Computations using Standard Notations

Computations using Standard Notations

- $P = F\left[\frac{1}{(1+i)^n}\right]$ $\quad P = F\,(^{P}/_{F}, i\%, n)$
- $F = P\,(1+i)^n$ $\quad F = P\,(^{F}/_{P}, i\%, n)$
- $P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$ $\quad P = A(^{P}/_{A}, i\%, n)$

Computations using Standard Notations

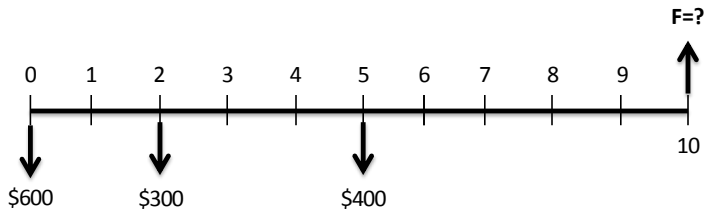
- $A = P\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$ $\qquad A = P({}^{A}/_{P}, i\%, n)$
- $A = F\left[\frac{i}{(1+i)^n-1}\right]$ $\qquad A = F\,({}^{A}/_{F}, i\%, n)$
- $F = A\left[\frac{(1+i)^n-1}{i}\right]$ $\qquad F = A\,({}^{F}/_{A}, i\%, n)$

Table D–16 Interest Factors for 10.00%

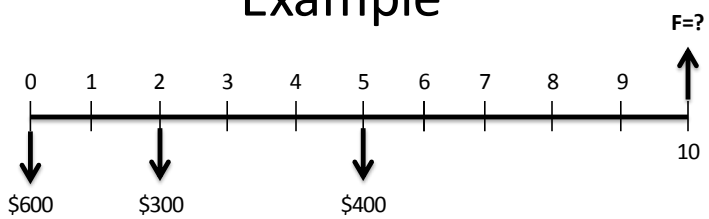
	Single Payment		Uniform Series			
	Compound-Amount Factor	Present-Worth Factor	Compound-Amount Factor	Sinking-Fund Factor	Present-Worth Factor	Capital-Recovery Factor
	Convert *P* to *F*	Convert *F* to *P*	Convert *A* to *F*	Convert *F* to *A*	Convert *A* to *P*	Convert *P* to *A*
N	(*F*/*P*,*i*,*N*)	(*P*/*F*,*i*,*N*)	(*F*/*A*,*i*,*N*)	(*A*/*F*,*i*,*N*)	(*P*/*A*,*i*,*N*)	(*A*/*P*,*i*,*N*)
1	1.1000	0.9091	1.0000	1.0000	0.9091	1.1000
2	1.2100	0.8264	2.1000	0.4762	1.7355	0.5762
3	1.3310	0.7513	3.3100	0.3021	2.4869	0.4021
4	1.4641	0.6830	4.6410	0.2155	3.1699	0.3155
5	1.6105	0.6209	6.1051	0.1638	3.7908	0.2638
6	1.7716	0.5645	7.7156	0.1296	4.3553	0.2296
7	1.9487	0.5132	9.4872	0.1054	4.8684	0.2054
8	2.1436	0.4665	11.4359	0.0874	5.3349	0.1874
9	2.3579	0.4241	13.5795	0.0736	5.7590	0.1736
10	2.5937	0.3855	15.9374	0.0627	6.1446	0.1627
11	2.8531	0.3505	18.5312	0.0540	6.4951	0.1540
12	3.1384	0.3186	21.3843	0.0468	6.8137	0.1468

Example

- If a woman deposits $600 now, $300 two years from now, and $400 five years from now, how much will she have in her account 10 years from now if the interest rate is 5% per year?



