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asset pricing, relative wealth, pricing kernel, risk free rate, option pricing

JEL: G12, G01, D03

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In the finance literature agents are portrayed as either mainly concerned about the risk in absolute wealth or in both absolute wealth and relative status. In contrast the current paper portrays agents concerned mainly with the risk in relative wealth. Objections to such a portrayal are responded to as follows. There is no discrepancy between preference for higher absolute wealth and preference for higher relative wealth in an economy characterized by certainty. In such a situation there is no conceivable situation where a higher level of one is not associated with a higher level of the other. The utility functions used in finance models are neither *direct* utility functions (the numeraire-single-consumption-good assumption does not render them as such) nor *indirect* utility functions (they cannot be characterized as such just because they are structured about wealth) in the sense used in the certainty case. These functions are essentially structures devised for characterizing attitudes towards risk which have their origins in attempts to solve the famous St.

Petersburg's Paradox. There is no logical rationale for limiting such characterization to concerns over uncertainty of absolute wealth. Faced with uncertainty, survival and fitness might be more important than riskiness of absolute wealth. Relative wealth is a better measure for capturing these concepts than absolute wealth. From another perspective, rational agents must realize that if the paper absolute wealth of all agents increases by about the same percentage, which might be much higher than the actual percentage change in the stream of future real output that is going to materialize, then this in no way can represent an improvement in welfare for any agent.

The current paper explores the impact on asset pricing when agents are mainly concerned about their relative wealth.

Some results from experimental economics are supportive of relative wealth as an important determinant of utility. For example, rejections of offers perceived as unfair in basic 'ultimatum game' experiments¹ can be explained as attempts to avoid deterioration in relative wealth positions. Also, people are found to care more about gains and losses than about absolute levels of wealth.² If one realizes that one agent's gain is probably another agent's loss then this is supportive of relative wealth as a main determinant of utility. Fliessbach et al (2007) find that activity in brain areas "... engaged in the prediction and registration of rewards ... increases with higher relative payments." Using brain scans they test pairs of men performing simple counting tasks. They find that the reward-relevant areas of a player's brain exhibit activation when the player gets the right answer. However, the highest activation occurs when the player gets the right answer and receives a reward while his co-player gets the wrong answer and receives nothing.

De Marzo et al (2008) argue that an agent's concern for relative status is likely to increase during periods of great economic/technological upheavals. They also argue that financial bubbles can be partially explained by agents' concerns over relative wealth because they tend to trade in the same direction as the rest of the crowd out of fear of losing their relative wealth position. Bubbles reflect a major form of

¹ In the 'ultimatum game' two players can divide a given sum of money among themselves. The division process proceeds as follows. A player offers the other a share. If the player receiving the offer accepts what is being offered, the money is split accordingly. If the offer is rejected both players get nothing (e.g. Armantier, 2006).

² See, for example, Kopcke et al (2004).

market inefficiency. Relative wealth concerns, however, can lead to market inefficiency in less drastic ways. An agent might be unwilling to react completely to a new piece of information until he/she sees how other agents are going to react. This need for reinforcement might result in delays in discounting the full effect of the information in prices.

Several researchers incorporate agents' concern for relative status in their respective models as a complement to the fundamental concern for the consumption/wealth process (e.g. Abel, 1990; Gali, 1994; Cole et al, 1995; Bakshi and Chen, 1996; DeMarzo et al, 2008). The role of the relative status variables is usually confined to capturing some partial concern of agents, such as 'keeping up with the Joneses' dispositions.

The rest of the paper is organized as follows. Section 1 develops the general restrictions of the framework and the Relative Wealth CAPM (RWCAPM). Section 2 shows that if, in a binomial state-of-nature framework, the binomial trees for basic securities satisfy the general restrictions presented in section 1 then the pricing of options on those basic securities using the RWCAPM is equivalent to their pricing using the Binomial Option Pricing Model (BOPM). Section 3 provides some concluding remarks.

1. THE MODEL

1.1 THE FUNDAMENTALS OF THE FRAMEWORK

The one-period³ (mathematically but not conceptually) economy consists of n risky assets, a risk-free asset, and m agents. Each agent maximizes the expected utility of end-of-period relative wealth.

Agent i 's maximization program is as follows:

$$\text{Max } E \left[U^i \left(\frac{\left\{ \sum_{j=1}^n \alpha_j^i \tilde{P}_{1j} + R^i (1 + r_f) \right\}}{\sum_{j=1}^n \tilde{P}_{1j}} \right) \right] + \gamma \left[W_0^i - R^i - \sum_{j=1}^n \alpha_j^i P_{0j} \right]$$

³ A continuous time model might be more sophisticated but is unnecessary, at least from this paper's vista, because maximized relative wealth at the end of one period puts the agent in the best possible position for the next period especially if portfolio rebalancing costs are minimal.

P_{1j} represents the end-of-period random payoffs of asset j . The \sim represents randomness.

R^i is the amount that agent i decides to invest in the risk-free asset whose rate of return is r_f .

W_0^i is the beginning-of-period wealth for agent i .

P_{0j} is the beginning-of-period price of asset j .

α_j^i is the fraction of asset j that agent i decides to hold.

γ is the Lagrangian multiplier.

Σ represents the summation operator.

E represents the expectation operator.

