Numerical simulations of the effect of an isotropic heat field on the entropy generation due to natural convection in a square cavity

Wael M. El-Maghlany a, Khalid M. Saqr b, Mohamed A. Teamah a,b,*

Abstract

Entropy generation associated with laminar natural convection in an infinite square cavity, subjected to an isotropic heat field with different intensities, was numerically investigated for different values of Rayleigh number. The numerical work was carried out using an in-house CFD code written in FORTRAN, which discretizes non-dimensional forms of the governing equations using the finite volume method and solves the resulting system of equations using Gauss-Seidal method utilizing a TDMA algorithm. Proper code validation was undertaken in order to establish the entropy generation calculations. It was found that the increase in the isotropic heat field intensity resulted in a corresponding exponential increase of the entropy augmentation number, and promoted high values of Bejan number within the flow. The entropy generation due to heat transfer was approximately one order of magnitude higher than the entropy generation due to fluid friction. The spatial uniformity of the Bejan number was more sensitive to the change in Rayleigh number than to the heat field intensity. The thermodynamic penalty of the isotropic heat field is shown by means of global integrals of the entropy source terms over the entire flow domain.

© 2014 Elsevier Ltd. All rights reserved.

1. Introduction

Natural convection at high Rayleigh number values is characterized by gravitational–buoyancy interaction, which overruns the viscous effects. The applications of natural convection in different engineering systems are indeed uncountable. The present study is focused on natural convection in an infinite square cavity, which can be reduced to a two-dimensional problem. This problem has several applications, which have been addressed by the authors in a number of recent studies [1–3]. The cooling systems of nuclear reactors are of the most important applications as reported by Jasmin Sudha and Velusamy [4]. The cooling system of Egypt Test and Research Reactor Number 2 (ETRR-2) is an example for such systems studied by El-Messiry [5]. Such system contains components that cool miniaturized parts of the reactor core assembly, which generate high-intensity isotropic heat fields within the cooling domain (i.e., cavity). The application of waste-heat recovery systems, such as thermoelectric systems [6] to these components requires an estimation of the thermodynamic behavior of the heat transfer process, which is radically affected by the presence of the isotropic heat field. Motivated by the entropy generation minimization concept of Bejan [7], the present work aims at investigating the effects of an isotropic heat field on entropy generation in enclosed natural convection flows induced by differential heating. First, we present a brief literature survey to highlight the problem significance and demonstrate the originality of results. Second, the mathematical model and numerical code, which adopts the finite volume method, are presented. The results are given in section three, followed by a discussion and conclusions section.

1.1. Literature review

This section presents a brief discussion of some important studies of entropy generation in cavities under natural convection conditions. Entropy generation in the presence of heat fields and sources was studied in a far less number of papers. Famouri and Hooman [8] showed that the heat transfer irreversibilities resulting from natural convection in a partitioned cavity increases monotonically with Nusselt number. Iliis et al. [9] studied the effect of geometry on the entropy generation in rectangular cavities. They demonstrated that the geometrical aspect ratio has a critical value, below which the local entropy generation increases with the increase of aspect ratio. When such critical aspect ratio is...
increased, the entropy generation decreases with higher values of aspect ratio. A significant number of the studies available on the topic were focused on natural convection in nanofluid. Erbay et al. [10] investigated the entropy generation in square enclosures at low Ra values using a numerical approach comparable to the approach used in the present work. They investigated the flow structure due to differentially heated walls and its effect on the entropy generation pattern for Prandtl numbers ranging from 0.01 to 1.0. They found that the flow configuration had a small effect on changing the active locations of entropy generation in the enclosure. In a recent study, Bouabid et al. [11] investigated the transient entropy generation due to natural convection in an inclined rectangular cavity using numerical analysis. They concluded that entropy generation increases with the increase of thermal Grashof number, irreversibility distribution ratio and aspect ratio of the cavity.

The use of entropy generation minimization method has become popular in heat transfer enhancement techniques. Entropy generation associated with forced convection in an elliptical tube enhanced with longitudinal fins was experimentally investigated by Ibrahim and Moawad [12]. Such research focused on the effects of fin location and geometrical parameters on entropy generation patterns. A similar research aimed at investigating the effect of baffles on the entropy generation in a circular tube was reported by Tandiroglu [13]. The latter work proposed nine correlations to investigate the effect of the isotropic heat field on the sources of entropy generation in a circular tube. The limitations of the scope of previous studies are dominated by natural convection [5,17]. The numerical work was conducted after rigorous verification and validation procedures, which proved that the code solves the governing equations with negligible numerical and modeling errors, respectively.

