Control Systems I

Lecture 8
Compensation-Bode Plot

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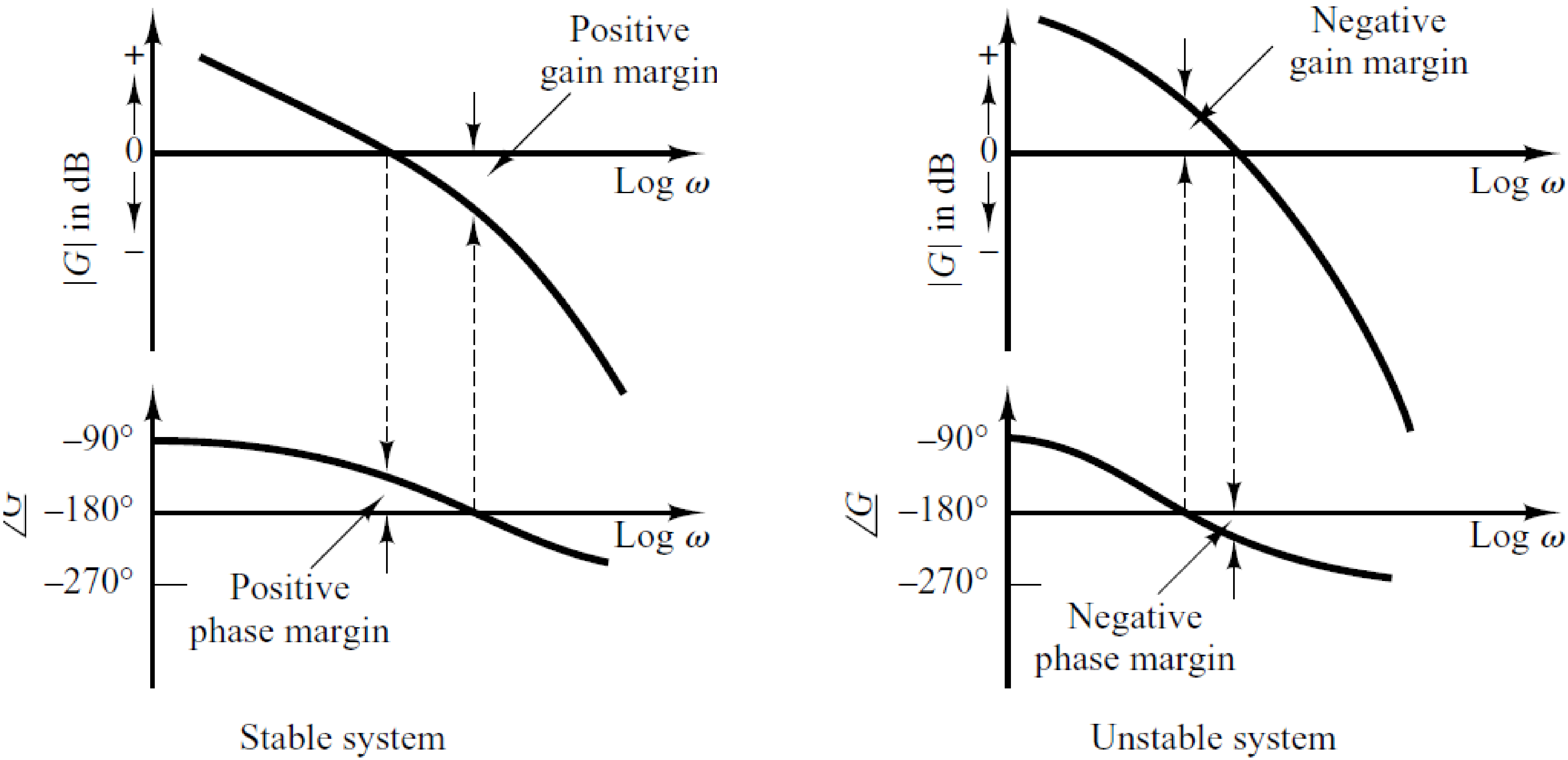
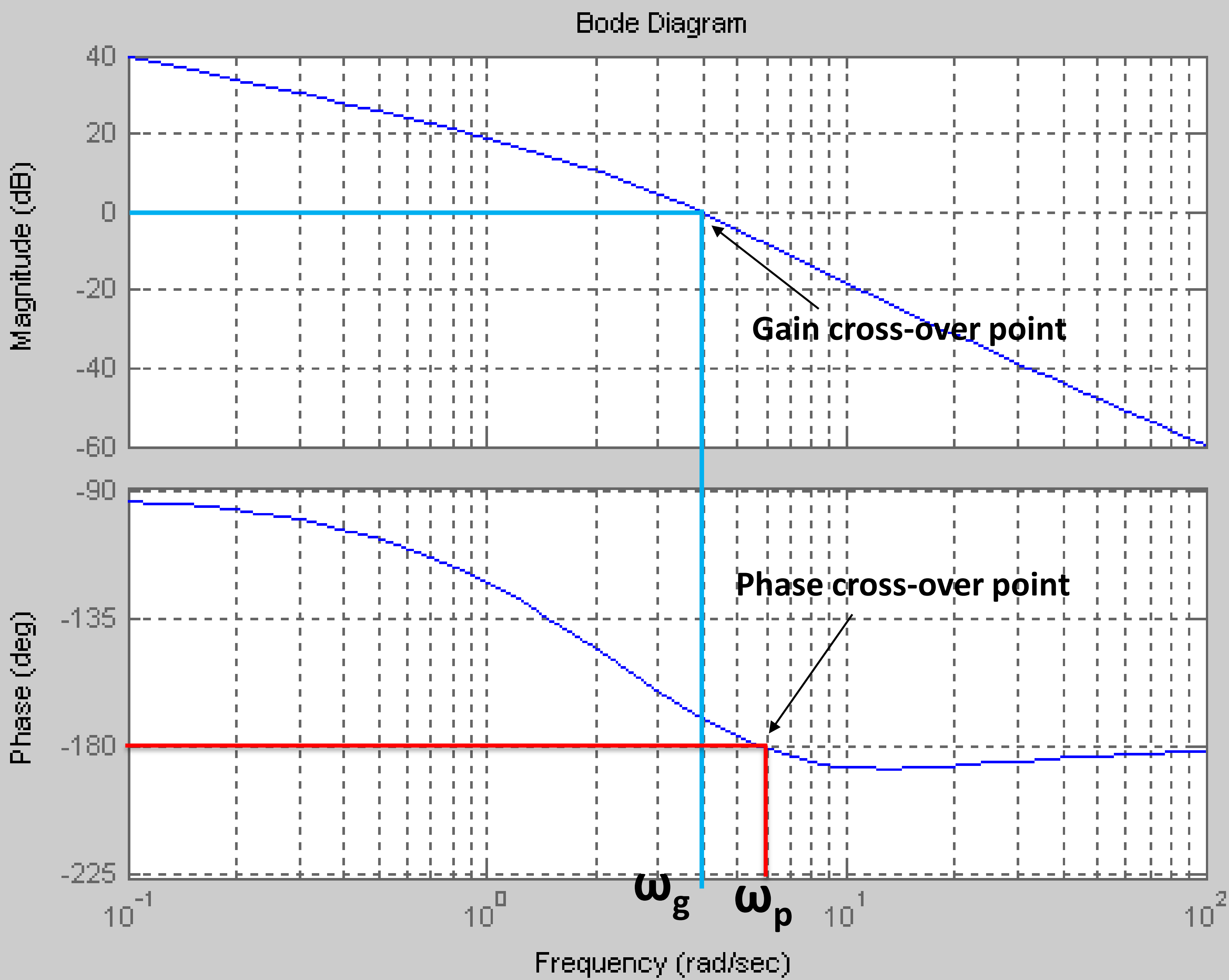
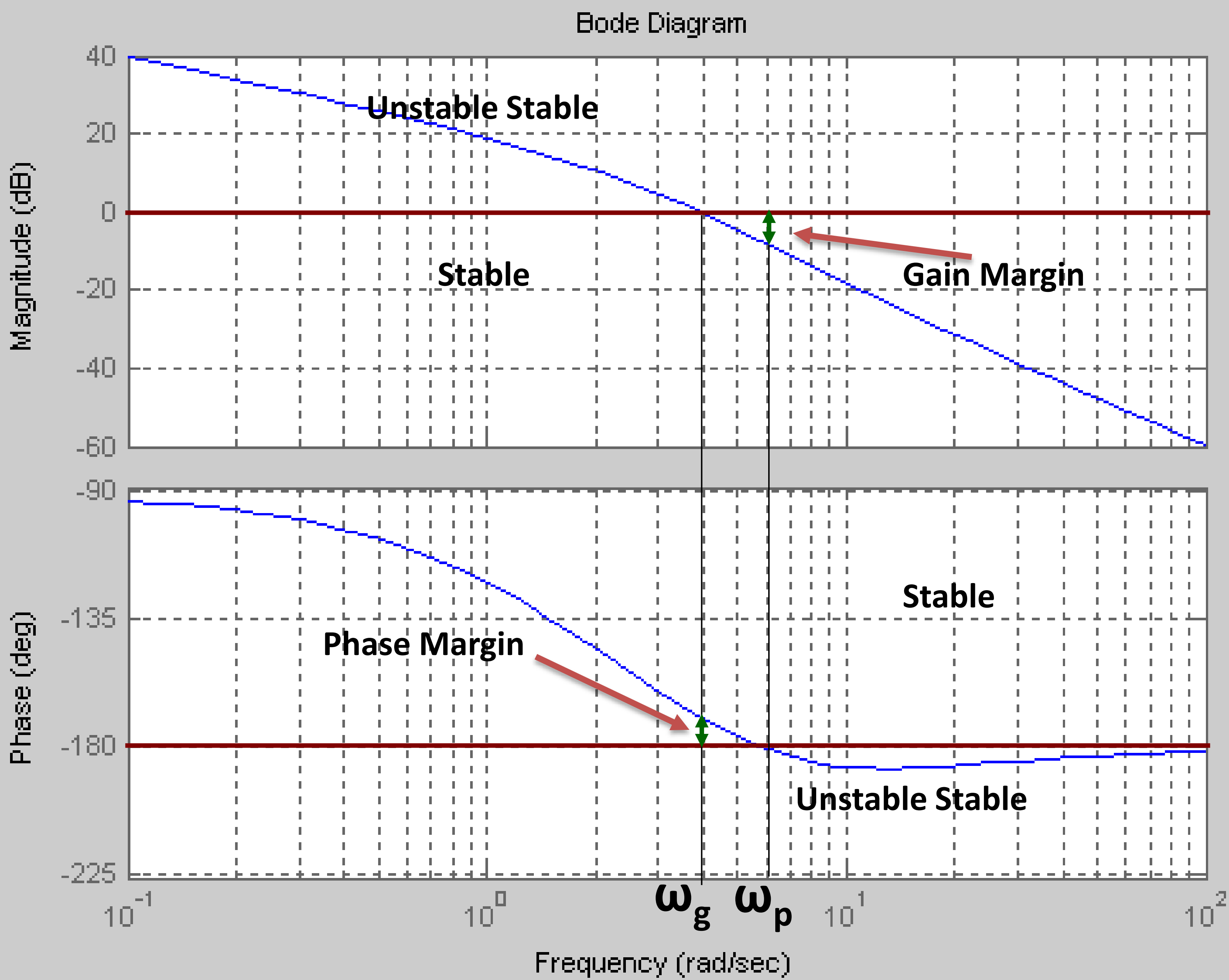
Relative Stability





