

Nonlinear Modeling of the Stochastic Errors of MEMS Inertial Sensors Utilized in Smart Phones

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Abstract—A robust nonlinear modeling technique based on Fast Orthogonal Search (FOS) is introduced to remove MEMS-based inertial sensor stochastic errors inside smart mobile phones that are used for several purposes including location based services and device usage classification. The proposed method is applied to MEMS-based gyroscopes and accelerometers. Results show that the proposed method models low-cost MEMS sensors errors with no need for de-noising techniques and, using smaller model order and less computation, outperforms traditional methods by two orders of magnitude.

Keywords—Inertial Navigation; Nonlinear System Identification; Fast Orthogonal Search; Adaptive Signal Processing

I. INTRODUCTION

Presently, GPS-enabled mobile devices offer various positioning capabilities to pedestrians, drivers, and cyclists. GPS provides absolute positioning information, but when signal reception is attenuated and becomes unreliable due to multipath, interference and signal blockage, augmentation of GPS with inertial navigation systems (INS) or the like is needed.

GPS/INS integrated navigation systems are extensively used [1], e.g. in mobile devices that require low-cost Micro-Electro-Mechanical-System (MEMS) inertial sensors (gyroscopes and accelerometers) due to their low cost, low power consumption, small size, and portability. The inadequate long – term performance of most commercially available MEMS-based INS limits their usefulness in providing reliable navigation solutions. MEMS are challenging in any consumer navigation system because of their large errors, extreme stochastic variance, and quickly changing error characteristics.

According to [2], the inertial sensor errors of a low cost INS consist of two parts: a deterministic part and a random part. The deterministic part includes biases and scale factors, which are determined by calibration and then removed from the raw measurements. The random part is correlated over time and is basically due to the variations in the INS sensor bias terms. Therefore, these errors must be modeled. This paper offers a robust method based on fast orthogonal search (FOS) to model the stochastic errors of low-cost MEMS sensors for smart mobile phones.

Many techniques have been used previously to de-noise and stochastically model the inertial sensor errors [2 - 5]. For example, several levels of wavelet decomposition have been used to de-noise the raw INS data and eliminate high frequency disturbances [2 - 4]. Modeling inertial sensor

errors using Autoregressive (AR) models was performed in [2], where the Yule-Walker, the covariance and Burg AR methods were used. The AR model parameters were estimated after reducing the INS sensor measurements noise using wavelet de-noising techniques.

In [5], FOS was used to augment a Kalman filter (KF) to enhance the accuracy of a low-cost 2D MEMS-based navigation system by modeling only the azimuth error. FOS is used in this paper to model the raw MEMS gyroscope and accelerometer measurement errors in the time domain. In this paper, the performance of FOS is compared to linear modeling techniques such as Yule-Walker, the covariance and Burg AR methods in terms of Mean Square Errors (MSE) and computational time.

II. PROBLEM STATEMENT

When the autocorrelation function of some of the noise sequences of MEMS measurements is studied, it has been shown that a first-order GM process may not be adequate in all cases to model such noise behavior. The shape of the autocorrelation sequence is often different from that of a first-order GM process, which is represented by a decaying exponential as shown in Figure 1.

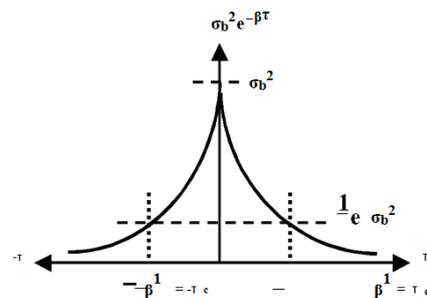


FIGURE 1 THE AUTOCORRELATION SEQUENCE OF A FIRST-ORDER GAUSS-MARKOV (GM) PROCESS

Most of the computed autocorrelation sequences follow higher order GM processes. An example of such computed autocorrelation sequences for one hour of static data of a MEMS accelerometer is shown in Figure 2. It clearly represents a higher-order GM process. These higher order GM processes can be modeled using an Autoregressive (AR) process of an appropriate order. In [2] it has been decided to model the randomness of the inertial sensor measurements using an AR process of order higher than one. With the present computational efficiency of microprocessor systems, efficient modeling of MEMS residual biases can be realized,

and thus, accurate prediction and estimation of such errors can be provided.