The main objective of the present work was to establish correlations between the isotropic heat field intensity and local and global entropy generation rates. A secondary objective was to investigate the effect of the isotropic heat field on the sources of entropy generation as represented by Bejan number. These objectives are very important for future research on waste heat recovery in relevant configurations, especially if an emphasis on the heat and mass transfer enhancement is required. The methodology followed to achieve these objectives, via numerical analysis, were greatly built on a comprehensive practical understanding of the verification and validation procedures required for CFD analysis as stipulated in [18].
2. Mathematical model and computational approach

2.1. Mathematical model

The system under consideration is shown in Fig. 1. The temperatures \(T_h\) and \(T_c\) are uniformly imposed along the vertical walls. The top and bottom surfaces are assumed to be adiabatic and impermeable. The working fluid is air (\(Pr = 0.7\)) which is subjected to a positive isotropic heat field of \((Q_a)\) volumetric intensity. Air is mathematically treated as Newtonian and incompressible fluid, and the flow is presumed to maintain hydrodynamic instability in the range of Rayleigh number investigated herein. Constant thermophysical properties are considered for the fluid, except for the density variation due to buoyancy forces, which was determined using the Boussinesq approximation. The dimensionless parameters used to formulate the non-dimensional form of the governing equations are:

\[
X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{UL}{\bar{a}}, \quad V = \frac{VL}{\bar{a}}, \quad P = \frac{PL^2}{\bar{a}^2}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad Ra = \frac{g \beta L^3 (T_h - T_c)}{\nu \bar{a}}, \quad Ec = \frac{\alpha^2}{L^2 \bar{c}_p \Delta T}, \quad Pr = \frac{\nu}{\bar{a}}, \quad \phi = \frac{Q_a L^2}{(\rho \bar{c}_p) \bar{a}}, \quad \Omega = \frac{\Delta T}{T_o}, \quad \xi = \frac{Ec Pr}{\Omega}
\]

(1)

The dimensionless governing equations can then be expressed in vector Cartesian form as:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]

(2)

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)
\]

(3)

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ra Pr \theta
\]

(4)

\[
\frac{\partial \theta}{\partial X} + \frac{\partial \theta}{\partial Y} = \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) + \phi \theta
\]

(5)

The dimensionless boundary conditions for vertical walls are:

\(U = V = 0, \ \theta = 1, \ \text{at} \ X = 0 U = V = 0, \ \theta = 0, \ \text{at} \ X = 1\)

The horizontal boundaries are

At \(Y = 0\) and \(Y = 1\), \(U = V = \frac{\partial \theta}{\partial Y} = 0.0\)

(6)

The stream function is calculated from

\[
U = \frac{\partial \psi}{\partial Y}, \quad V = -\frac{\partial \psi}{\partial X}
\]

(7)

2.1.1. Entropy calculation

In order to calculate the volumetric entropy production rate, the source terms of the entropy transport equation were directly solved as explained below. Given the thermal and velocity fields resulting from the numerical solution of the governing equations, the entropy transport equation was solved. According to Moore [19] and Adeyinka and Naterer [20] the local entropy generation rate \(S_{\text{loc}}\) is composed of thermal dissipation irreversibility \(S_{\text{th}}\) and viscous dissipation irreversibility \(S_{\text{f}}\). These terms can be expressed in dimensionless form, as:

\[
S_{\text{th}} = \frac{S_{\text{th}}}{S_a} = \left( \frac{\partial \theta}{\partial X} \right)^2 + \left( \frac{\partial \theta}{\partial Y} \right)^2 + \frac{\phi \theta}{\Omega}
\]

(8)

\[
S_{\text{f}} = \frac{S_{\text{f}}}{S_a} = Ec Pr \left[ 2 \left( \frac{\partial U}{\partial X} \right)^2 + 2 \left( \frac{\partial V}{\partial Y} \right)^2 + \left( \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right)^2 \right]
\]

(9)

\[
S_a = S_{\text{th}} + S_{\text{f}}
\]

(10)

An important measure of the entropy field is Bejan number (Be) which is defined as the ratio between entropy generations due to thermal irreversibilities to the total entropy generation as follow

\[
Be = \frac{S_{\text{th}}}{S_a} = \frac{S_{\text{th}}}{S_{\text{th}} + S_{\text{f}}}
\]

(11)

Bejan number ranges from 0 to 1. Accordingly, Be = 1 is the limit at which the heat transfer irreversibility dominates, Be = 0 is the opposite limit at which the irreversibility is dominated by fluid friction effects, and Be = 0.5 is the case in which the heat transfer and fluid friction entropy generation rates are equal. Entropy generation rate \(S_{\text{th}}\) would be integrated over the whole domain to obtained the global entropy generation in the square cavity as follow

\[
S_t = \int_0^1 \int_0^1 S_{\text{th}} \, dX \, dY
\]

