Fractal Barker Codes for Efficient Channel Estimation for Wavelet OFDM Systems

Radwa K. Hamad, Ehab F. Badran and Amira I. Zaki
Department of Electronics and Communication Engineering, Arab Academy for Science and Technology
Alexandria, Egypt
radwa_hamad,ebadran,amzak10@aast.edu

Said E. El-Khamy
Department of Electrical Engineering, Faculty of Engineering, Alexandria University
Alexandria, Egypt
elkhamy@ieee.org

Abstract

In this paper a new wideband channel estimation technique is presented. This new technique is based on using fractal Barker codes and the multi-scale decomposition of the wavelet filter bank. In this technique new Barker codes named fractal Barker codes are generated and used as inputs to the different stages of wavelet based orthogonal frequency division multiplexing (WOFDM) transmitter. This results in a wideband fractal output training sequence at the transmitter. This fractal training signal is used to accurately estimate the characteristics of a multipath frequency selective fading channel. In doing so, the complex path gains can be obtained with accuracy through the use of only a few samples of the transmitted and received training sequences. The delays for each path can be calculated using the transmitted and received fractal Barker codes. This channel estimation technique allows for accurate calculations without the need for pilot interpolations as with other conventional techniques. The performance of this new synthesized pilot signal is tested with different wavelet filters for its generation in the transmitter. The simulations and results show how the fractal training signal is generated and how the wideband and fractal nature of the introduced training sequence results in a channel estimation technique with a reasonable percentage of error in addition to the simplicity of the technique as no pilot interpolation is needed.

1. Introduction

WOFDM has the same advantages as the conventional OFDM but out performs it because of the fact that the usage of a mother wavelet to achieve the orthogonality of the subcarriers eliminates the need for the cyclic prefix and in doing so increases the bandwidth available for actual data transmission and thus improves the bandwidth efficiency as shown in [1] (transactions).

The need for channel estimation is essential making it a popular research field and explains the reason for existence of a wide range of channel estimation techniques. Accurate calculation of the channel's path gains and individual path delays introduced by multipath frequency selective channels allows for correct reception of data at the receiving end. In [2] (proceedings) a different method for the generation of Barker code based pilot signal for WOFDM channel estimation was introduced. Despite the advantages of the method presented in [2], it only succeeded to estimate the delay for a single path channel but failed to perform the accurate estimation for a channel with different multipath components. The spectral properties of the generated pilot signal in [2] were not wideband or fractal. In this paper, however, a new method of generation of fractal Barker codes that results in fractal pilot signal that is of a Multi-band/wideband nature.

The paper organization is as follows, first the WOFDM system is reviewed. The concept of fractal Barker codes is introduced and is examined thoroughly along with its unique spectral and fractal properties and the proposed system modifications in section III along with complete explanation of the method with which the complete channel transfer function will be deduced. Section IV will introduce and discuss the simulation results. Finally, the last section will be dedicated to listing the conclusions.

2. WOFDM Basic Principles

OFDM is regarded as a multi-carrier transmission technique which is particularly useful for frequency selective channels and high data rates explained in [3] (transactions). This technique transforms a frequency-selective wideband channel into a group of non-selective narrowband channels, making it perfect for use by systems that are subjected to channels with large delay spreads. The transmitted OFDM signal maintains its orthogonality in the frequency domain. The assurance that the orthogonality of each of the used subcarriers is sustained through the use of a cyclic prefix in the beginning of each of the transmitted OFDM symbol. This cyclic prefix uses a part of the bandwidth which could otherwise be used to transmit data and this is considered a waste of useable bandwidth thus decreasing the bandwidth efficiency.