Bode Diagram
Magnitude (dB)
40
20
0
-20
-40
-60
Gain cross-over point
Phase (deg)
-90
-135
-180
-225
Phase cross-over point
ω_g
ω_p
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/sec)



Bode Diagram
Magnitude (dB)
40
20
0
-20
-40
-60
Unstable Stable
Stable
Gain Margin
Phase (deg)
-90
-135
-180
-225
Stable
Phase Margin
Unstable Stable
10^-1
10^0
ω_g
ω_p
10^1
10^2
Frequency (rad/sec)

Compensation design using the bode diagram

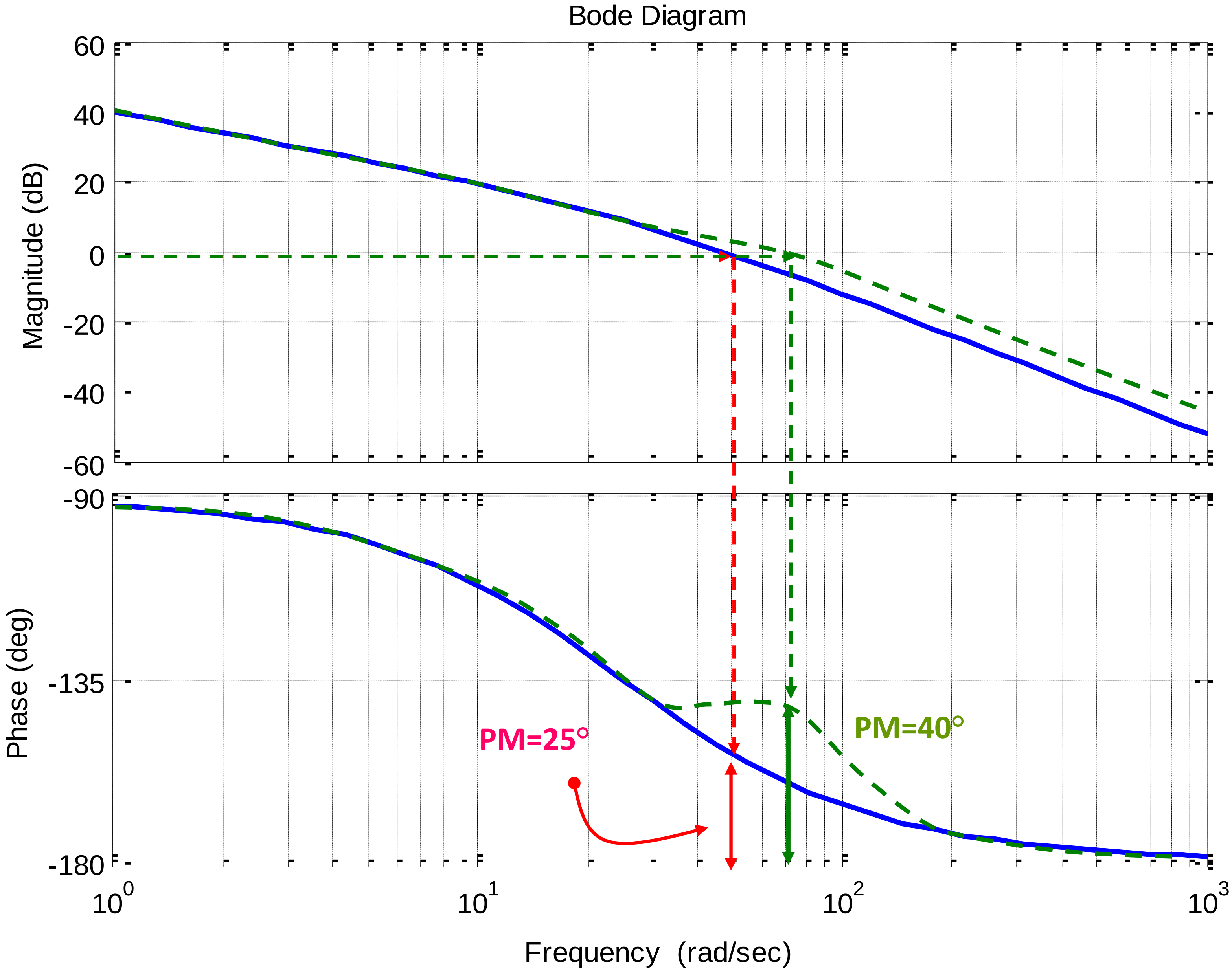
The compensator $G_c(s)$ can be written

$$G_c(s) = \frac{K(s+z)}{(s+p)}$$

The phase-lead compensation when $z<p$.

The phase-lag compensation when $z>p$.