(12)

Similarity, average Bejan number can be obtained as follow

\[
Be_{av} = \frac{\int_0^1 \int_0^1 S_{\text{th}} \, dX \, dY}{\int_0^1 \int_0^1 S_{\text{th}} \, dX \, dY}
\]

(13)

2.2. Numerical solver details

The in-house CFD code used in the present work solves the governing equations using the finite volume approach proposed by Patankar [21]. The discretization scheme of the governing equations used in the code is based on spatial central differencing. The discretized equations are solved by the Gauss–Seidel method. The iteration method is a line-by-line procedure, which is a combination of the direct method and the resulting Tri Diagonal Matrix Algorithm (TDMA). This code, which was developed in FORTRAN language, was used by the authors in numerous other studies, where more detailed description of the solver and its validation...
measures were presented [2,22–24]. The computational approach included numerical error verification and physical model validation. Firstly the grid size was examined. To resolve the thin boundary layer near the solid boundaries, the size of 5 control volumes close to all boundaries were 1/4 the size of the central control volumes. Three grid sizing with different resolution (30 × 30, 60 × 60 and 100 × 100) were constructed, solved and verified for numerical errors. The deviations between the results for local Nusselt number on the left vertical wall obtained for domain (60 × 60) and (100 × 100) were less than 0.1%, as depicted in Fig. 2. Therefore, throughout this study, the number of grids (60 × 60) was used. The convergence of the iteration is determined by the change in the average Nusselt number as well as other dependent variables through one hundred iterations to be less than 0.01% from its initial value. Fig. 3 shows the convergence and stability of the solution.

2.3. Code validation

In order to check on the validity of the numerical technique employed for the solution of the problem considered in the present study, it was validated with Ilis et al. [9] for square cavity. Fig. 4 plots the iso contours with Pr = 0.7, φ = 0.0, ζ = 10⁻⁴ and Ra = 10³. (a) for present code and (b) Ilis et al. [9]. The figure shows very good agreement between the present code and their results.

3. Results and discussions

3.1. Effects of Rayleigh number and heat field intensity on the flow structure

Fig. 5 shows isocontours of the streamline function for different values of Rayleigh number and the heat field intensity. The effect of the heat field intensity on the streamline function is marginal compared to the effect of Rayleigh number. At low Ra (i.e. Ra = 10⁵) the flow is characterized by central weak rotation, which gets stronger with the increase of Rayleigh number to 10⁶. Further increasing Ra results in a stretching in the central rotation to generate two adjacent and interacting vortices, as shown for Ra = 10⁷. When the value of the heat field intensity increases from 2 to 4, the two vortices concentrate into one vortex near to the cold wall for Ra = 10⁵. For lower values of Rayleigh number, the increase of heat field intensity did not have a significant effect on the flow. This can be attributed to the isotropy of the heat field, which does not differentially contribute to the gravitational–buoyancy force balance causing the flow.

3.2. Effects of Rayleigh number and heat field intensity on the temperature distribution

The temperature response to the change in Rayleigh number and the heat field intensity values is depicted by isothermal lines in Fig. 6. At low Ra (Ra = 10⁵), the conduction regime is dominant and the heat field intensity has strong effect on the temperature distribution, at φ = 0 (no heat field intensity) weak heat transfer rate between the hot left side to the right cold side, this led to sharp heat gradient in the cavity between the two vertical sides within the cavity. On the other hand the isotherms are parallel lines from adiabatic bottom wall to adiabatic upper wall (conduction domain) near the left hot wall depart gradually as the heat field intensity increases. The increasing of heat field intensity at this low Rayleigh number led to high temperature level within all the cavity parts (φ = 8). As the Rayleigh number increases to Ra = 10⁷, the convection mode is pronounced and the effect of heat field intensity on the temperature level in the cavity decreases. When the heat field intensity is increased two orders of magnitude (Ra = 10¹²), the natural convection effect strongly high up and consequently the temperature field exhibits minimal response to the increase of heat field intensity.