978-1-880843-98-7 / copyright ISCA, CATA 2015
March 9-11, 2015, Honolulu, Hawaii, USA

WOFDM works the same way as the OFDM in terms of principle but instead of using inverse fast Fourier transform (IFFT) in order to generate the orthogonal subcarriers as is the case in OFDM, the inverse discrete wavelet transform (IDWT) is used instead. The advantage of doing so is that the maintaining of the orthogonality of the subcarriers is automatically achieved due to the nature of the wavelet families and does not need the addition of any cyclic prefix. This means that the entire OFDM block can be used for data transmission and so bandwidth is more efficiently employed as illustrated in [4] (proceedings). The OFDM symbols generated by the IFFT have frequencies which are multiples of the center frequency f_o , thus the OFDM output components s (p) are obtained as follows

$$s(p) = d(p)e^{jk\,2®f_o t} \quad (1)$$

where d(p) denotes the p^{th} data sample. WOFDM achieves orthogonality using the orthogonality property of the wavelet family which states that such a family can be obtained by translating and scaling the wavelet mother function $\Psi(t)$ as shown [5] (proceedings), [6] (transactions)

$$\Psi_{j,k}(t) = ¢^{-j/2}\Psi(¢^{-j}\cdot t - k²) \quad (2)$$

Equation (2) corresponds to a sampled version of a wavelet family, where δ is the scaling factor and τ is the translation factor. Since $\Psi_{j,k}(t)$ forms an orthonormal, family any signal s(t) can be written as a weighted sum of wavelet functions (4), the weights being given by the wavelet coefficients ($w_{j,k}$) as stated

$$s(t) = \sum_{j\in Z} w_{j,k}\Psi_{j,k}(t) \quad (3)$$

The signal s(t) can be interpreted as a WOFDM symbol. Any wavelet from a certain level j can be composed as a weighted sum of scaled version of the mother wavelet function from the previous level j-1. In equation (3) $w_{j,k}$ are the wavelet coefficients. The discrete version of the signal, s[n] is calculated by performing the IDWT. At each iteration, an up sampling operation is performed with a factor 2 and followed by two quadrature mirror filters. The output of each iteration x[n] can be described by

$$x[n] = \sum_{k} (w[k]h[n-2k] + a[k]g[n-2k]) \quad (4)$$

We can consider that the data we have to transmit is a set of coefficients as follows

$$s[n] = \{[a_L],[w_L],[w_{L-1}],.....[w1]\} \quad (5)$$

The input data which is usually a set of modulated data bits is considered as the wavelet coefficients in WOFDM systems [7]. The data sequence is brought to the input of the IDWT processor, whose output will be the discrete version of the WOFDM symbol. After each iteration the number of wavelet coefficients halves (because each iteration will rely on a down-sampling and filtering couple). Thus, considering that the data vector from (5) has *N* samples, then half of them will be stored in $[w_1]$ and they will be transmitted in the channel using the upper half of the dedicated bandwidth.

Next, $[w_2]$ contains one quarter of the symbols to be transmitted. Finally, considering the last iteration performed 2^{J-L} symbols will be grouped in the approximations vector and an equal number will be stored in the details vector. The number of signal samples composing a WOFDM is limited to N samples. Since WOFDM relies on an IDWT modulator, it follows that the transmission performance can be influenced by the parameters of this modulator such as the choice of wavelet filter. Figure 1 shows the complete block diagram of a conventional WOFDM system.

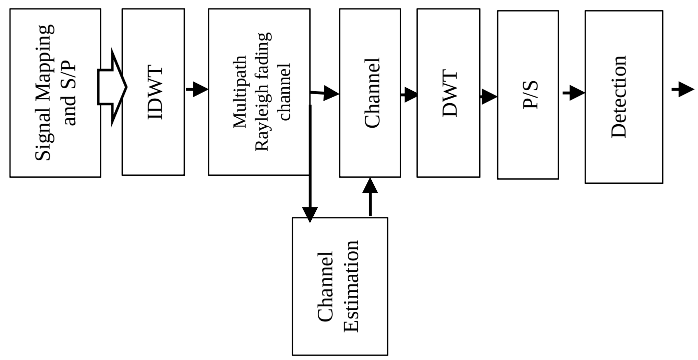

Figure 1: Block diagram of a WOFDM system

3. The Proposed System Model

The proposed system model introduces changes to more than one constituent of the conventional WOFDM system. The first proposed change is in the pilot generation stage where fractal Barker 13 codes replace conventional pilots. These fractal Barker codes then replace conventional inputs in both the IDWT and the DWT blocks [7] (book).