Note that the end-of-period random relative wealth⁴ for agent i , RW_1^i , equals:

$$RW_1^i = \frac{\left\{ \sum_{j=1}^n \alpha_j^i \tilde{P}_{1j} + R^i (1 + r_f) \right\}}{\sum_{j=1}^n \tilde{P}_{1j}}$$

The first order conditions with respect to R^i , and α_j^i , respectively, are as follows:

$$E \left[\frac{\partial U^i}{\partial RW_1^i} \frac{(1 + r_f)}{\sum_{j=1}^n \tilde{P}_{1j}} \right] - \gamma = 0 \quad (1)$$

$$E \left[\frac{\partial U^i}{\partial RW_1^i} \frac{\tilde{P}_{1j}}{\sum_{j=1}^n \tilde{P}_{1j}} \right] - \gamma P_{0j} = 0 \quad (2)$$

From equations (1) and (2):

$$E \left[\frac{\partial U^i}{\partial RW_1^i} \frac{(\tilde{P}_{1j} - P_{0j}(1 + r_f))}{\sum_{j=1}^n \tilde{P}_{1j}} \right] = 0 \quad \forall j \quad (3)$$

Multiplying both sides by $\frac{\sum_{j=1}^n P_{0j}}{P_{0j}}$

$$E \left[\frac{\partial U^i}{\partial RW_1^i} \frac{(\tilde{R}_j - r_f)}{(1 + \tilde{R}_m)} \right] = 0 \quad \forall j \quad (4)$$

R_j and R_m are the random returns of asset j and the market portfolio respectively.

Using definition of covariance:

⁴ To simplify notation the \sim symbol, which represents randomness, is not placed over the RW_1^i variable.

$$E\left[\frac{\partial U^i}{\partial RW_1^i}\right] E\left[\frac{\tilde{R}_j - r_f}{1 + \tilde{R}_m}\right] = -Cov\left(\frac{\partial U^i}{\partial RW_1^i}, \frac{\tilde{R}_j - r_f}{1 + \tilde{R}_m}\right) \quad (5)$$

The following list shows $\frac{\partial U^i}{\partial RW_1^i}$ for various utility functions defined over end-of period relative wealth:

| | | |
|----------------------------|---|--|
| <i>Quadratic Utility</i> | $U^i = a^i RW_1^i - \frac{A^i}{2} RW_1^{i2}$ | $\frac{\partial U^i}{\partial RW_1^i} = a^i - A^i RW_1^i$ |
| <i>Exponential Utility</i> | $U^i = -Exp(-A^i RW_1^i)$ | $\frac{\partial U^i}{\partial RW_1^i} = A^i Exp(-A^i RW_1^i)$ |
| <i>Narrow Power U</i> | $U^i = \frac{A^i}{A^i - 1} (RW_1^i)^{\left(\frac{1}{A^i}\right)}$ | $\frac{\partial U^i}{\partial RW_1^i} = (RW_1^i)^{\left(\frac{1}{A^i}\right)}$ |
| <i>Extended Power U</i> | $U^i = \frac{1}{A^i - 1} (a^i + A^i RW_1^i)^{\left(\frac{1}{A^i}\right)}$ | $\frac{\partial U^i}{\partial RW_1^i} = (a^i + A^i RW_1^i)^{\left(\frac{1}{A^i}\right)}$ |
| <i>Logarithmic U</i> | $U^i = a \ln(RW_1^i)$ | $\frac{\partial U^i}{\partial RW_1^i} = \frac{a}{RW_1^i}$ |

Let's begin the analysis assuming each agent has quadratic utility as follows:

$$U^i = a^i RW_1^i - \frac{A^i}{2} (RW_1^i)^2$$

Then:

$$\frac{\partial U^i}{\partial RW_1^i} = a^i - A^i RW_1^i, \quad \text{and} \quad \frac{\partial^2 U^i}{\partial RW_1^{i2}} = -A^i$$

Therefore, from equation (5):

$$E[a^i - A^i RW_1^i] E\left[\frac{\tilde{R}_j - r_f}{1 + \tilde{R}_m}\right] = A^i Cov\left(RW_1^i, \frac{\tilde{R}_j - r_f}{1 + \tilde{R}_m}\right) \quad (6)$$

Agent's i absolute risk aversion parameter, φ^i , is given by:

$$\varphi^i = -E\left[\frac{\partial^2 U^i}{\partial RW_1^{i2}}\right] \Big/ E\left[\frac{\partial U^i}{\partial RW_1^i}\right] = A^i \Big/ E[a^i - A^i RW_1^i]$$

Therefore, from equation (6):

$$(\varphi^i)^{-1} E\left[\frac{\tilde{R}_j - r_f}{1 + \tilde{R}_m}\right] = Cov\left(RW_1^i, \frac{\tilde{R}_j - r_f}{1 + \tilde{R}_m}\right) \quad (7)$$

Summing equation (7) across all m agents in the economy and noting that since

$\sum_{i=1}^m RW_1^i = 1$ then the aggregated covariance term equals zero and noting that

$\sum_{i=1}^m (\varphi^i)^{-1} \neq \text{zero}$:

$$E \left[\frac{\tilde{R}_j - r_f}{1 + \tilde{R}_m} \right] = \text{zero} \quad (8)$$

Another form of equation (8) is:

$$E \left[\frac{(\tilde{P}_{1j} - P_{0j}(1 + r_f))}{\sum_{j=1}^n \tilde{P}_{1j}} \right] = 0$$

Because multiplying both sides by $\frac{\sum_{j=1}^n P_{0j}}{P_{0j}}$ leads to the original form.