phase-lead compensation



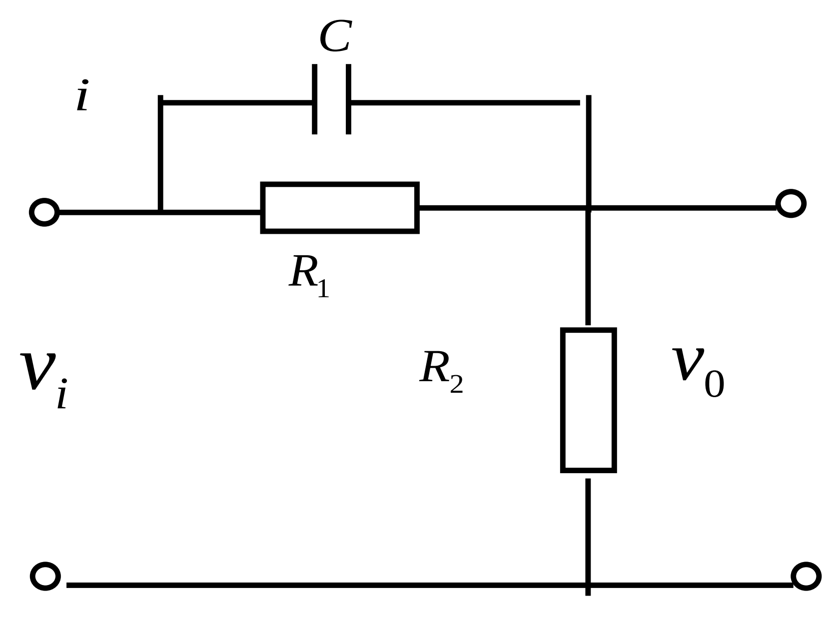
Bode Diagram
Magnitude (dB)
60
40
20
0
-20
-40
-60
Phase (deg)
-90
-135
-180
PM=25°
PM=40°
10^0
10^1
10^2
10^3
Frequency (rad/sec)

Phase -lead compensation

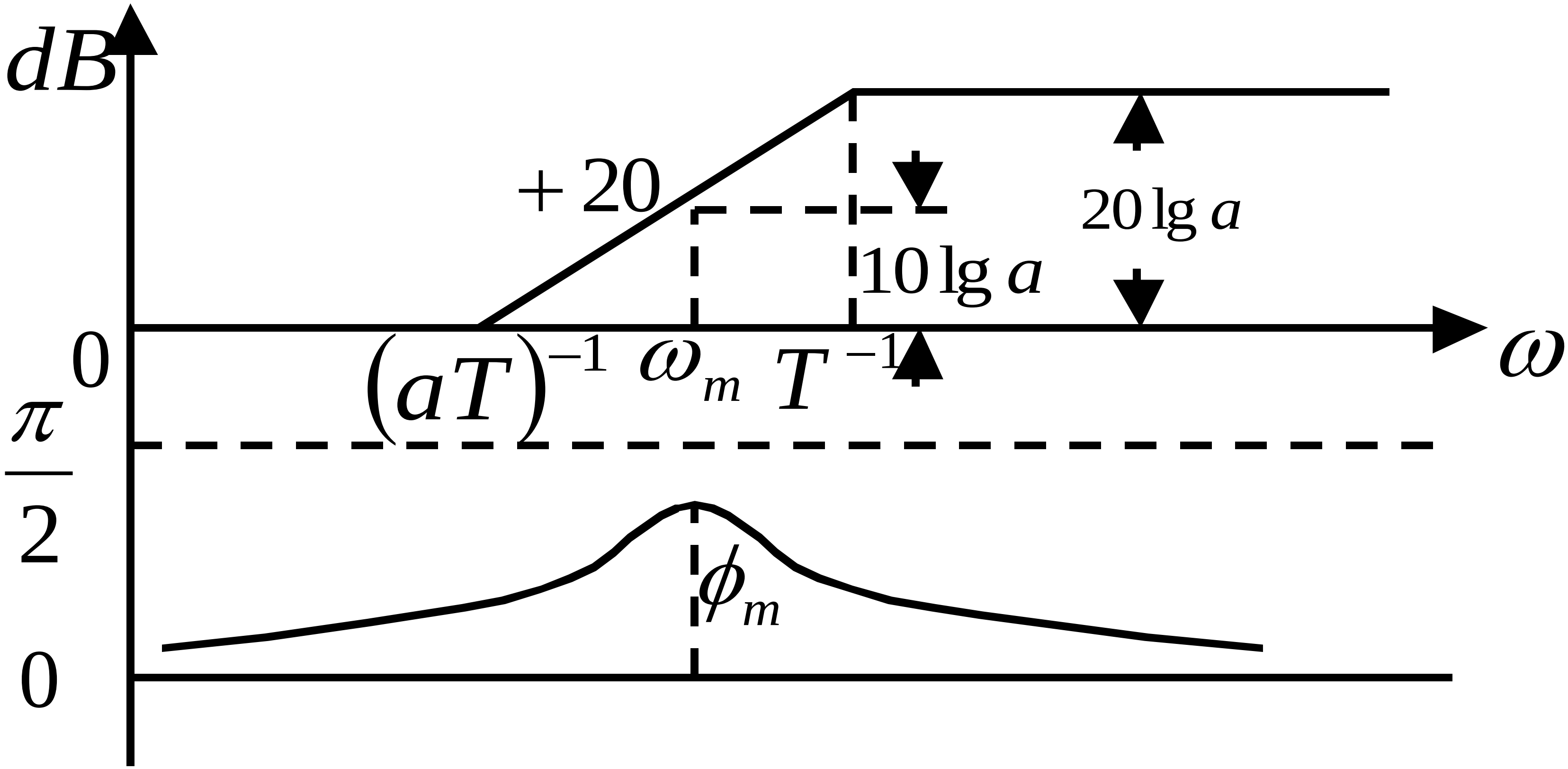
$$G_c(s) = \frac{1 + aTs}{1 + Ts} \quad (a > 1)$$