A. Fractal Barker codes

Barker codes are finite length codes that have optimum correlation characteristics for the peak of the autocorrelation of a Barker code of any length is equal to the length of that Barker code as shown in [8],[9] and [10] (transactions). The fractal Barker codes used in this paper are fractal Barker codes of length 13. The first fractal version of the Barker 13 code is of a vector of length 64 and that is first pilot signal and is named pilot_1 and is shown in figure 2(a). The second scaled version is a vector of length 128 and is named pilot_2 and is shown in figure 2(b), the third and fourth fractal versions are of two vectors of length 256 and 512 respectively and are named pilot_3 and pilot_4 and are shown in figures 2(c) and 2(d) respectively. The construction of a Barker code vector of length 128 is achieved by decreasing the sampling time of each sample in the 64 length code vector by half. This procedure is repeated when the Barker codes of lengths 256 and 512 are constructed as well. Figure 2 shows each fractal version in the time domain. This novel method of generating this fractal versions of an otherwise conventional code provided the spectrum shown in figure 3.

The spectrum of each of the fractal Barker codes is inversely proportional to its sampling time. The spectrum of the first constituent of the pilot signal which is the Barker code vector of length 64is shown in figure 3a and has half the bandwidth of the spectrum of the Barker code vector of length 128 as shown in figure 3b, figure 3c shows that the spectrum of the Barker code vector of length 64 has one fourth the bandwidth of that of the Barker code vector of length 256 and one eighth the bandwidth of the Barker code vector of length 512 as shown in figure 3d.

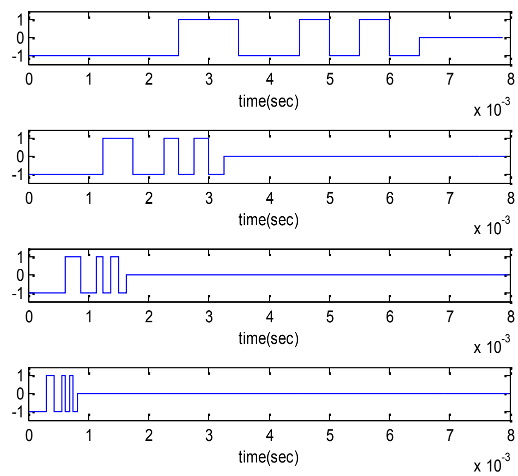

Figure 2 : (a, b, c, d): Time domain pilot signals (Fractal Barker codes)

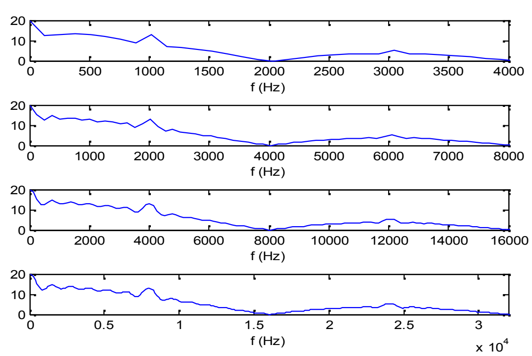

Figure 3: Frequency spectrum of the fractal Barker codes

B. Proposed System changes

The block diagram of this paper's proposed system model is shown in figure 5 but the conventional IDWT and DWT blocks are replaced by the two blocks shown in figures 6 and 7 respectively. For the proposed algorithm the fractal Barker codes explained in section III and shown in figure 2 are the inputs at the different stages at the transmitting end $w_4(n)$, $w_4(n)$, $w_3(n)$, $w_2(n)$ and $w_1(n)$ are shown in figure 6. Where, the Barker code vector of length 64 is the input marked $w_4(n)$ The Barker code vector of length 128 is the input marked $w_3(n)$ *and* the Barker code vector of length 256 is the input marked $w_2(n)$ and a Barker code vector of length 512 is the input marked $w_1(n)$. This arrangement generates a 1024 bit training sequence signal $s(n)$ for transmission in the channel [11] (proceedings)explain the signal channel's response. Figure 5 shows the reconstruction process and the retrieval of the training sequence using this paper's proposed DWT filter banks. The IDWT and DWT blocks play a crucial role in the shape of the output signal and that shape depends on the mother wavelet used. In the proposed system model four different wavelet filter were used and the spectrum of the output pilot in each case was studied thoroughly in order to determine the spectrum best suited for channel estimation. The quadrature mirror filters shown in figure 5 were changed four times during the simulations. The four studied filters were Daubechies 4, biorthogonal 3.3, Symlet 6 and Haar filters.