From definition of covariance:

$$E[\tilde{R}_j] - r_f = \frac{-Cov\left(\tilde{R}_j, \frac{1}{1 + \tilde{R}_m}\right)}{E\left[\frac{1}{1 + \tilde{R}_m}\right]} \quad (9)$$

The intuition for equation (9) is as follows: $1 + \tilde{R}_j$ is the random payoff at time one from an investment of one dollar in asset j at time zero. $\frac{1}{1 + \tilde{R}_m}$ is the price that

would be recognized, at time one, to have been paid at time zero corresponding to a one dollar payoff from the market portfolio when a given value of \tilde{R}_m materializes.

As the covariance between $1 + \tilde{R}_j$ and $\frac{1}{1 + \tilde{R}_m}$ increases (becomes less negative)

asset j will be more valuable because the payoffs from investing in j would be a better counter to the prices that would have been paid for a given payoff from

the market portfolio. $E\left[\frac{1}{1 + \tilde{R}_m}\right]$, in the denominator of the R.H.S., reconciles the

time frames of the two elements of the covariance because, as shown in subsection 1.2, its inverse is equal to $1 + r_f$.

Multiplying both sides of equation (9) by the weight of asset j in the market portfolio and summing over all n assets we get:

$$E[\tilde{R}_m] - r_f = \frac{-Cov\left(\tilde{R}_m, \frac{1}{1+\tilde{R}_m}\right)}{E\left[\frac{1}{1+\tilde{R}_m}\right]} \quad (10)$$

$$E\left[\frac{1}{1+\tilde{R}_m}\right] = \frac{-Cov\left(\tilde{R}_m, \frac{1}{1+\tilde{R}_m}\right)}{E[\tilde{R}_m] - r_f} \quad (11)$$

Substituting in equation (9) we get:

$$E[\tilde{R}_j] - r_f = \left\{ \frac{Cov\left(\tilde{R}_j, \frac{1}{1+\tilde{R}_m}\right)}{Cov\left(\tilde{R}_m, \frac{1}{1+\tilde{R}_m}\right)} \right\} (E[\tilde{R}_m] - r_f) \quad (12)$$

Equation (12) is the Relative Wealth Capital Asset Pricing Model (RWCAPM).

In brief form the RWCAPM is expressed as follows:

$$E[\tilde{R}_j] - r_f = \chi_{jm} (E[\tilde{R}_m] - r_f) \quad (13)$$

$$\chi_{jm} = \left\{ \frac{Cov\left(\tilde{R}_j, \frac{1}{1+\tilde{R}_m}\right)}{Cov\left(\tilde{R}_m, \frac{1}{1+\tilde{R}_m}\right)} \right\} \quad (14)$$

What about the other utility functions in the list or even any other not listed? If all agents have the same utility function (e.g. they all have an extended power function with a^i and A^i the same across all agents) then the RWCAPM holds approximately.

The reason is that $\sum_{i=1}^m RW_1^i = 1$, i.e. it is fixed. It can be shown with numerical examples that for the condition:

$$E[f(\tilde{X}^i)]E[\tilde{Y}] = -Cov[f(\tilde{X}^i), \tilde{Y}]$$

to apply to each member i of the population (which is the scheme of equation 5)

when $\sum_{i=1}^m \tilde{X}^i$ is constant then $E[\tilde{Y}]$ is usually very small compared to the absolute

value of any single value that Y can assume. That is, $E[\tilde{Y}]$ is *effectively* zero.

All of the above analysis is done starting from the position of individual agents. Many models in finance are built around the construct of a ‘representative agent’ who owns all the wealth and consumes everything. This simplifying construct could be somewhat meaningful when concern is about random absolute wealth. However, it is not very useful when analyzing concern for relative wealth. A representative agent’s relative wealth is not random, it is always one. Nevertheless, for the sake of completeness, it is stated here that carrying out the analysis from a representative agent’s perspective also leads to the RWCAPM. Looking at equation (5) from a representative agent’s perspective it is noted that RW_1 is fixed and hence also is $\frac{\partial U}{\partial RW_1}$ (regardless of the type of utility function) thus the covariance term on the R.H.S. vanishes. The result is equation (8) leading to the RWCAPM.

It is interesting that equation (9) also obtains in the case of a representative agent with logarithmic preferences over absolute wealth. For this case the following equation replaces equation (5)

$$E[\tilde{R}_j - r_f] = \frac{-Cov\left(\frac{\partial U}{\partial W_1}, \tilde{R}_j\right)}{E\left[\frac{\partial U}{\partial W_1}\right]} \quad \text{where } \frac{\partial U}{\partial W_1} = \frac{a}{W_1}$$

Multiplying both the numerator and denominator by the value of aggregate wealth at time zero leads to equation (9) and hence to the RWCAPM. It is difficult though to ascertain the significance of this fact. Maximizing the expected logarithm of wealth is argued for by many researchers (e.g. Kelley, 1956; Bell and Cover, 1980; Evstigneev et al, 2008) as the best strategy for survival and dominance⁵. However, this strategy is advocated from the perspective of individual agents not from that, of course, of a representative agent.