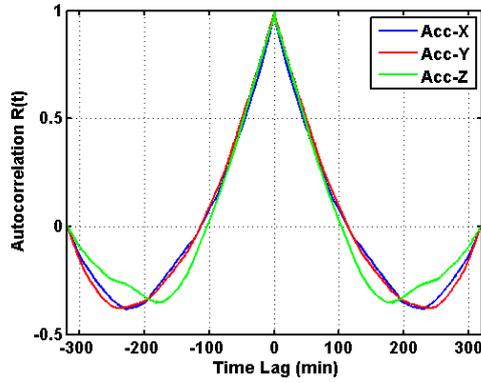


FIGURE 2 THE COMPUTED AUTOCORRELATION SEQUENCE FOR A MEMS ACCELEROMETER DATA

III. MODELING METHODS OF AR PROCESSES

The Autoregressive Moving Average (ARMA) modeling is based on the mathematical modeling of a time series of measurements assuming that each value of such series is dependent on: (a) a weighted sum of the “previous” values of the same series (AR part) and, (b) a weighted sum of the “present and previous” values of a different time series (MA part). The ARMA process can be described using a pole-zero (AR-MA) transfer function system $H(z)$ as follows:

$$H(z) = \frac{Y(z)}{W(z)} = \frac{B(z)}{A(z)} = \frac{\sum_{k=0}^q b_k z^{-k}}{1 + \sum_{k=1}^p a_k z^{-k}} \quad (1)$$

where $W(z)$ is the z -transform of the input $w(n)$, $Y(z)$ is the z -transform of the output $y(n)$, p is the order of the AR process, q is the order of the MA process and a_1, a_2, \dots, a_p and b_1, b_2, \dots, b_q are the AR and MA process parameters (weights), respectively. The AR process is a special case of an ARMA process, where q in Equation 1 will be zero and thus $H(z)$ will be only an all-pole transfer function of the form:

$$H(z) = \frac{Y(z)}{W(z)} = \frac{B(z)}{A(z)} = \frac{b_0}{1 + \sum_{k=1}^p a_k z^{-k}} \quad (2)$$

Therefore, the name “Autoregressive” comes from the fact that each signal sample is regressed on (or predicted from) the previous values of itself. In the time domain, the above AR transfer function relationship can be obtained after applying the inverse z -transform for Equation (2).

The resultant equation is written as:

$$y(n) = -\sum_{k=1}^p a_k y(n-k) + b_0 w(n)$$

$$y(n) = -a_1 y(n-1) - a_2 y(n-2) - \dots - a_p y(n-p) + b_0 w(n) \quad (3)$$

The above input-output relationship in both frequency and time domains is shown in Figure 3.

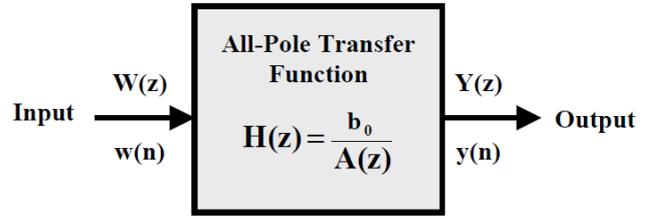


FIGURE 3 THE INPUT-OUTPUT RELATIONSHIP OF AN AUTOREGRESSIVE (AR) PROCESS

The problem in this case is to determine the values of the AR model parameters (predictor coefficients) a_k that optimally represent the random part of the inertial sensor biases. This is performed by minimizing the error $e(n)$ between the original signal $y(n)$ represented by the “AR process” of Equation 3 and the estimated signal $\hat{y}(n)$, which is estimated by an “AR model” of the form:

$$\hat{y}(n) = -\sum_{k=1}^p a_k y(n-k) \quad (4)$$

The cost function for this minimization problem is the energy E of $e(n)$, which is given as:

$$E = \sum_{n=1}^N e^2(n)$$

$$= \sum_{n=1}^N [y(n) - \hat{y}(n)]^2$$

$$= \sum_{n=1}^N \left[-\sum_{k=1}^p a_k y(n-k) + b_0 w(n) + \sum_{k=1}^p a_k y(n-k) \right]^2$$

$$= \sum_{n=1}^N b_0^2 w^2(n) = \min \quad (5)$$

where N is the total number of data samples. In this case, $w(n)$ is a sequence of stationary uncorrelated sequences (white noise) with zero mean and unity variance.

Therefore, the resultant energy from Equation 5 $[\sum_{n=1}^N b_0^2 w^2(n)]$ will be b_0^2 . Therefore, b_0^2 represents the estimated variance σ_w^2 of the white noise input to the AR model, or more generally, the prediction mean-square error σ_e^2 . This is due to the fact that the AR model order p is completely negligible with respect to the MEMS data sample size N .

Several methods have been reported to estimate the a_k parameter values by fitting an AR model to the input data. Three AR methods are considered in this paper, namely: the Yule-Walker method, the covariance method and Burg’s method. In principle, all of these estimation techniques should lead to approximately the same parameter values if fairly large data samples are used [6].

A. The Yule-Walker Method

The Yule-Walker method, which is also known as the autocorrelation method determines first the autocorrelation sequence $R(\tau)$ of the input signal (inertial sensor residual bias in our case). Then, the AR model parameters are optimally computed by solving a set of linear normal equations. These normal equations are obtained using the formula:

$$\frac{\partial E}{\partial a_k} = 0 \quad (6)$$

which leads to the following set of normal equations:

$$Ra = -r \quad \leftrightarrow \quad a = -R^{-1}r \quad (7)$$

where

$$a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} \quad (8a)$$

$$r = \begin{bmatrix} R(1) \\ R(2) \\ \vdots \\ R(p) \end{bmatrix} \quad (8b)$$

and

$$R = \begin{bmatrix} R(0) & R(1) & \dots & R(p-1) \\ R(1) & R(0) & \dots & R(p-2) \\ \vdots & \vdots & \dots & \vdots \\ R(p-1) & R(p-2) & \dots & R(0) \end{bmatrix} \quad (8c)$$

If the mean-square error σ_e^2 is also required, it can be determined by:

$$\begin{bmatrix} R(0) & R(1) & \dots & R(p-1) \\ R(1) & R(0) & \dots & R(p-2) \\ \vdots & \vdots & \dots & \vdots \\ R(p-1) & R(p-2) & \dots & R(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_p \end{bmatrix} = \begin{bmatrix} \sigma_e^2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (9)$$

Equations 7 and 9 are known as the Yule-Walker equations [7-10]. Instead of solving Equation 9 directly (i.e. by first computing R^{-1}), it can efficiently be solved using the Levinson-Durbin (LD) algorithm which proceeds recursively to compute a_1, a_2, \dots, a_p and σ_e^2 . The LD algorithm is an iterative technique that computes the next prediction coefficient (AR parameter) from the previous one. This LD recursive procedure can be summarized in the following [7]:

$$E_0 = R(0) \quad (9a)$$

$$\gamma_k = -\frac{R(k) + \sum_{i=1}^{k-1} a_{i,k-1} R(k-i)}{E_{k-1}} \quad 1 \leq k \leq p \quad (9b)$$

$$a_{k,k} = \gamma_k \quad (9c)$$

$$a_{i,k} = a_{i,k-1} + \gamma_k a_{k-i,k-1} \quad 1 \leq i \leq k-1 \quad (9d)$$

$$E_k = (1 - \gamma_k^2) E_{k-1} \quad (9e)$$

Equations 9a-9e are solved recursively for $k = 1, 2, \dots, p$ and the final solution for the AR parameters is provided by:

$$a_i = a_{i,p} \quad 1 \leq i \leq p \quad (9f)$$

Therefore, the values of the AR prediction coefficients in the Yule-Walker method are provided directly based on minimizing the forward prediction error $e_f(n)$ in the Least-Squares sense. The intermediate quantities γ_k represented by Equation 9b are known as the reflection coefficients. In Equation 9e, both energies E_k and E_{k-1} are positive, and thus, the magnitude of γ_k should be less than one to guarantee the stability of the all-pole filter.