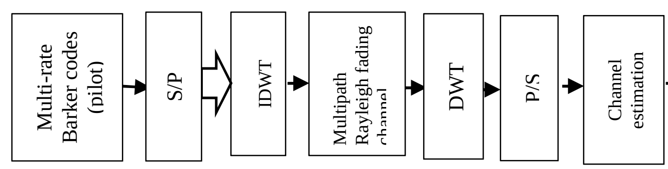

Figure 4: Proposed WOFDM System Model

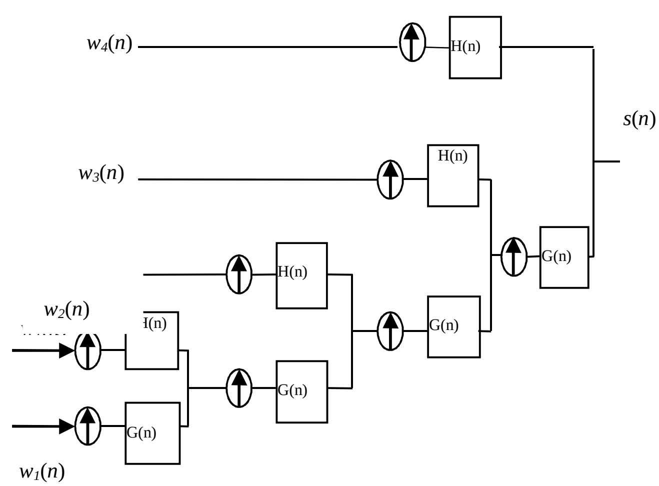

Figure 5: Fractal Barker codes (pilots) as inputs to the conventional IDWT block

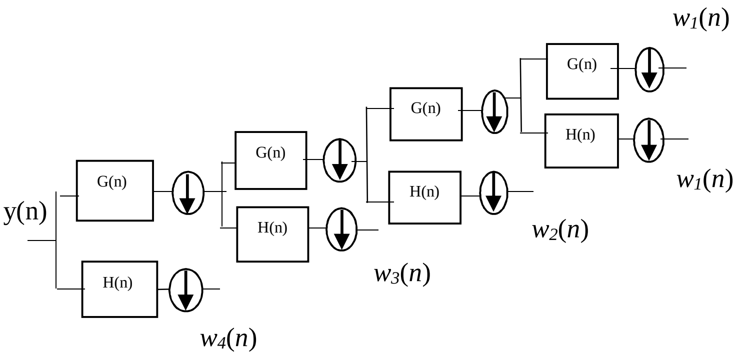

Figure 6: Reception of the Fractal Barker codes using DWT

4. Simulation Results

The proposed fractal training sequence is subjected to a two path slow fading Rayleigh channel having a Doppler shift of 50 Hz and sampled with a frequency of 128 KHz. The desired spectrum of the training sequence needs to be fractal and wideband, several types of wavelet filters were examined at the transmitter to select the wavelet filter that produces the best fractal, wideband training sequence.

The results obtained when Haar, Bior3.3, Symlet 6 and Daubechies 4 filters were used are plotted in figures 6, 7, 8 and 9 respectively. It can be deducted easily from these figures that the Daubechies 4 filter generated the best training spectrum in terms of clear peaks and fractal characteristics. It has an almost constant fractal measure (main lobe to minor lobe ratio). Its first main lobe to side lobe gave a ratio of 0.36, its second main lobe to side lobe ratio gave a ratio of 0.32, while the third and fourth main lobes to side lobe ratios gave 0.36 and 0.33 respectively