⁵ Sinn and Weichenrieder (1993) write “...nature links the generational risks not according to an additive, but according to a multiplicative function,” and a multiplicative function can be transformed into an additive one by taking logarithms.

1.2 FURTHER DETAILS

1.2.1

It is evident from equation (9) that $1/(1 + \tilde{R}_m)$ represents the pricing kernel (or stochastic discount factor) M_t underlying the model. Equation (9) is a special case of the well known relationship for a pricing kernel:

$$E[\tilde{R}_j - r_f] = -\frac{\text{Cov}(M_1, (\tilde{R}_j - r_f))}{E[M_1]} \quad (15)$$

From equation (9) using definition of covariance:

$$E[\tilde{R}_j] - r_f = \frac{-E\left[\frac{\tilde{R}_j}{1 + \tilde{R}_m}\right] + E[\tilde{R}_j]E\left[\frac{1}{1 + \tilde{R}_m}\right]}{E\left[\frac{1}{1 + \tilde{R}_m}\right]} \quad (16)$$

$$r_f = \frac{E\left[\frac{\tilde{R}_j}{1 + \tilde{R}_m}\right]}{E\left[\frac{1}{1 + \tilde{R}_m}\right]} \quad (17)$$

Similarly from equation (10) above:

$$E[\tilde{R}_m] - r_f = \frac{-E\left[\frac{\tilde{R}_m}{1 + \tilde{R}_m}\right] + E[\tilde{R}_m]E\left[\frac{1}{1 + \tilde{R}_m}\right]}{E\left[\frac{1}{1 + \tilde{R}_m}\right]} \quad (18)$$

$$r_f = \frac{E\left[\frac{\tilde{R}_m}{1 + \tilde{R}_m}\right]}{E\left[\frac{1}{1 + \tilde{R}_m}\right]} = \left\{ \frac{1}{E\left[\frac{1}{1 + \tilde{R}_m}\right]} \right\}^{-1} \quad (19)$$

Equation (19) provides a relationship between the risk-free rate and moments of the probability distribution of market returns. Equation (19) can be rewritten as follows:

$$1 + r_f = \frac{1}{E\left[\frac{1}{1 + \tilde{R}_m}\right]} \quad \text{where} \quad 0 < E\left[\frac{1}{1 + \tilde{R}_m}\right] \leq 1 \quad (20)$$

Thus the simple gross risk-free return is the (weighted) harmonic mean⁶ of the probability distribution of simple gross market returns.

Equation (20) is derived based on the premise that the risk-free rate is set mainly by direct interaction between economic agents in their investment activities. Obviously, this is not the case. Institutional intervention (by central banks, treasury departments, and financial intermediaries) has great impact on setting this rate. Against this more realistic backdrop equation (20) may be construed as providing a guide for setting the risk-free rate given an analysis of market perceptions. The interventions might be looked upon as a force channeling the equilibrium in a certain direction. An intervention based on changing the level of the risk-free-rate may or may not be successful in changing expectations of market performance. If it is unsuccessful an intervention might end up mainly changing the perceived market risk. The following example seeks to clarify this point.

Suppose the currently anticipated performance of the market is captured by the following probability distribution (equal probabilities) of simple gross market returns:

0.8, 0.9, 1, 1.1, 1.2, 1.3, 1.4. This implies that:
 $E[1 + \tilde{R}_m] = 1.1$, $E[\tilde{R}_m] = 0.1$, $Std.Dev.(\tilde{R}_m) = 0.2$, $r_f = 6.27\%$, $Sharpe\ ratio = 0.19$

Also suppose that the central bank reasons that lowering the risk-free rate to 3.9% would boost the economy. This boost should be reflected in changed anticipations of market performance.

Success Scenario:

The intervention would be successful if the probability distribution of simple gross market returns changes to: 0.63, 0.72, 1.1294, 1.3106, 1.4, 1.42, 1.44.

$E[1 + \tilde{R}_m] = 1.15$, $E[\tilde{R}_m] = 0.15$, $Std.Dev.(\tilde{R}_m) = 0.3165$, $r_f = 3.9\%$, $Sharpe\ ratio = 0.35$

Failure Scenario:

The intervention would be relatively unsuccessful if the probability distribution of simple gross market returns changes to: 0.67, 0.87, 1.06, 1.13, 1.22, 1.32, 1.45.

⁶ The weights of the harmonic mean are the relevant probability measures. $\frac{1}{1 + \tilde{R}_m}$ can be interpreted as the present value of one dollar to be received at the end of the period using R_m as the discount factor. $E\left[\frac{1}{1 + \tilde{R}_m}\right]$ is the mean of these present values.

$$E[1 + \tilde{R}_m] = 1.1, E[\tilde{R}_m] = 0.1, Std.Dev.(\tilde{R}_m) = 0.247, r_f = 3.9\%, Sharpe\ ratio = 0.25$$

With the help of equation (20), equation (8) can be recast as

$$E\left[\frac{\tilde{P}_{1j}}{\sum_{j=1}^n \tilde{P}_{1j}}\right] - \frac{P_{0j}}{\sum_{j=1}^n P_{0j}} = zero$$

That is, in equilibrium the expected relative contribution of asset j to aggregate wealth at time one is equal to its certain relative contribution at time zero. This is the result of interactions between agents all of whom assess utility over relative wealth.