However, the Yule-Walker method performs adequately only for long data records [11]. The inadequate performance in case of short data records is usually due to the data windowing applied by the Yule-Walker algorithm. Moreover, the Yule-Walker method may introduce a large bias in the AR estimated coefficients since it does not guarantee a stable solution of the model [8].

B. The Covariance Method

The covariance method is similar to the Yule-Walker method in that it minimizes the forward prediction error in the Least-Squares sense but it does not consider any windowing of the data. Instead, the windowing is performed with respect to the prediction error to be minimized. Therefore, the AR model obtained by this method is typically more accurate than the one obtained from the Yule-Walker method [12].

Furthermore, it uses the covariance $C(\tau_i, \tau_j)$ instead of $R(\tau)$. In this case, the Toeplitz structure of the normal equations used in the autocorrelation method is lost, and hence the LD algorithm cannot be used for the computations. To achieve an efficient C^{-1} in this case, Cholesky factorization is usually utilized [11].

The method provides more accurate estimates than the Yule-Walker method especially for short data records. However, the covariance method may lead to unstable AR models since the LD algorithm is not used for solving the covariance normal equations [6].

C. Burg's Method

The Burg's method was introduced in 1967 to overcome most of the drawbacks of the other AR modeling techniques by providing both stable models and high resolution (i.e. more accurate estimates) for short data records. Burg's method tries to make the maximum use of the data by defining both a forward and a backward prediction error terms, $e_f(n)$ and $e_b(n)$. The energy to be minimized in this case (E_{Burg}) is the sum of both the forward and backward prediction error energies, i.e.

$$E_{Burg} = \sum_{n=1}^N [e_f^2(n) + e_b^2(n)] = \min \quad (10)$$

where e_f and e_b are defined as:

$$e_f = y(n) + a_1 y(n-1) + a_2 y(n-2) + \dots + a_p y(n-p) \quad (11a)$$

$$e_b = y(n-p) + a_1 y(n-p+1) + a_2 y(n-p+2) + \dots + a_p y(n) \quad (11b)$$

The forward and backward prediction error criteria are the same, and hence, they have the same optimal solution for the model coefficients [9]. Considering the energies in Equation 9e to be E_{Burg} , the forward and backward prediction errors can, therefore, be expressed recursively as:

$$e_{f^k}(n) = e_{f^{k-1}}(n) + \gamma_k e_{b^{k-1}}(n-1) \quad (12a)$$

$$e_{b^k}(n) = e_{b^{k-1}}(n-1) + \gamma_k e_{f^{k-1}}(n) \quad (12b)$$

These recursion formulas form the basis of what is called Lattice (or Ladder) realization of a prediction error filtering (see Figure 4).