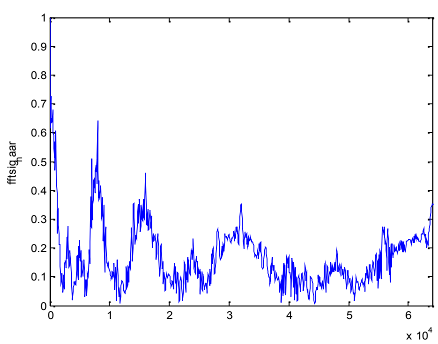

Figure 7: The spectrum of the constructed pilot when Haar filter is used

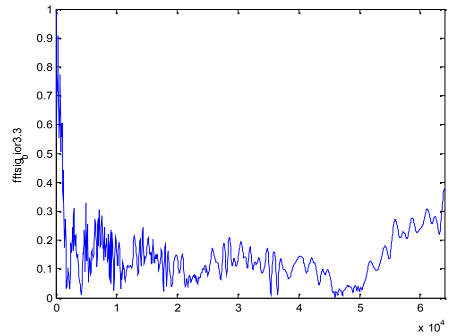

Figure 8: The spectrum of the constructed pilot when Bior3.3 filter is used

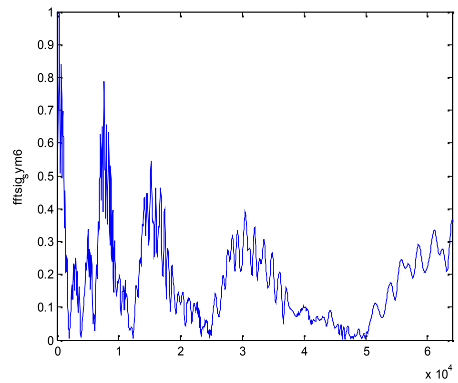

Figure 9: The spectrum of the constructed pilot when Symlet 6 filter is used

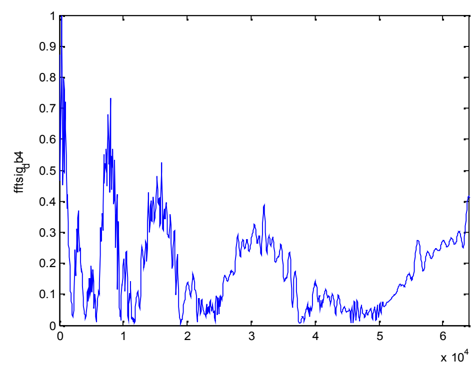

Figure 10: The spectrum of the constructed pilot when Daubechies 4 filter is used

The training sequence obtained when the Daubechies 4 filter is used for the remainder of the presented simulations because it has the best fractal measure. The spectrum of the combined signal shows that these novel multi-scaling technique used in the generation of the Barker codes created a wideband fractal spectrum that is ideal for channel estimation. The generated pilot allows for detection of all the characteristics of a fading multipath channel without the need for any pilot interpolation. This is due to the wideband nature of the pilot signal. Figure 10 shows the transmitted (top) and received (bottom) spectrums. It can be seen that despite of the multipath effect of the channel and the Doppler introduced by that channel, the received signal maintained its fractal relations and wideband characteristics even in the case of severe multipath.

Difference between path delays (in symbol time)	Path 1 delay percentage error	Path 2 delay percentage error
0.5	19.215%	21.92%
1	3.25%	3.52%
2	2.702%	2.546%
3	2.61%	2.632%
4	2.014%	1.798%
5	1.837%	1.545%
6	1.621%	1.541%
7	1.504%	1.493%
8	1.398%	1.441%
9	1.39%	1.42%

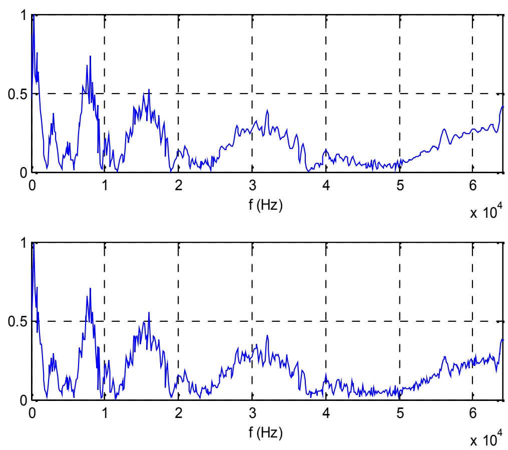

Figure 10: The spectrum of transmitted and received training sequences

Once the training signal is received, the reciprocal of the process at the transmitting end is done and the signal goes through the DWT wavelet filter bank shown in figure 5 in order to obtain each of the fractal Barker codes once more. Figure 11 shows the output of the DWT different stages [12] (proceedings) and [13] (transactions) show the partial channel estimation after the DWT process.