A general restriction of the model is:

$$E\left[\frac{r_f}{1 + \tilde{R}_m}\right] = E\left[\frac{\tilde{R}_m}{1 + \tilde{R}_m}\right] = E\left[\frac{\tilde{R}_j}{1 + \tilde{R}_m}\right] = \frac{r_f}{1 + r_f} \quad \forall j \quad (21)$$

The last equality in equation (21) results from equation (20). Thus

$$E\left[\frac{1 + r_f}{1 + \tilde{R}_m}\right] = E\left[\frac{1 + \tilde{R}_m}{1 + \tilde{R}_m}\right] = E\left[\frac{1 + \tilde{R}_j}{1 + \tilde{R}_m}\right] = 1 \quad (22)$$

The expected value of the ratio of payoffs from a one dollar investment in any asset and a one dollar investment in the market portfolio is the same for all assets and equals one. This is the result of relative wealth being the emphasis of agents and their need for reassurance that their individually chosen portfolio composition will not adversely affect their relative wealth position.

Equations (20) and (22) show that $\frac{1}{1 + \tilde{R}_m}$, as a pricing kernel M_1 , satisfies the

following additional well known relationships:

$$E[M_1] = \frac{1}{(1 + r_f)} \quad (23)$$

$$E[M_1(1 + \tilde{R}_j)] = 1 \quad (24)$$

Assume agents recognize that they will redo their optimization programs at the start of each period. Thus equation (20) is expected to prevail at the start of each period. This makes it possible to recast the unbiased (or the liquidity-premium augmented) expectations hypothesis of the term structure of interest rates. The

current forward rates are derived (partially) through agents' anticipations of the harmonic mean of the probability distribution of future simple gross market returns.

1.2.2

A Taylor expansion of $\frac{1}{1+\tilde{R}_m}$ around zero yields:

$$\frac{1}{1+\tilde{R}_m} = 1 - \tilde{R}_m + \tilde{R}_m^2 - \tilde{R}_m^3 + \tilde{R}_m^4 - \tilde{R}_m^5 + \dots$$

The pricing kernel underlying the RWCAPM is “highly nonlinear” and decreasing in R_m . These characteristics of a pricing kernel are advocated by several researchers (e.g. Dittmar, 2002). Using the Taylor expansion in equation (12) indicates that the excess return of asset j does not depend only on the covariance of its return with R_m , as in the basic CAPM, but also on the coskewness, cokurtosis, and all the higher co-moments. All of these co-moments⁷, are encompassed by covariance with $\frac{1}{1+\tilde{R}_m}$. Noting that the covariance in the denominator of χ_{jm} is

always negative, it can be seen that a market populated by agents mainly concerned about their relative wealth shows aggregate preference for low covariance, cokurtosis and all other even co-moments, between R_j and R_m , while it prefers high co-skewness and all other odd co-moments - in line with Scott and Horvath (1980). Chi for an asset, χ_{jm} , can be greater than, equal, or less than its beta β_{jm} depending on the interplay between the co-moments beyond covariance in both the numerator and denominator of χ_{jm} . The relationship between the two parameters, which results from simple statistical manipulations, is as follows:

$$\chi_{jm} = \frac{\frac{E[\tilde{R}_m]E\left[\frac{\tilde{R}_j}{1+\tilde{R}_m}\right] - E[\tilde{R}_j\tilde{R}_m]}{E\left[\frac{1}{1+\tilde{R}_m}\right]Var(\tilde{R}_m)} + \beta_{jm}}{\frac{E[\tilde{R}_m]E\left[\frac{\tilde{R}_m}{1+\tilde{R}_m}\right] - E[\tilde{R}_m^2]}{E\left[\frac{1}{1+\tilde{R}_m}\right]Var(\tilde{R}_m)} + 1} \quad (25)$$

⁷ The importance of higher moments, especially skewness and kurtosis, has been highlighted by many researchers, e.g. Kraus and Litzenberger (1976) and Jondeau and Rockinger (2006).

Thus χ_{jm} equals β_{jm} if:

$$E[\tilde{R}_j \tilde{R}_m] = \frac{E[\tilde{R}_m] E\left[\frac{\tilde{R}_j}{1+\tilde{R}_m}\right]}{E\left[\frac{1}{1+\tilde{R}_m}\right]} \quad \text{and} \quad E[\tilde{R}_m^2] = \frac{E[\tilde{R}_m] E\left[\frac{\tilde{R}_m}{1+\tilde{R}_m}\right]}{E\left[\frac{1}{1+\tilde{R}_m}\right]}$$

Or, equivalently, if:

$$\text{Cov}\left(\tilde{R}_j \tilde{R}_m, \frac{1}{1+\tilde{R}_m}\right) = \text{Cov}\left(\tilde{R}_m, \frac{\tilde{R}_j}{1+\tilde{R}_m}\right) \quad \text{and} \quad \text{Cov}\left(\tilde{R}_m^2, \frac{1}{1+\tilde{R}_m}\right) = \text{Cov}\left(\tilde{R}_m, \frac{\tilde{R}_m}{1+\tilde{R}_m}\right)$$

If these conditions are satisfied then the effects of the co-moments between R_j and R_m , beyond covariance, cancel each other out in both the numerator and denominator of χ_{jm} .

