

\( p_{11} \) and \( p_{12} \) are components of the strain-optic tensor and \( \nu \) is the Poisson's ratio.

The strain sensitivity \( S_{\varepsilon} \) is given by

\[
S_{\varepsilon} = \frac{\Delta \lambda_{B}}{\varepsilon} = \lambda_{B} (1 - p_{\text{eff}}), \quad (13)
\]

**II.2.2. Temperature Sensitivity**

A change in temperature shifts the Bragg wavelength in two ways. First, thermal expansion (or contraction) changes the grating period \( \Lambda \), and thus changes the Bragg wavelength. In addition, the effective fiber refractive index, \( n_{\text{eff}} \), is temperature dependent, and thus also affects the value of the
Bragg wavelength. Thus, the temperature-induced change in center wavelength of a fiber Bragg grating is given by

$$\Delta \lambda_B = 2 \left( \Lambda \frac{\partial n_{\text{eff}}}{\partial T} + n_{\text{eff}} \frac{\partial \Lambda}{\partial T} \right) \Delta T.,$$

(14)

Substituting $\frac{\partial n_{\text{eff}}}{\partial T} = \zeta_{\text{eff}}$, where $\zeta$ is the thermo-optic coefficient, and $\frac{\partial \Lambda}{\partial T} = \alpha \Lambda$, where $\alpha$ is the coefficient of thermal expansion of the fiber gives

$$\Delta \lambda_B = 2 n_{\text{eff}} \Lambda \zeta \Delta T + 2 n_{\text{eff}} \Lambda \alpha \Delta T.,$$

(15)

which, upon substitution of the Bragg condition, gives

$$\Delta \lambda_B = \lambda_B (\zeta + \alpha) \Delta T..$$

(16)

The temperature sensitivity, $S_T$, is defined as

$$S_T = \frac{\Delta \lambda_B}{\Delta T} = \lambda_B (\zeta + \alpha).$$

(17)

**III. RESULTS AND DISCUSSION**

One will design a tunable multi-wavelength EDFL that produces wavelengths of a band of 13 nm centered about 1550 nm. By the same procedures, one can select the appropriate tuning band for any other communication system. In order to select the appropriate method for tuning, we have to analyze and discuss these methods of tuning from the practical view.

**III.1. Tuning by an applied strain**

For a typical germanosilicate optical fiber $p_{11} = 0.113$, $p_{12} = 0.252$, $v = 0.16$, and $n_{\text{eff}} = 1.48$ [6]. Using these parameters and (13), the anticipated strain sensitivity at ~1550 nm is 12 nm / 1%. For continuous tuning, the Bragg wavelength has to be shifted from 1550 nm. Positive and negative shifts of 6.5 nm through tensile and compressive axial strains are required to cover all the tuning band in our design. Strain sensitivity that gives ±0.542% axial strain is required to cover this bandwidth, Fig. 1. For a fiber diameter 10 µm and germanosilicate modulus of elasticity $E \approx 72.5$ GPa, the force required to achieve 0.542% of axial strain is

$$F = \sigma A_f = E \frac{\pi}{4} d^2 \approx 0.031N.$$  

Such a force is relatively small and can easily be achieved using a conventional stepper motor [10].

**III.2. Tuning by a temperature variation**

For a typical germanosilicate optical fiber the values of thermal expansion coefficient, $\alpha$, and thermo-optic coefficient, $\zeta$, are $0.55 \times 10^{-6} \text{C}^{-1}$ and $8.6 \times 10^{-6} \text{C}^{-1}$, respectively [6]. Substituting these values into (17), the temperature sensitivity at ~1550 nm is 1.4 nm / 100 °C. For continuous tuning, the Bragg wavelength has to be shifted from 1550 nm. Positive and negative shifts of 6.5 nm through an increase and decrease in temperature with respect to the room temperature (25 °C) are required to cover the CFBG bandwidth in our design. Temperature sensitivity that gives $\Delta T = \pm 464.29$ °C is required to cover this bandwidth. Such a temperature difference is essentially impossible to apply in practice. For example, temperature of about 500 °C will damage the fiber coating and may even erase the FBG [10]. Practically, the temperature can be changed from 25 °C to 200 °C easily [11]. This temperature difference range can tune the Bragg wavelength continuously over a range of 2.45 nm, Fig. 2.
liquid nitrogen (77 K) to enable stable lasing at multiple wavelengths and pumped by a laser diode at 980 nm [5].