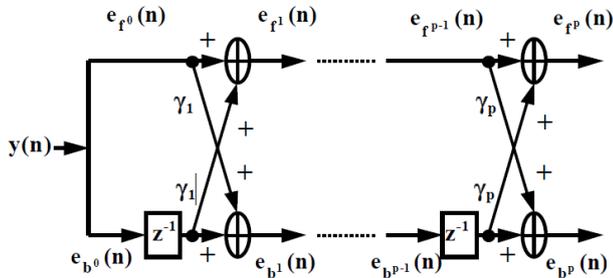


FIGURE 4 THE FORWARD-BACKWARD PREDICTION ERROR LATTICE FILTER GENERAL STRUCTURE

As has been shown for the Yule-Walker method, the accuracy of the estimated parameters a_1, a_2, \dots, a_p and σ_e^2 depends mainly on accurate estimates of the autocorrelation sequence $R(\tau)$. However, this can be rarely achieved due to the pre-windowing of data [12] or the existence of large measurement noise [10]. To avoid the difficulties of the computation of the autocorrelation sequences, Burg in his method estimates first the reflection coefficients γ_k using another formula instead of equation 9b. This formula is derived by substituting Equation 12 into Equation 13 and setting the derivative of E_{Burg} with respect to γ_k (instead of a_k in the Yule-Walker and covariance methods) to zero. This leads to the form:

$$\gamma_k = \frac{-2 \sum_{n=1}^N [e_{f^{k-1}}(n) e_{b^{k-1}}(n-1)]}{\sum_{n=1}^N e_{f^{k-1}}^2(n) + \sum_{n=1}^N e_{b^{k-1}}^2(n-1)} \quad (13)$$

which shows clearly that the magnitude of γ_k is forced (guaranteed) to be less than one, and thus the obtained model is guaranteed to be stable. Both Equations (12 and 13) form the recursive structure of the Burg's Lattice filter, which is shown in Figure 4 with the initial conditions of $e_{f^0}(n) = e_{b^0}(n) = y(n)$. Finally, the prediction coefficients a_k are obtained by constraining them to satisfy Equation 9d in the LD algorithm.

Therefore, the utilization of Equations 9d and 13 together will always ensure the stability of the Burg's method solution [8]. Moreover, the utilization of both forward and backward prediction errors usually yields better estimation results than using the forward prediction approach used in the previous two methods. Finally, it has been reported by [13] that Burg's method generally provides better residual estimates than the Yule-Walker method.

IV. FAST ORTHOGONAL SEARCH METHOD

FOS is an efficient general purpose method of building models of time-series and of systems with unknown structure [14]. It operates by searching through a list of pre-designated candidate functions and iteratively adding the term that lowers the MSE of the model by the greatest amount. The model developed by FOS can take the form:

$$y(n) = \sum_{m=0}^M a_m P_m(n) + e(n) \quad (13)$$

where $P_0(n) = 1$ and the $P_m(n)$ are the model terms selected from the set of candidate functions. These model terms can involve the system input x and output y , and cross-products and powers thereof:

$$P_m(n) = y(n-l_1) \dots y(n-l_i) x(n-k_1) \dots x(n-k_j) \\ m \geq 1, i \geq 0, j \geq 0, \forall i > 0: 1 \leq l_i \leq L, \forall j > 0: 0 \leq k_j \leq K \quad (14)$$

In (14), K and L are the maximum lags in the input and output, respectively, $i=0$ denotes the absence of y -factors, and $j=0$ denotes the absence of x -factors. In (13), the a_m are the associated coefficients which best fit the output in the mean-square sense. Finally, $e(n)$ is the model error, M is the number of (non-constant) terms in the model. In our application to gyroscope and accelerometer MEMS devices, FOS finds a linear or a nonlinear autoregressive (AR) model, hence $y(n)$ is the measured device output, while the $P_m(n)$ contain no x -factors so K has no meaning. FOS selects model terms by implicitly creating an orthogonal function corresponding to a given candidate term, enabling FOS to systematically and efficiently test each possible candidate term $P_m(n)$ to select the best one [14]. The selected candidate added to the model is the one causing maximum reduction of MSE. Model building continues until the stopping criteria [14] have been met, enabling an accurate and concise model to be constructed from the given training data.

V. EXPERIMENTAL RESULTS

The data were collected by a low cost MEMS-based inertial measurement unit (IMU CC-300, Crossbow) for one hour to obtain stochastic error models of both gyroscopes and accelerometers. Figure 5 shows one hour of sampled Accelerometer-Y and Gyro-Y acquired at 200 Hz, other inertial sensors gave similar results.