$$a = \frac{R_1 + R_2}{R_2}$$

$$T = \frac{R_1 R_2}{R_1 + R_2} C$$



Bode diagram of the phase-lead network



$$G_c(s) = \frac{1 + aTs}{1 + Ts} \quad (a > 1)$$

$$\omega_m = \frac{1}{T\sqrt{a}}$$

$$\phi_m = \sin^{-1}\frac{a-1}{a+1}$$

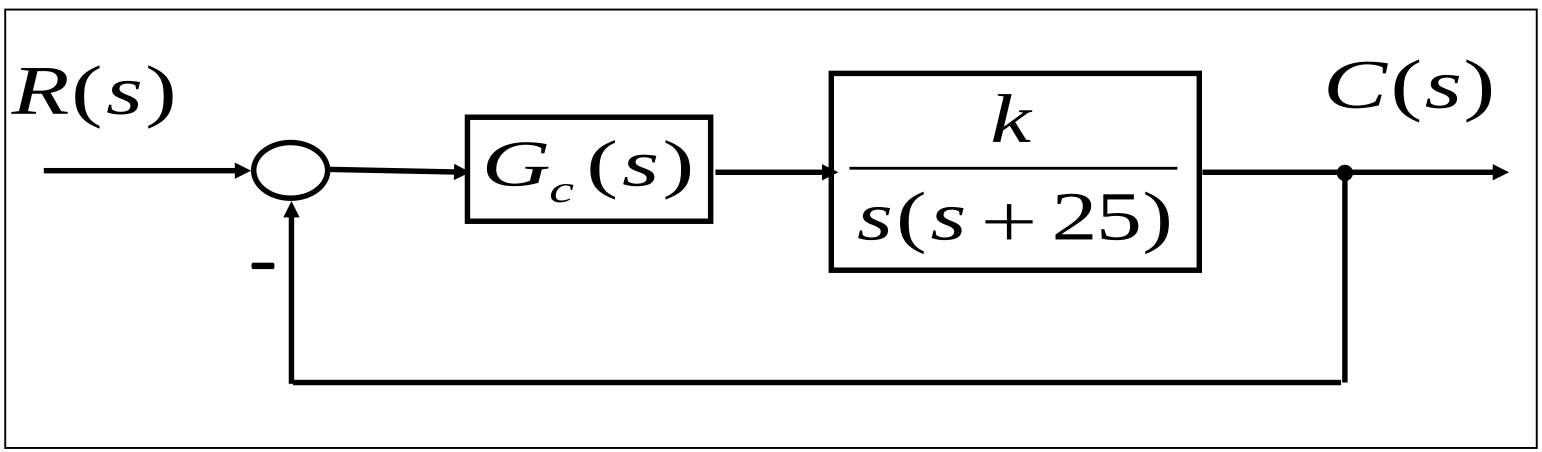
$$20\lg\left|G_c(j\omega_m)\right| = -10\lg a$$

Phase -lead compensation Steps:

1. Evaluate the uncompensated system phase margin when error constants are satisfied.
2. Allowing for a small amount of safety , determine the necessary additional phase lead, ϕ_m .
3. Evaluate a form Eq. $\phi_m = \sin^{-1} \frac{a-1}{a+1}$
4. Evaluate -*10loga* and determine the frequency ω_m .
5. Evaluate T from Eq. $\omega_m = \frac{1}{T\sqrt{a}}$
6. Check the result, and repeat the steps if necessary.

Example

Design a lead controller such that the phase margin be 45° and the ramp error constant be 100.



Example

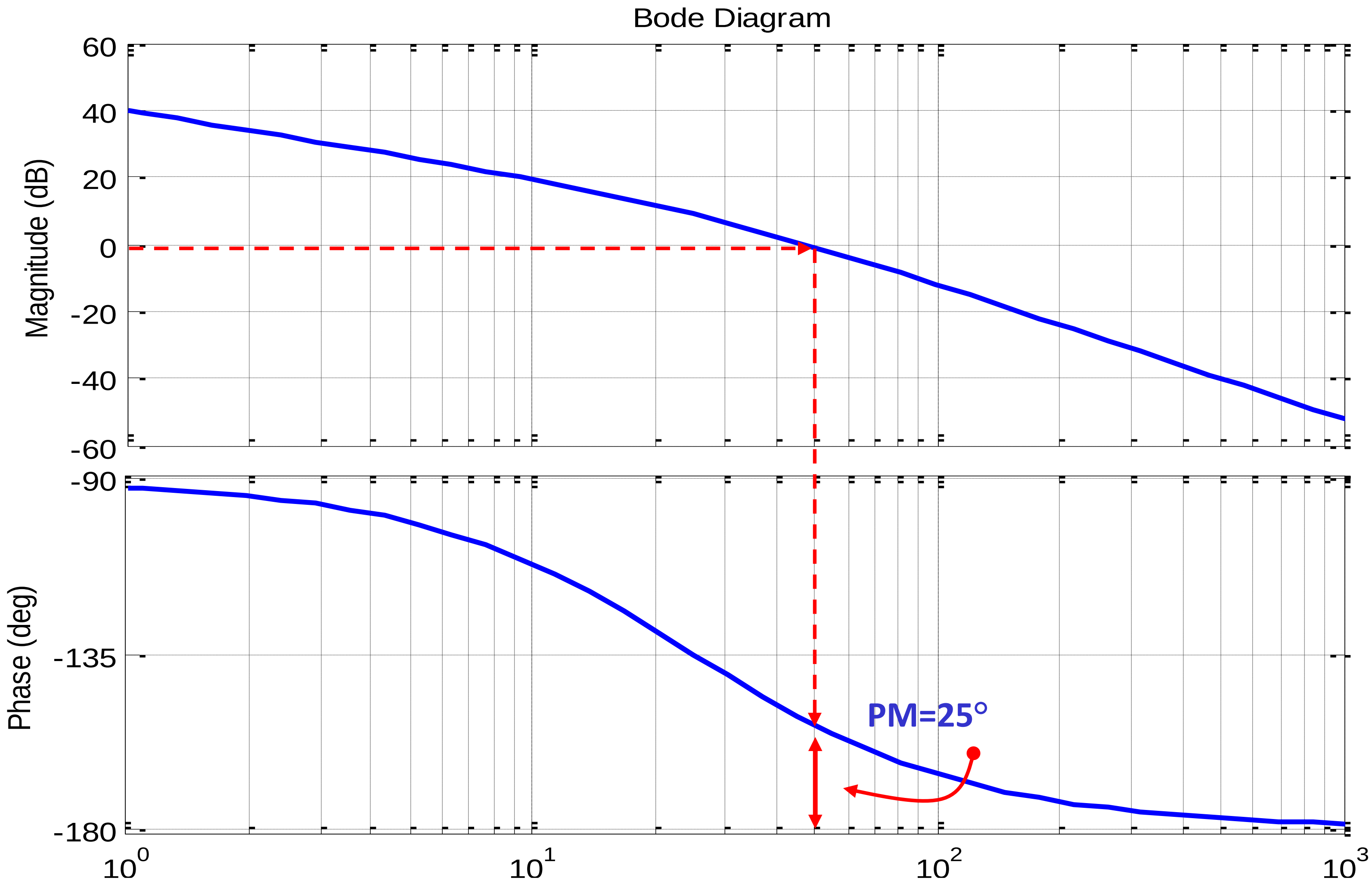
Step 1: Consider $G_c(s) = \frac{a\tau s + 1}{\tau s + 1}$ with $a > 1$ as a phase-lead controller.