Using the mentioned transfer matrix method, one can get the reflectivity of the CFBG, Fig. 4. Each sub-section has a different pitch and increases linearly from $z = 0$ to $z = L$. The analysis of the CFBG is based on the following characteristics [1, 6, 11]:

1- Length = $L = 5$ cm.
2- Average reflectivity = -3 dB over 13 nm band centered about 1550 nm.
3- Centered wavelength = $\lambda_c = 1550$ nm.
4- Chirped value = 2.6 nm/cm.
5- Amplitude of index modulation = $\Delta n = 2.7 \times 10^{-4}$.
6- Core diameter = 10 $\mu$m with NA = 0.133.
7- Number of sub-sections = $N = 100$.
8- Effective refractive index = $n_{eff} = 1.48$.

To avoid the cross talk in the tunable multi-wavelength EDFL, one has to make a spacing of 0.8 nm between each tuning range of the cascaded apodized FBGs. For a three output wavelengths, three cascaded apodized FBGs must be used. The Bragg wavelengths of these FBGs are 1545.4, 1550 and 1554.6 nm with tuning ranges of 1543.5–1547.3 nm, 1548.1–1551.9 nm and 1552.7–1556.5 nm, respectively. Each FBG covers a tuning range of 3.8 nm with a spacing of 0.8 nm. A tuning range of 3.8 nm can be covered only using an applied axial strain to shift the Bragg wavelengths of the cascaded apodized FBGs. A -0.158%–0.158% axial strain range is used to cover each tuning range of the FBGs.

Using the described transfer matrix method for the apodized FBG, the reflectivity of the cascaded apodized FBGs at different values of the applied strain is shown in Fig. 5. Similar to the CFBG, the analysis of the cascaded apodized Bragg gratings is based on the following characteristics [1, 6, 11, 12]:

1- Length = $L = 10$ cm.
2- Maximum reflectivity = $R_{max} \geq 99 \%$.
3- Blackman profile is used for apodization.
4- Amplitude of index modulation = $\Delta n = 4.344 \times 10^{-5}$.
5- Core diameter = 10 $\mu$m with NA = 0.133.
6- Effective refractive index = $n_{eff} = 1.48$.

One can increase the number of the output wavelengths to five with a 0.8 nm spacing between each tuning range of the cascaded FBGs. In this case, five of the cascaded apodized FBGs must be used. Each FBG covers a tuning range of 1.96 nm with a spacing of 0.8 nm. This tuning range is small enough to be covered by a temperature variation to shift the Bragg wavelengths of the cascaded FBGs. It can also be covered using an applied axial strain. According to the design using
temperature variation, the Bragg wavelengths of the cascaded FBGs are 1543.5, 1546.26, 1549.02, 1551.78 and 1554.54 nm with tuning ranges of 1543.5–1545.46 nm, 1546.26–1548.22 nm, 1549.02–1550.98 nm, 1551.78–1553.74 nm and 1554.54–1556.5 nm, respectively. A 25–165 °C temperature range is used to cover each tuning range of the FBGs. The laser configuration is illustrated in Fig. 6.

Fig. 6 The schematic of tunable five-wavelength EDFL using temperature.

For the design using an applied axial strain, the Bragg wavelengths of the cascaded FBGs are 1544.48, 1547.24, 1550, 1552.76 and 1555.52 nm with tuning ranges of 1543.5–1545.46 nm, 1546.26–1548.22 nm, 1549.02–1550.98 nm, 1551.78–1553.74 nm and 1554.54–1556.5 nm, respectively. A -0.082%–0.082% axial strain range is used to cover each tuning range of the FBGs. The laser configuration is illustrated in Fig. 7.

Fig. 7 The schematic of tunable five-wavelength EDFL using strain.

The reflectivity of the cascaded FBGs at different values of temperatures and applied strains are displayed in Figs. 8 and 9, respectively. The exact values of the total output power and 3-dB linewidth of the lasing output spectra can be obtained experimentally using an optical analyzer. The main advantage of this configuration is the nearly constant output power and constant 3-dB linewidth expected to the output lasings with different wavelengths.

Fig. 8 Reflectivity of the cascaded FBGs at different values of temperature.
IV- CONCLUSION

In this work, a novel design for a tunable multi-wavelength laser using a CFBG, cascaded AFBGs and an EDF has been proposed. Applied axial strain is used to design a tunable three-wavelength EDFL with a tuning range of 3.8 nm for each output wavelength. Similarly, temperature variations and applied axial strains can be used to design a tunable five-wavelength EDFL with a tuning range of 1.96 nm for each output wavelength. Following the same procedure, one can design a tunable multi-wavelength EDFL with any number of output wavelengths, but keeping in mind that, as the number of output wavelengths increases, one loses a multiple of 0.8 nm spacing between the tuning ranges to avoid the cross talk.

REFERENCES