Whether or not χ_{jm} equals β_{jm} depends on the return distribution of each individual asset and its interplay with the return distribution of the market portfolio. In this paper's framework there is no dominant parameter whose adjustment leads to complete congruence with the basic CAPM for *all* assets as is the case, for example, in Gali's (1994) 'basic equivalence result' which states that "...equilibrium asset prices and returns in an economy with externalities are identical to those of an externality-free economy with a properly adjusted degree of risk aversion." In fact, as discussed below, risk aversion parameters play no observable role in aggregate in the relative wealth framework.

1.2.3

It is interesting to ponder the absence of an aggregate risk aversion factor in equation (9). Such a factor is very much present in the corresponding equation that materializes when the outcome variable of interest is wealth rather than relative wealth. The analysis in Appendix B shows that the aggregate ARA factor acts in two offsetting ways. First, it is a pricing factor for the covariance between an asset's returns and the reciprocals of the simple gross market returns. Second, it is a reducing factor for the magnitude of this same covariance. These offsetting actions lead to a constant effective aggregate ARA (equal to 1). This fits well with the fact that the aggregate relative wealth invested in risky assets is constant at a value of one (thus aggregate relative risk aversion equals aggregate ARA). The analysis in the Appendix B also shows that the absence of the risk aversion factor occurs not

only at the aggregated level but also for the individual agent when there is a single risky asset and a single risk-less asset.

Thus in the relative wealth framework risk aversion is mainly a motivator at the individual agent level for calibration of risk premiums *across* assets until each asset satisfies equation (9). The importance of this calibration is intuitively the result of relative wealth being the emphasis of each agent and his need for reassurance that his individually chosen portfolio composition will not adversely affect his relative wealth position. In the relative wealth framework the aggregate risk aversion factor is not assigned the job of determining the market risk premium since the risk free rate is determinable endogenously as shown in sub-subsection 1.2.1.

1.2.4

The current paper is theoretical in nature and no empirical tests are attempted but some remarks concerning such testing are made next.

Empirical estimates for χ_{jm} for an asset j can be obtained by regressing

$$\tilde{R}_j \text{ on } \frac{1}{1 + \tilde{R}_m} \quad \text{and regressing } \tilde{R}_m \text{ on } \frac{1}{1 + \tilde{R}_m} \quad \text{and dividing the}$$

slope coefficient of the first regression by the slope coefficient of the second regression.

In empirical undertakings it might be a good idea to include a measure of the returns to human capital in conjunction with the usual return on the market portfolio in the estimates of the return on aggregate wealth⁸. Dittmar (2002) notes that: "... pricing kernels improve substantially relative to the case in which human capital is not included in the measure of aggregate wealth."

Finally, the RWCAPM might perform better empirically with data related to time periods that correspond to financial crisis and/or economic upheavals. These are the periods during which the RWCAPM is likely to come closer to representing the structure of asset pricing. Several researchers note that during periods of financial instability assets' return distributions exhibit large deviations from normality and/or symmetry (e.g. Pownall and Koedijk, 1999; Lillo and Mantegna, 2000; Consigli, 2002). As shown with numerical examples in Appendix A, for skewed distributions

⁸ R_m in all the equations of this paper would thus represent the return on aggregate wealth.

the χ_{jm} factor is significantly higher than the β_{jm} factor for negatively skewed distributions and significantly lower for positively skewed distributions.

2. EQUIVALENCE OF RWCAPM AND BOPM

In a binomial state-of-nature framework if the binomial trees for basic securities (including the risk-free bond) satisfy the general restriction presented in equations (21) and (22) then the pricing of options on those basic securities using the RWCAPM is equivalent to their pricing using the Binomial Option Pricing Model (BOPM).

Let the following represent the binomial tree for basic security j :

P_{1jU} the higher price at end of period , probability of this state is q

P_{0j}

P_{1jD} the lower price at end of period , probability of this state is $(1-q)$

The price of a call option C on security j is determined using the BOPM as follows:

$$C_0 = \frac{\left(\frac{(1+r_f) - \frac{P_{1jD}}{P_{0j}}}{\frac{P_{1jU}}{P_{0j}} - \frac{P_{1jD}}{P_{0j}}} \right) C_{1U} + \left(\frac{\frac{P_{1jU}}{P_{0j}} - (1+r_f)}{\frac{P_{1jU}}{P_{0j}} - \frac{P_{1jD}}{P_{0j}}} \right) C_{1D}}{(1+r_f)}$$

Where:

C_{1U} is the higher option payoff and C_{1D} is the lower option payoff.

Algebraic manipulation leads to:

$$C_0 = \left(\frac{P_{0j} - \frac{P_{1jD}}{(1+r_f)}}{P_{1jU} - P_{1jD}} \right) C_{1U} + \left(\frac{\frac{P_{1jU}}{(1+r_f)} - P_{0j}}{P_{1jU} - P_{1jD}} \right) C_{1D}$$

On the other hand, the price of the call option using the RWCAPM is determined as follows:

$$C_0^* = q \frac{C_{1U}}{1+R_{mU}} + (1-q) \frac{C_{1D}}{1+R_{mD}}$$

Where:

R_{mU} and R_{mD} are the returns of the market portfolio in the state with probability q and the state with probability $(1-q)$ respectively.