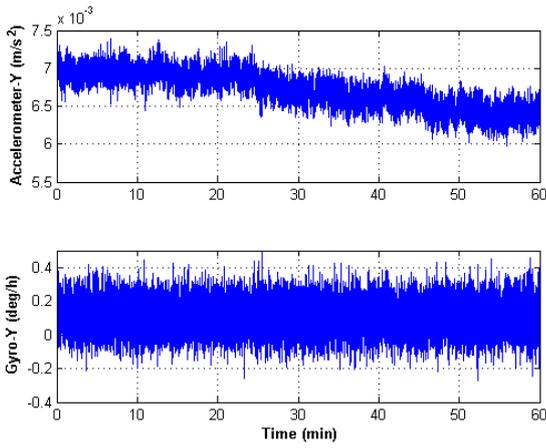


FIGURE 5 ACCELEROMETER-Y AND GYRO-Y SPECIFIC FORCE MEASUREMENTS

FOS is applied directly on the raw inertial sensor 200 Hz data without any pre-processing or de-noising. Traditional methods like Yule Walker, Covariance, and Burg perform poorly on the raw data, so we first applied wavelet de-noising of up to 4 levels of decomposition that resulted in band limiting the spectrum of the raw inertial sensor data to 12.5 Hz. Therefore, unlike FOS, the other 3 methods operate on the de-noised version of the same data. After de-noising, AR model parameters were then estimated as well as the corresponding prediction MSE for all sensors using Yule-Walker, Covariance, and Burg methods.

For FOS, the raw INS data were divided into three data sets for model training, evaluation, and prediction stages. The first 3 minutes of the INS raw data were utilized for model training, which uses the FOS algorithm to identify several possibly nonlinear AR equations. Different models can be obtained by changing the maximum delay L in the output, and the degree of output cross-products (CP). The next 3 minutes of the data were used for the evaluation stage. Here, models are compared and the best one, fitting the real output with minimum MSE, is chosen. In the prediction stage, the output and MSE of the chosen model is computed over the remaining (novel) raw INS data.

Figure 6 shows prediction MSE of Accelerometer-Y samples by Yule-Walker, Covariance, Burg, and FOS methods. For FOS, an AR model of order 3 or 4 suffices, and MSE decreases when the degree of the cross-product terms is raised to 2 (non-linear model). An AR model of order 7 or 8 is required for Burg or Covariance method, and order 9 or 10 for Yule – Walker. Large AR model order complicates the estimation method (like KF) used for the INS/GPS integration.

Table 1 shows that, for both model orders 1 and 10, FOS without pre-processing provides lowest MSE and consequently lowest position error. FOS takes less computation time than Yule – Walker, covariance, and Burg methods. Increasing the cross-product degree to 2 for FOS improves model accuracy and lessens position error by an order of magnitude (for $L = 10$) but increases computation time.

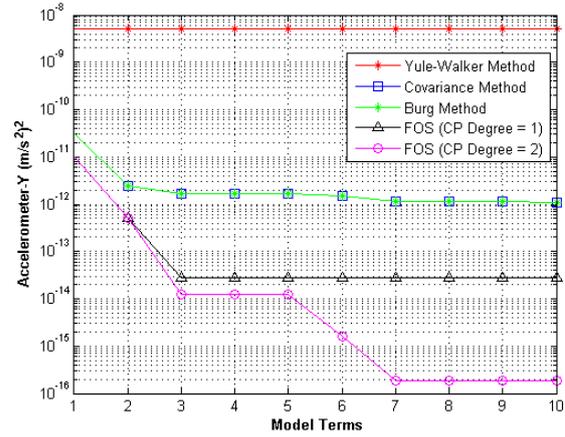


FIGURE 6 ACCELEROMETER-Y PREDICTION MSE USING YULE-WALKER, COVARIANCE, BURG AND FOS METHODS. FOR CP DEGREE=1, ONLY LINEAR CANDIDATE TERMS UP TO A MAXIMUM OUTPUT LAG $L=10$ WERE ALLOWED. FOR CP DEGREE=2, BOTH LINEAR AND $Y(N-L_1)Y(N-L_2)$ CANDIDATES WERE ALLOWED, UP TO MAXIMUM OUTPUT LAG $L=10$

TABLE 1 PERFORMANCE SUMMARY OF AR MODELS OBTAINED BY YULE-WALKER, BURG AND FOS OVER ONE HOUR OF NOVEL ACCELEROMETER – Y MEASUREMENTS. SEE FIG 2 FOR MEANING OF CP DEGREE.