Step 2: find k.

$$k_v = \lim_{s \to 0} sG_c(s)\frac{k}{s(s+25)} = 100 \qquad k = 2500$$

$$G(s) = \frac{2500}{s(s+25)} = \frac{100}{s(s/25+1)}$$

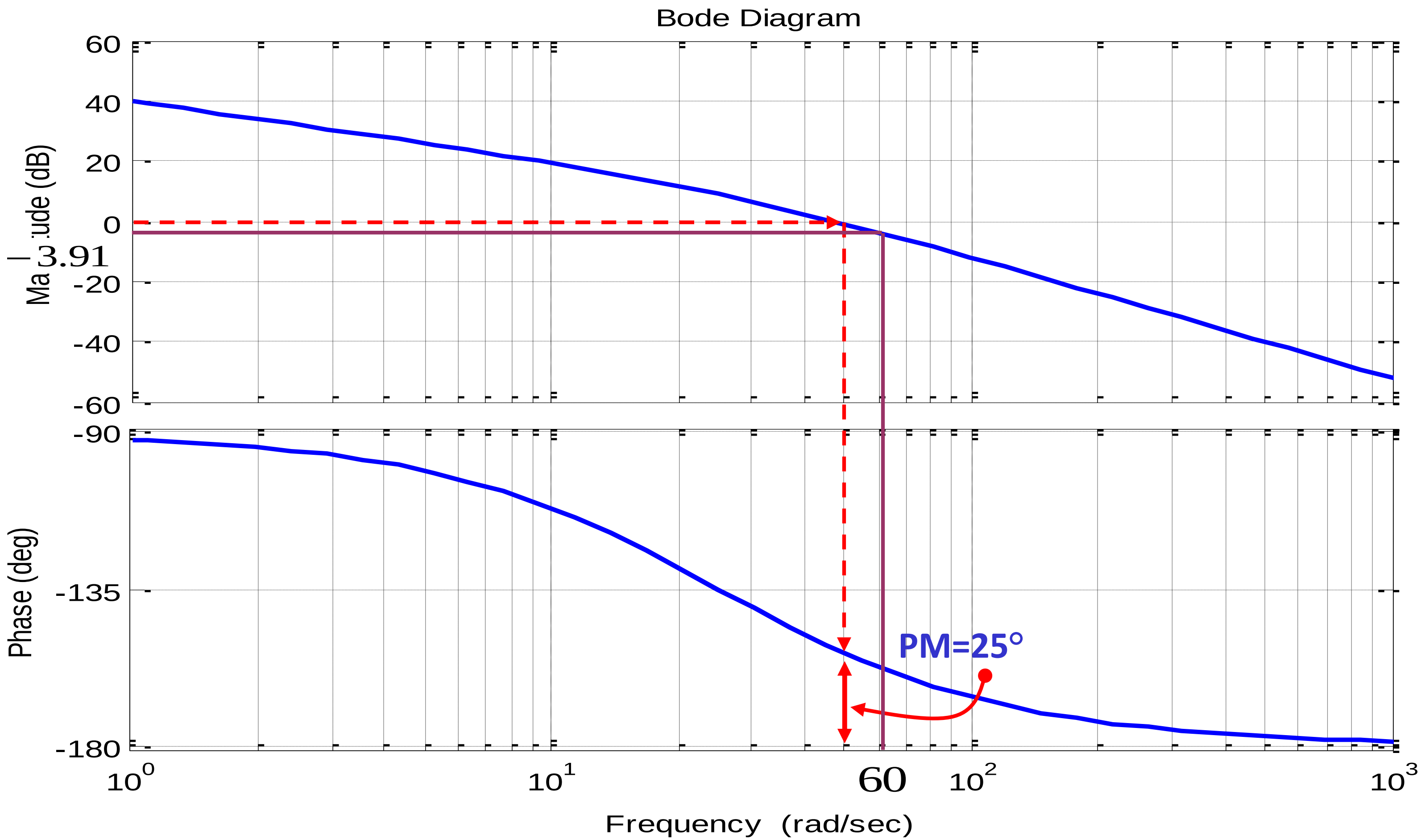
Step 3: Sketch the Bode plot for G(s).