Thus:

$$C_0 = C_0^* \quad \text{iff} \quad \frac{q}{1 + R_{mU}} = \frac{P_{0j} - \frac{P_{1jD}}{(1+r_f)}}{P_{1jU} - P_{1jD}} \quad \text{and} \quad \frac{1-q}{1 + R_{mD}} = \frac{\frac{P_{1jU}}{(1+r_f)} - P_{0j}}{P_{1jU} - P_{1jD}}$$

Since basic securities satisfy the general restriction in equations (21) and (22)

$$P_{0j} = q \frac{P_{1jU}}{1 + R_{mU}} + (1-q) \frac{P_{1jD}}{1 + R_{mD}}$$

$$\frac{1}{1 + r_f} = q \frac{1}{1 + R_{mU}} + (1-q) \frac{1}{1 + R_{mD}}$$

Plugging the above two expressions in $\frac{P_{0j} - \frac{P_{1jD}}{(1+r_f)}}{P_{1jU} - P_{1jD}}$ and $\frac{\frac{P_{1jU}}{(1+r_f)} - P_{0j}}{P_{1jU} - P_{1jD}}$ leads to

the proof that pricing using the BOPM is equivalent to that using the RWCAPM.

3. CONCLUSION

The present paper theoretically explores the implications for asset pricing if economic agents build their utility assessments mainly around relative wealth. This portrayal need not be conceptually problematic. There is no conceivable state of nature wherein higher relative wealth does not imply higher absolute wealth. The difference between relative wealth concerns and absolute wealth concerns materializes only in case of uncertainty due to probability weighing. Faced with uncertainty, survival and fitness might be more important than riskiness of absolute wealth. Relative wealth is a better measure for capturing these concepts than absolute wealth. Elevated concern for relative wealth is also likely to exist in periods of financial crisis and/or economic upheavals. The severe deviations from normality and/or symmetry that return distributions exhibit during such periods could be caused by herd behavior motivated by increasing concern for relative wealth positions.

All the well known simplifying assumptions of the basic one-period CAPM are retained but the results are different. Instead of the covariance of an asset's return with the return on the aggregate wealth being the driver of that asset's risk premium now it is the covariance of the asset's return with the reciprocal of the simple gross return on aggregate wealth. The reciprocal of the simple gross return on aggregate wealth represents the pricing kernel underlying the model.

Several areas of possible future developments come to mind: investigation of the implications of the model for portfolio compositions, exploration of the case wherein some agents assess their utility mainly over relative wealth whereas others assess their utility mainly over absolute wealth, and analyzing the case of heterogeneous preferences.

APPENDIX A

This appendix provides an example that invokes a simple economy to illustrate the mathematics of the model.

Assume there are two assets, A and B , in the economy. The weight of asset A in the market portfolio is 0.5 and that of asset B is 0.5. The following table shows the probability distributions (equal probabilities) of the simple gross returns of assets A , B , and the market portfolio.

| Asset A | Asset B | Market Portfolio |
|----------------------------------|-------------------------------------|-------------------------------------|
| 0.6 | 0.75 | 0.675 |
| 0.9 | 0.8 | 0.85 |
| 1.1 | 0.9 | 1 |
| 1.25 | 0.95 | 1.1 |
| 1.35 | 1.2 | 1.275 |
| 1.45 | 1.55 | 1.5 |
| 1.5 | 2.295 | 1.8977 |
| Mean = 1.1643 Std. Dev. = 0.3 | Mean = 1.2065 Std. Dev. = 0.5123 | Mean = 1.1854 Std. Dev. = 0.3834 |

Note that the first cell in the column for asset A and the last cell in the column for asset B were adjusted so that the economy satisfies equation (22). The return distribution for asset A is skewed to the left. That for asset B is skewed to the right. The following results can be easily obtained.

$$E\left[\frac{1}{1+\tilde{R}_m}\right] = 0.935 \quad r_f = 0.0695$$

$$E[\tilde{R}_A] - r_f = 0.0948 \quad E[\tilde{R}_B] - r_f = 0.137 \quad E[\tilde{R}_m] - r_f = 0.1159$$

$$\frac{E[\tilde{R}_A] - r_f}{E[\tilde{R}_m] - r_f} = 0.818 \quad \frac{E[\tilde{R}_B] - r_f}{E[\tilde{R}_m] - r_f} = 1.182$$

The chi and beta factors for both assets are as follows:

$$\chi_{Am} = 0.818 \quad \beta_{Am} = 0.6568$$

$$\chi_{Bm} = 1.182 \quad \beta_{Bm} = 1.2936$$

APPENDIX B

Equation (3) is restated below:

$$E \left[\frac{\partial U^i}{\partial RW_1^i} \frac{(\tilde{P}_{1j} - P_{0j}(1+r_f))}{\sum_{j=1}^n \tilde{P}_{1j}} \right] = 0 \quad \forall j$$

From the definition of covariance:

$$\begin{aligned} E \left[\frac{\partial U^i}{\partial RW_1^i} \frac{1}{\sum_{j=1}^n \tilde{P}_{1j}} \right] E[\tilde{P}_{1j} - P_{0j}(1+r_f)] &= -Cov \left(\frac{\partial U^i}{\partial RW_1^i} \frac{1}{\sum_{j=1}^n \tilde{P}_{1j}}, (\tilde{P}_{1j} - P_{0j}(1+r_f)) \right) \\ \left(E \left[\frac{\partial U^i}{\partial RW_1^i} \right] E \left[\frac{1}{\sum_{j=1}^n \tilde{P}_{1j}} \right] + Cov \left(\frac{\partial U^i}{\partial RW_1^i}, \frac{1}{\sum_{j=1}^n \tilde{P}_{1j}} \right) \right) E[\tilde{P}_{1j} - P_{0j}(1+r_f)] &= -Cov \left(\frac{\partial U^i}{\partial RW_1^i} \frac{1}{\sum_{j=1}^n \tilde{P}_{1j}}, \tilde{P}_{1j} \right) \end{aligned}$$