Modeling Technique	Model MSE (m/s^2) ²	Corresponding Position Error (m)	Computational Time (s)
Model Order (Maximum output lag L) 1			
Yule-Walker	5×10^{-9}	458	0.25
Covariance/Burg	3×10^{-11}	35	0.23
CP Degree=1 FOS	1×10^{-11}	20	0.13
CP Degree=2 FOS	1×10^{-11}	20	0.36
Model Order (Maximum output lag L) 10			
Yule-Walker	5×10^{-9}	458	0.69
Covariance/Burg	2×10^{-12}	9	0.45
CP Degree=1 FOS	3×10^{-14}	1	0.30
CP Degree=2 FOS	2×10^{-16}	0.09	0.60

Similarly, Figure 7 shows the prediction MSE of Gyro-Y samples by using Yule-Walker, the Covariance, Burg and FOS methods.

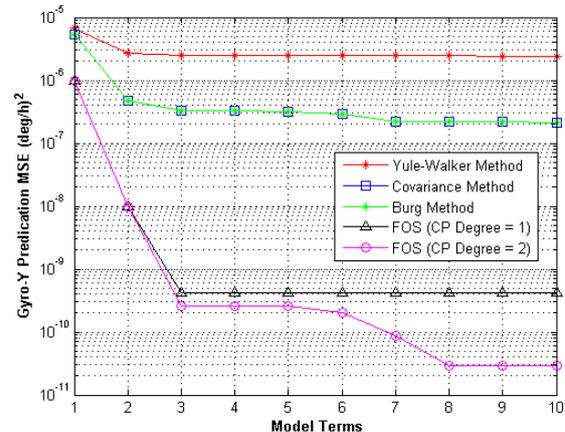


FIGURE 7 GYRO-Y PREDICTION MSE USING YULE-WALKER, COVARIANCE, BURG AND FOS METHODS. SEE FIG. 2 FOR MEANING OF CP DEGREE

TABLE 2 PERFORMANCE SUMMARY OF BOTH AR YULE-WALKER AND BURG MODELS AND FOS MODEL OVER ONE HOUR OF NOVEL GYRO – Y MEASUREMENTS. SEE FIG 2 FOR MEANING OF CP DEGREE

Modeling Technique	Model MSE (deg/h) ²	Corresponding Position Error (m)	Computational Time (s)
Model Order (Maximum output lag L) 1			
Yule-Walker	7×10^{-6}	978	0.40
Covariance/Burg	5×10^{-6}	826	0.23
CP Degree=1 FOS	1×10^{-6}	369	0.09
CP Degree=2 FOS	1×10^{-6}	369	0.35
Model Order (Maximum output lag L) 10			
Yule-Walker	2×10^{-6}	523	0.44
Covariance/Burg	2×10^{-7}	165	0.28
CP Degree=1 FOS	4×10^{-10}	7	0.18
CP Degree=2 FOS	3×10^{-11}	2	0.6

Clearly FOS achieves minimum MSE error with lower model order. Table 2 summarises the performance of Yule-Walker, Covariance, Burg, and FOS with cross-product degree set to 1 (linear model) and 2 (nonlinear model), for different model orders 1 and 10 over one hour of Gyro-Y measurements. FOS performs best in speed and accuracy.

VI. CONCLUSION

For either gyroscope or accelerometer case, the FOS model surpasses those obtained by traditional methods. FOS lowers MSE and corresponding position error, dispenses with pre-processing or de-noising, and produces a smaller model order in less time.

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