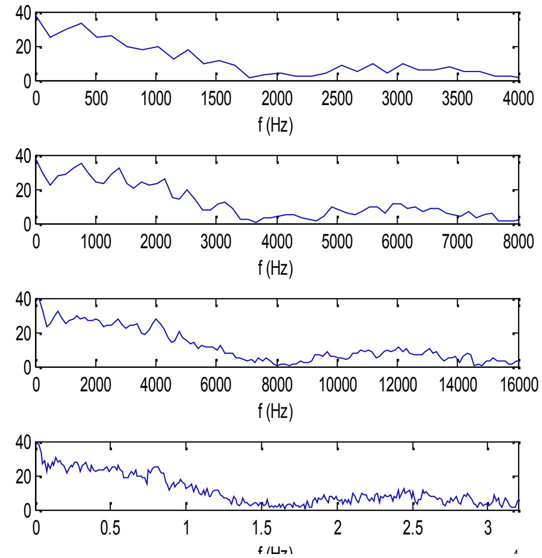

Figure 11: The output of the different stages of the DWT

After the obtainment of the fractal Barker codes at the DWT stage, each of these received fractal codes is then correlated with its transmitted counterpart. Each of these four correlations results in three distinctive peaks each of them corresponding to one of the three path of the channel. The difference between the locations at which each of these peaks occur at each individual correlation is used to obtain the delays introduced by the paths of the channel. A combination method between the results obtained at each scale is used to further improve the result and obtain the least percentage of error when calculating the path delays. The results of each of these correlations are shown in figure 12.

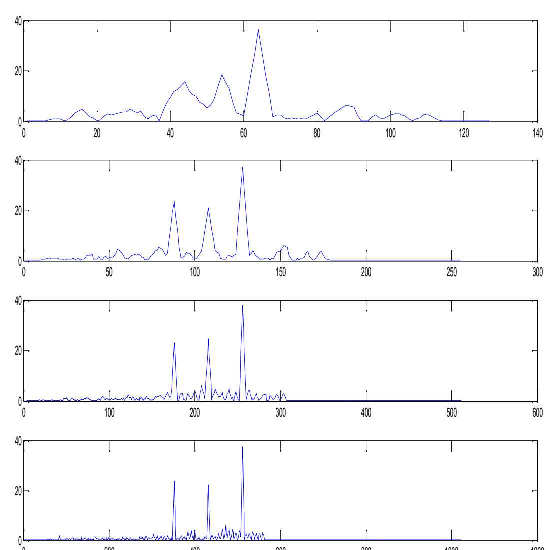
Figure 12: The correlation between each of transmitted and received fractal Barker codes

The delay introduced by path 1 at each scale is obtained by subtracting the locations of the first two peaks and multiplying that difference by the sampling time. However, the delay introduced by path 2 at each scale is obtained by subtracting the locations of the first and third peak and multiplying the difference by the sampling time. While the difference obtained from the correlation of the two 64 length Barker codes is directly multiplied by the sampling time for each of the two delays, however the differences obtained from the correlation of each of the Barker 128, Barker 256 and Barker 512 length codes is divided by 2, 4 and 8 respectively.

Then the delays for paths one and two obtained from each of the scales are then tested in an attempt to find the estimated delay values for each of the paths that yield minimum error for each of the two paths. Table 1 shows the percentages of error obtained for each of the two paths versus the difference between the paths delays.

The proposed estimation technique of the complex path gains were also obtained through the usage of a few samples from the transmitted and received fractal pilot. The path gains can be obtained by solving the following equation.

$$\begin{bmatrix} y(0)\\ y(1)\\ y(2)\\ \vdots \\ y(N)\end{bmatrix} = \begin{bmatrix} s(0)&0&0&\cdots&0\\ s(1)&s(0)&0&\cdots&0\\ s(2)&s(1)&s(0)&\cdots&0\\ \vdots&\vdots&\vdots&\cdots&\vdots\\ s(N)&s(N-1)&s(N-2)&\cdots&s(0)\end{bmatrix}\begin{bmatrix} g(0)\\ g(1)\\ g(2)\\ \vdots\\ g(N)\end{bmatrix} \quad (6)$$

Where y(n) are the received signal samples and s(n) are the transmitted signal samples. g(0)….g(n) are the required path gains, in this paper we only have two paths and in solving these N equations there will only be two values for the complex path gains. Table II shows the percentage errors obtained when estimating the path gains introduced by the channel.