Assuming quadratic utility:

$$\begin{aligned} \left(E[a^i - A^i RW_1^i] E \left[\frac{1}{\sum_{j=1}^n \tilde{P}_{1j}} \right] - A^i Cov \left(RW_1^i, \frac{1}{\sum_{j=1}^n \tilde{P}_{1j}} \right) \right) E[\tilde{P}_{1j} - P_{0j}(1+r_f)] &= \\ -a^i Cov \left(\frac{1}{\sum_{j=1}^n \tilde{P}_{1j}}, \tilde{P}_{1j} \right) + A^i Cov \left(\frac{RW_1^i}{\sum_{j=1}^n \tilde{P}_{1j}}, \tilde{P}_{1j} \right) \end{aligned}$$

Divide both sides by A^i :

$$\begin{aligned} \left([\varphi^i]^{-1} E \left[\frac{1}{\sum_{j=1}^n \tilde{P}_{1j}} \right] - Cov \left(RW_1^i, \frac{1}{\sum_{j=1}^n \tilde{P}_{1j}} \right) \right) E[\tilde{P}_{1j} - P_{0j}(1+r_f)] &= \\ -\frac{a^i}{A^i} Cov \left(\frac{1}{\sum_{j=1}^n \tilde{P}_{1j}}, \tilde{P}_{1j} \right) + Cov \left(\frac{RW_1^i}{\sum_{j=1}^n \tilde{P}_{1j}}, \tilde{P}_{1j} \right) \end{aligned}$$

Aggregating across all n agents:

$$\begin{aligned} \left[\sum_{i=1}^m \varphi^{i-1} \right] E \left[\frac{1}{\sum_{j=1}^n \tilde{P}_{1j}} \right] E[\tilde{P}_{1j} - P_{0j}(1+r_f)] &= \\ -Cov \left(\frac{1}{\sum_{j=1}^n \tilde{P}_{1j}}, \tilde{P}_{1j} \right) \left(\sum_{i=1}^m \frac{a^i}{A^i} \right) + Cov \left(\frac{1}{\sum_{j=1}^n \tilde{P}_{1j}}, \tilde{P}_{1j} \right) & \\ = Cov \left(\frac{1}{\sum_{j=1}^n \tilde{P}_{1j}}, \tilde{P}_{1j} \right) \left(-\sum_{i=1}^m \frac{a^i}{A^i} + 1 \right) \end{aligned}$$

$$E[\tilde{P}_{1j} - P_{0j}(1+r_f)] = \left[\sum_{i=1}^m \varphi^{i-1} \right]^{-1} \left(-\sum_{i=1}^m \frac{a^i}{A^i} + 1 \right) \frac{\text{Cov}\left(\frac{1}{\sum_{j=1}^n \tilde{P}_{1j}}, \tilde{P}_{1j} \right)}{E\left[\frac{1}{\sum_{j=1}^n \tilde{P}_{1j}} \right]}$$

Dividing both sides by P_{0j} and multiplying R.H.S. by $\frac{\sum_{j=1}^n P_{0j}}{\sum_{j=1}^n P_{0j}}$

$$E[\tilde{R}_j - r_f] = \left[\sum_{i=1}^m \varphi^{i-1} \right]^{-1} \left(-\sum_{i=1}^m \frac{a^i}{A^i} + 1 \right) \frac{\text{Cov}\left(\frac{1}{1+\tilde{R}_m}, \tilde{R}_j \right)}{E\left[\frac{1}{1+\tilde{R}_m} \right]}$$

Thus equation (9) is obtained since $\sum_{i=1}^m \varphi^{i-1} = \left(\sum_{i=1}^m \frac{a^i}{A^i} - 1 \right)$

Next assume that there is a single risky asset and one risk-free asset. The minimum risk premium needed to induce an agent to invest all his wealth in the risky asset⁹ can be found from the following relationship which is a direct result of equation (3) applied to this special case

$$E\left[\frac{\partial U^i}{\partial R W_1^i} \left(\frac{W_0^i(1+\tilde{R})}{V_0(1+\tilde{R})} \right) \frac{(\tilde{R} - r_f)}{V_0(1+\tilde{R})} \right] \geq 0 \quad \text{where } W_0^i \text{ is the invested wealth of agent } i.$$

Since there is no uncertainty about the (positive) marginal utility

$$E\left[\frac{(\tilde{R} - r_f)}{(1+\tilde{R})} \right] \geq 0 \quad \text{and using the definition of covariance}$$

$$E[\tilde{R} - r_f] \geq \frac{-\text{Cov}\left(\tilde{R}, \frac{1}{(1+\tilde{R})} \right)}{E\left[\frac{1}{(1+\tilde{R})} \right]}$$

⁹ The development here follows steps similar to Huang and Litzenberger (1988).

This is similar to equation (9).

An intuitive explanation for the disappearance of the risk aversion factor in is included in sub-section 1.2.3.

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