3.3. Effects of Rayleigh number and heat field intensity on the entropy generation and Bejan number

The entropy response represented in the local Bejan number to the change in Rayleigh number and the heat field intensity values is depicted in Fig. 7. The Bejan number is used to evaluate the total entropy generation due to heat transfer in the entire domain. Considerable variation of heat transfer and fluid friction irreversibilities in the cavity is depicted with the change of Rayleigh number and heat field intensity. For cavity with low Rayleigh number values (conduction domain i.e., Ra = 10⁵), the heat transfer...
irreversibility is strongly dominant and the total entropy generation increases with the increase of heat field intensity. This is reflected in the local Bejan number values, which approaches unity in all cavity parts mainly at high heat field intensity. As the Rayleigh number increases with small heat field intensity, the heat transfer rate increases and consequently the heat transfer irreversibility decreases due to low temperature gradient variation. On the contrary, the increasing of Rayleigh number led to strengthen vortices flow within the cavity, consequently fluid friction irreversibility is dominant and total entropy generation increases led to decreasing in local Bejan number, this effect at high Rayleigh number is vanished gradually as the heat field intensity increases. The entropy response represented in the average Bejan number to the change in Rayleigh number and the heat field intensity values is depicted in Fig. 8. For low Ra values, the entropy generation is due to heat transfer irreversibility led to average Bejan number closed to unity for all heat field intensity values. As the Ra number increases the entropy generation due to heat transfer irreversibility decreases led to decreasing in average Bejan number. For constant high Rayleigh number, as the heat field intensity increases the average Bejan number increases but with decelerated value. The variations of total entropy generation for various Rayleigh number and heat field intensity are illustrated in Fig. 9. The entropy generations due to heat transfer irreversibility play more important role than fluid friction irreversibility in the cavity with natural convection heat transfer noticeably in the presence of heat field intensity. The results show at small values of heat field intensity, in the range of 0–4 (moderate and low values of nuclear reactor heat generation), as Rayleigh number increases the convective flow starts inside the cavity which increases the viscous entropy generation.

**Fig. 4.** Iso contours for $\phi = 0$, $\zeta = 10^{-4}$, $Pr = 0.7$ and $Ra = 10^3$ (a) Present code and (b) Ilis et al. [9].
Fig. 5. Isocontours of the streamline function as function of $Ra$ and heat field intensity.
Fig. 6. Contours of temperature isothermal lines for different values of Ra and $\phi$. 
The strength of flow increases with the Ra, Consequently both viscous and total entropy generation inside the cavity increases. At high values of heat field intensity, in the range of 6–10, for low values of Ra number the presence of high heat field intensity increases the fluid temperature inside the cavity. The phenomenon of increasing the fluid temperature inside the cavity more than both hot and cold walls was observed by Teamah and El-Maghlany [2]. They also reported that, the fluid temperature decreases with
increasing the value of Rayleigh number. The increase of fluid temperature leads to a high thermal entropy generation. As the Ra number increases the convective flow is pronounced and reduces the temperature of the fluid inside the cavity and the rate of thermal entropy generation is reduced. For further increase in Ra, the flow strength is increased as well as a lower temperature of the fluid is reached. The decrease in fluid temperature reduces the thermal entropy generation. On the other hand the rate of entropy generation due to viscous flow slightly increases and the total rate of entropy generation decreases. For more increase in the Rayleigh number, the rate of entropy generation due to viscous flow strongly increases and the total entropy generation increases again. So, it is recommended for high heat field intensity operation to increase the Rayleigh number values by controlling the cold side temperature value (heat sink) to enhance the heat transfer and also reducing the value of total entropy generation. Fig. 10 represents the entropy generation augmentation factor, this factor represent total entropy generation as a percentage of the zero heat field intensity operation. It is well to minimize this factor to value of unity; this will be achieved by increasing the Rayleigh number value by lowering the heat sink cold side temperature.

3.4. Correlations for average total entropy generation

The predicted values of average total entropy generation over the range of the investigated Rayleigh number and the heat field intensity values are correlated. The correlations are listed in Table 1. The maximum deviations for all correlations are within 7% which is more applicable than the numerical values.
**4. Conclusion**

Entropy generation in natural convection was found to be drastically affected by the superposition of an isotropic heat field. Such effect was found to be global and local, and it was interpreted by depicting the entropy augmentation number as well as Bejan number, respectively. Entropy generation rates showed global responses to the change of the heat source intensity as well as the Rayleigh number. However, the response of the entropy generation field to the change of Rayleigh number was found to rather qualitative compared to the similar change due to the increase in the heat field intensity. The latter seemed to increase the local rates of entropy generation rather than changing their spatial distribution. In addition, the analysis of the average Bejan number revealed that the effect of the heat field is qualitatively equal to the effect of Rayleigh number on the contribution of heat transfer irreversibilities to the entropy generation rates. CFD based empirical were derived by nonlinear regression to describe entropy generation rates in natural convection as a function of Rayleigh number and heat field intensity.

The future work of the present problem should investigate different cavity aspect ratios as well as anisotropic heat fields. The change in aspect ratio would represent different cooling configurations. The anisotropic heat fields would represent the regions where the core is effective in the cooling process (i.e. regions closer to the core). In addition, further analysis is needed for the flow induced by the natural convection in terms of its structure, in order to provide sufficient data for possible waste-heat recovery systems.

**References**