Step 4: Find the system PM and if it is not sufficient choose the required phase by: $\varphi_m = \text{Desired PM} - \text{Existed PM} + \Delta$

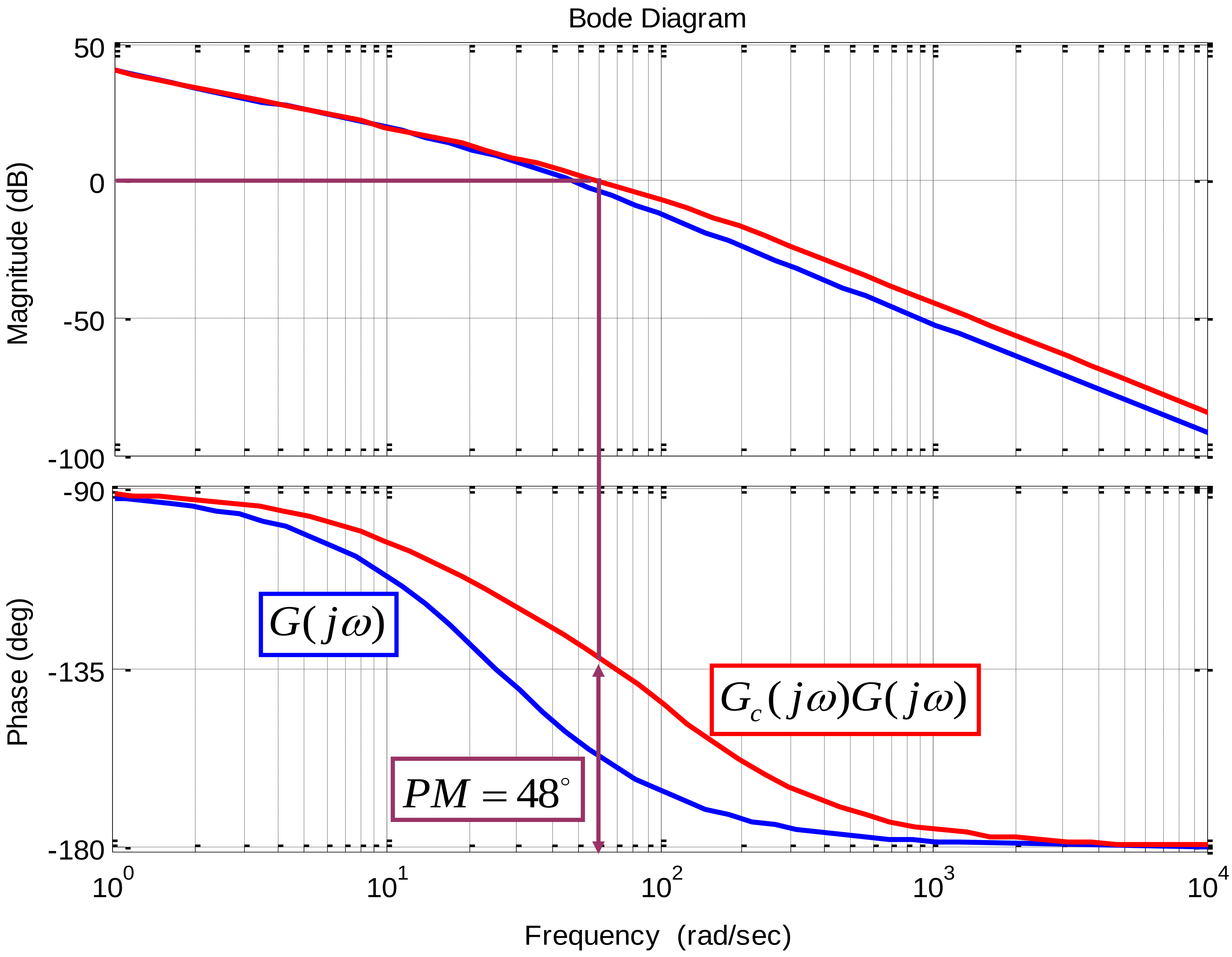
$\varphi_m = 45^\circ - 25^\circ + 5^\circ = 25^\circ$ $\quad \sin 25 = \dfrac{a-1}{a+1}$ $\quad a = 2.46$

Step 5: Put the center of the controller in the new gain crossover frequency: $20\log|G(j\omega)|_{\omega=\omega_c^{new}} = -10\log(a) = -3.91$



$\omega_c^{new} = 60$ $\omega_c^{new} = \dfrac{1}{\tau\sqrt{a}}$ $\tau = 0.0106$ $G_c(s) = \dfrac{a\tau s+1}{\tau s+1} = \dfrac{0.0261s+1}{0.0106s+1}$