TABLE 1: PATH GAINS PERCENTAGE ERRORS

Path Type	**Percentage error in path gain**
Direct	**0.562%**
First	**0.7%**
Second	**1.525%**

5. Conclusion

In this paper a new fractal Barker code based pilot technique was introduced which uses the advantages of Barker codes mixed with the advantages of both wavelets and fractals. The fractal Barker codes created were used as inputs to the WOFDM IDWT transmitter. The generated wideband fractal pilot signal made the channel estimation a relatively simpler task since no pilot interpolation is needed for an accurate estimation. The simulations were done when different wavelet filters were used in the pilot synthesizing phase. The Daubechies four proved to be the best filter for the task because it generated a pilot signal with an almost constant fractal measure. The generated fractal pilot proved to be resilient and the received pilot maintained its fractal nature at the receiving end. The channel delays were then estimated from the correlations between the transmitted and received fractal Barker codes at every scale after the reception of the fractal Barker codes. After the path delays are estimated at every scale , a novel combination scheme is introduced which is used in order to increase the accuracy of the delay estimations for each of the multipath components by taking advantage of the fractal nature of the codes. The complex path gains were then estimated and the results were compared with the ones used when creating the channel model and the error proved to be very reasonable.

References

[1] C. J. Ahn, S. Takahashi. Performance Evaluation of an OFDM using Unitary Matrix Frequency Modulation for Power Line Communications over Impulsive Noise Channels. *WSEAS Transactions on Communications*, Issue 3, Volume 5, pp. 454-459, March 2006.

[2] R.K.Hamad, E.F.Badran, A.I.Zaki, S.El.Khamy. Channel Estimation Using Fractal Barker Codes for WOFDM Based Systems. *Proceedings of the 29 CATA conference*, pp.121-126, Vegas Nevada, March 2014

[3] M. Mahmoud, M. Riad, OFDM System under the Combined Effect Analysis of the Phase Noise, Carrier Frequency Offset, Doppler Spread, and Amplifier Nonlinearity of the SEL. *WSEAS Transactions on Communications*, Issue 7 , Volume 7, pp. 719-728, July 2008.

[4] M. Oltean, M. Nafornita. Efficient Pulse Shaping and Robust Data Transmission Using Wavelets. *Proceedings of the third IEEE International Symposium on Intelligent Signal Processing, WISP 2007*, pp. 43-48, Alcala de Henares, Spain, October 2007.

[5] M. K. Lakshmanan, H. Nikookar. *A review of wavelets for digital wireless communication*. Wireless Personal Communications, Springer Netherlands, Volume 37, No. 3-4, pp. 360-430, May 2006.

[6] M. K. Ozdemir, H. Arslan. Channel estimation for wireless OFDM systems. *IEEE Communication Surveys*, Vol. 9, No 2, 2007.

[7] Abbas Hasan Kattoush, Waleed A. Mahmoud, S. Nihad. *The performance of multiwavelets based OFDM system under different channel conditions*. Digital Signal Processing, 2009

[8] S. EL. Khamy. On generation, correlation and Spectrum measurement of Barker codes. *IEEE INT.J.ELECTRONICS*, VOL.50, NO. 5, 353-361, 1981

[9] S.EL. Khamy. Identification of dispersive communication channels using Barker codes. *IEEE INT.J.ELECTRONICS*, VOL.50, NO.4, 299-304, 1981

[10] S. EL. Khamy. On generation and measurement of combined Barker codes. *IEEE INT. J. ELECTRONICS*, VOL.50, NO.6, 429-436, 1981.

[11] Marius Oltean, Miranda Nafornita. Errors per Scale Statistics for a Wavelet OFDM Transmission in Flat Fading Channels. *6th IEEE International Symposium on Intelligent Signal Processing*, pp. 119-124, Aug 2009.

[12] Volkan Kumbasar, Oguz Kucur. Alamouti Coded Wavelet based OFDM for Multipath Fading Channels. *Wireless Telecommunications Symposium*, pp. 1-5, April 2009.

[13] Sinem Coleri, Mustafa Ergen,Anuj Puri, Ahmad Bahai. A study of channel Estimation in OFDM systems. *IEEE*. 2002