Example



$F = F_1 + F_2 + F_3$

= $600(F/P, 5%,10) + $300(F/P,5%,8) + $400(F/P,5%,5)

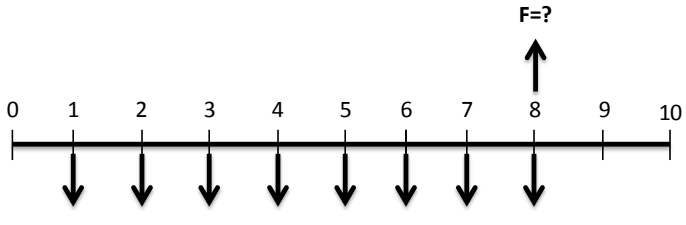
TABLE D–11 Interest Factors for 5.00%

	Single Payment		Uniform Series			
	Compound-Amount Factor	Present-Worth Factor	Compound-Amount Factor	Sinking-Fund Factor	Present-Worth Factor	Capital-Recovery Factor
	Convert P to F (F/P,i,N)	Convert F to P (P/F,i,N)	Convert A to F (F/A,i,N)	Convert F to A (A/F,i,N)	Convert A to P (P/A,i,N)	Convert P to A (A/P,i,N)
N						
1	1.0500	0.9524	1.0000	1.0000	0.9524	1.0500
2	1.1025	0.9070	2.0500	0.4878	1.8594	0.5378
3	1.1576	0.8638	3.1525	0.3172	2.7232	0.3672
4	1.2155	0.8227	4.3101	0.2320	3.5460	0.2820
5	1.2763	0.7835	5.5256	0.1810	4.3295	0.2310
6	1.3401	0.7462	6.8019	0.1470	5.0757	0.1970
7	1.4071	0.7107	8.1420	0.1228	5.7864	0.1728
8	1.4775	0.6768	9.5491	0.1047	6.4632	0.1547
9	1.5513	0.6446	11.0266	0.0907	7.1078	0.1407
10	1.6289	0.6139	12.5779	0.0795	7.7217	0.1295
11	1.7103	0.5847	14.2068	0.0704	8.3064	0.1204
12	1.7959	0.5568	15.9171	0.0628	8.8633	0.1128

F = $600*1.6289 +$300*1.4775 + $400*1.2763 = $1931.09

Example

- How much money would a man have in his account after 8 years if he deposited $1,000 per year for 8 years at 14% per year starting 1 year from now?



$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

TABLE D–20 Interest Factors for 14.00%

	Single Payment		Uniform Series			
	Compound-Amount Factor	Present-Worth Factor	Compound-Amount Factor	Sinking-Fund Factor	Present-Worth Factor	Capital-Recovery Factor
	Convert P to F $(F/P,i,N)$	Convert F to P $(P/F,i,N)$	Convert A to F $(F/A,i,N)$	Convert F to A $(A/F,i,N)$	Convert A to P $(P/A,i,N)$	Convert P to A $(A/P,i,N)$
N						
1	1.1400	0.8772	1.0000	1.0000	0.8772	1.1400
2	1.2996	0.7695	2.1400	0.4673	1.6467	0.6073
3	1.4815	0.6750	3.4396	0.2907	2.3216	0.4307
4	1.6890	0.5921	4.9211	0.2032	2.9137	0.3432
5	1.9254	0.5194	6.6101	0.1513	3.4331	0.2913
6	2.1950	0.4556	8.5355	0.1172	3.8887	0.2572
7	2.5023	0.3996	10.7305	0.0932	4.2883	0.2332
8	2.8526	0.3506	13.2328	0.0756	4.6389	0.2156
9	3.2519	0.3075	16.0853	0.0622	4.9464	0.2022
10	3.7072	0.2697	19.3373	0.0517	5.2161	0.1917

$13,232.8

Example

- A unit of mechanical equipment has an initial cost of 10,000 LE and annual maintenance expenditure is expected to be 200 LE for its eight years of life. If interest is 6% and the equipment has no salvage value, what is its equivalent annual cost, excluding labor, fuel, etc?

Nominal & Effective Interest Rates

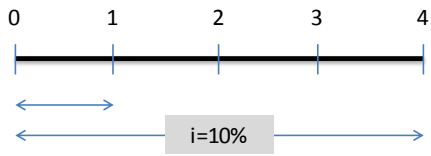
Nominal & Effective Interest Rates

- 10% year compounded quarterly or semi annual in such case the effective interest rate per year will be greater than 10%

- $i_{eff} = (1 + \frac{i}{m})^m - 1$

Example

- If a loan of 1,000LE is made a nominal interest rate of 10% per year, compounded quarterly, what is the effective interest rate?



10.4%

Example

- If a woman deposits 1000 LE now, and 3000LE 4 years from now and 1500 LE 6 years from now at an interest rate of 12% compounded semiannually, how much money will she have in her account 10 years from now?

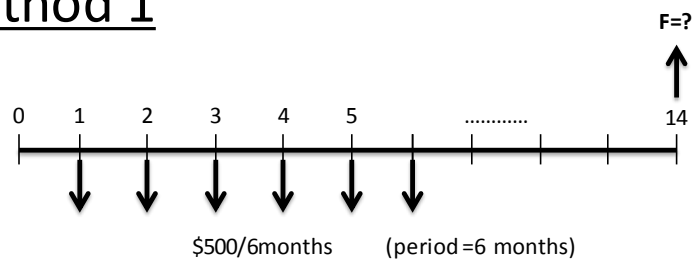
11,634.5 LE

Example

- If a man deposits 500 LE every 6 months for 7 years, how much money will he have in his account after he makes his last deposit if the interest rate is 20% per year compounded quarterly?

14,244.55 LE

Method 1

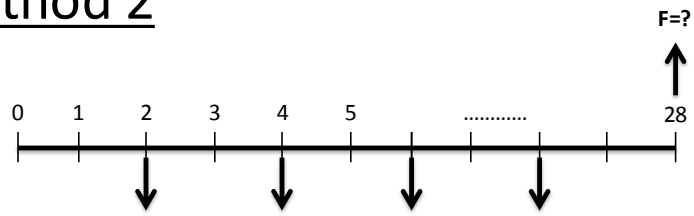


i% per year = 20%
i% per 6 months = 20%/2 = 10% per period → (nominal)
Compounding quarterly (every 3 months) → **twice** in the 6 months
Then i effective = $(1+i/m)^m - 1$ = 10.25% (m=2)

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

A=500, i = 10.25% and n =14 → F =$14,244.55

Method 2

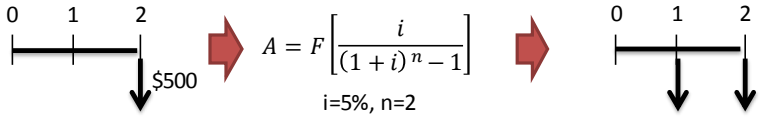


$500/6months every 2 periods

i% per year = 20%

i% per period= 20%/4 = 5% per period

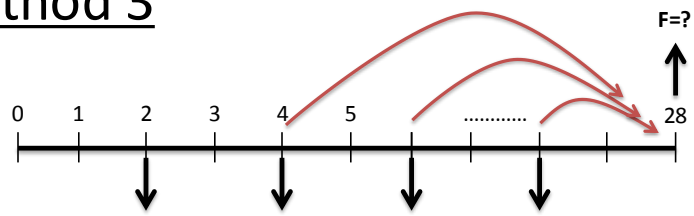
Covert money from 500 every 6 months to money every period (3 months)



$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

A=243.9, i = 5% and n =28 → F =$14,244.55

Method 3



$500/6months every 2 periods

i% per year = 20%

i% per period= 20%/4 = 5% per period

$$F = P(1+i)^n$$

P=$500, i=5%, n = 2, 4, 6, ….26

F = 500 * (Factor $_{n=2}$ +Factor $_{n=4}$ +…+Factor $_{n=26}$)

TABLE D–11 Interest Factors for 5.00%

	Single Payment		Uniform Series			
	Compound-Amount Factor	Present-Worth Factor	Compound-Amount Factor	Sinking-Fund Factor	Present-Worth Factor	Capital-Recovery Factor
	Convert P to F $(F/P,i,N)$	Convert F to P $(P/F,i,N)$	Convert A to F $(F/A,i,N)$	Convert F to A $(A/F,i,N)$	Convert A to P $(P/A,i,N)$	Convert P to A $(A/P,i,N)$
N						
1	1.0500	0.9524	1.0000	1.0000	0.9524	1.0500
2	1.1025	0.9070	2.0500	0.4878	1.8594	0.5378
3	1.1576	0.8638	3.1525	0.3172	2.7232	0.3672
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10	1.6289	0.6139	12.5779	0.0795	7.7217	0.1295
11	1.7103	0.5847	14.2068	0.0704	8.3064	0.1204
12	1.7959	0.5568	15.9171	0.0628	8.8633	0.1128
...						
n =26						

F = $500 * (Sum of factors from 2 to 26 @ step =2)

Economic Comparisons

Money based – P, A, F, B/C
Interest - i
Time - n