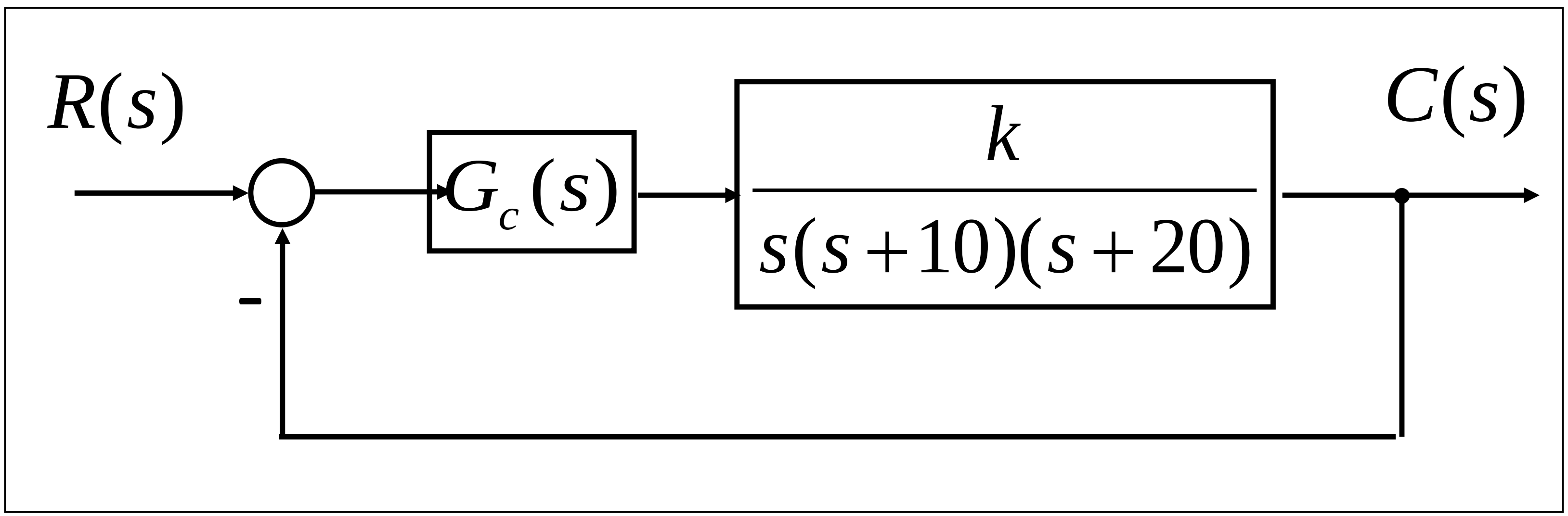
Step 6: Check the controller.



phase-lag compensation

Example 2

Design a lag controller for the following system such that the phase margin be 45° and the ramp error constant be 100.



Step 1: Consider $G_c(s) = \frac{a\tau s + 1}{\tau s + 1}$ with $a < 1$ as a phase-lag controller.

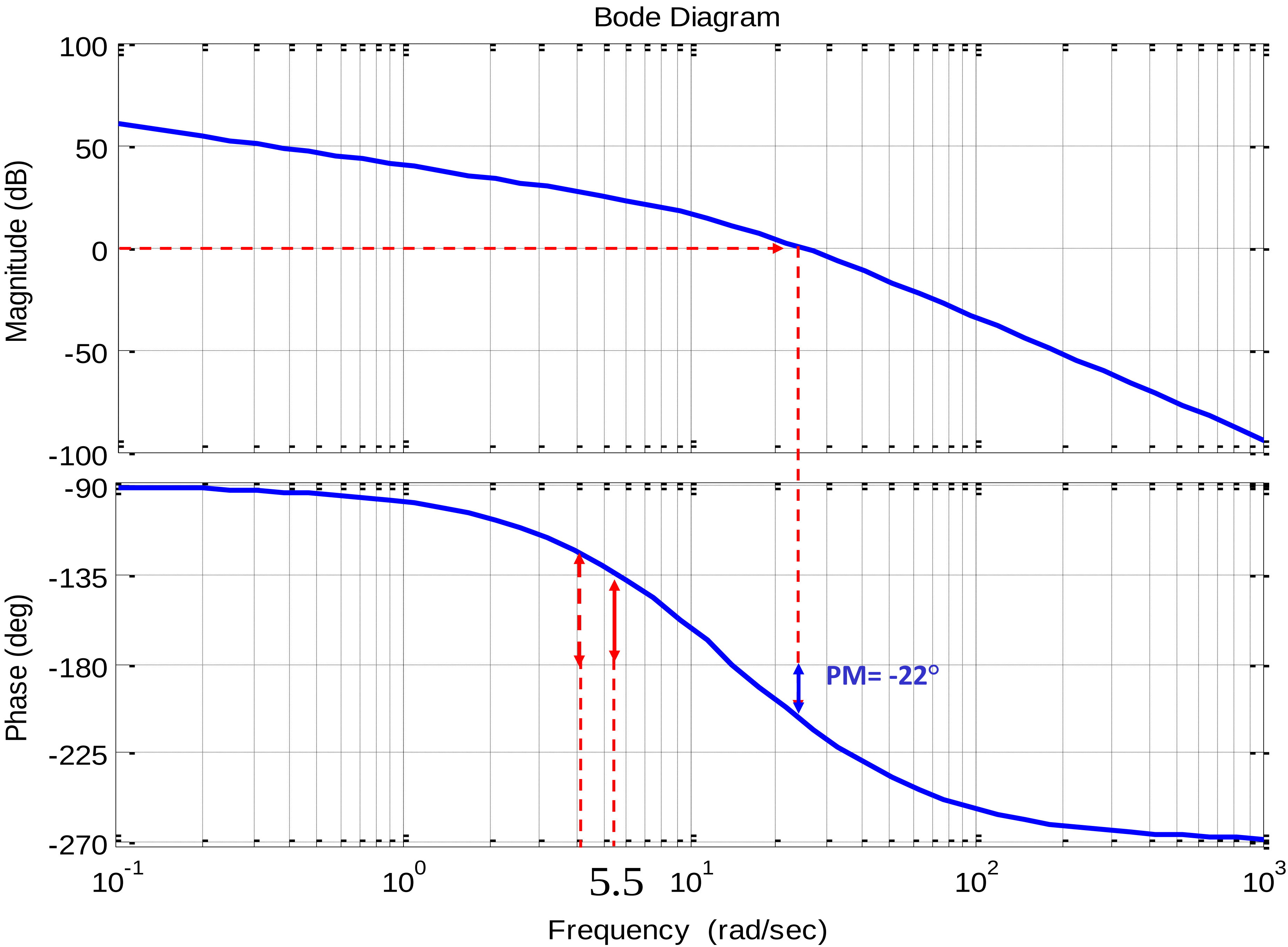
Step 2: Find k

$$k_v = \lim_{s \to 0} sG_c(s) \frac{k}{s(s+10)(s+20)} = 100$$

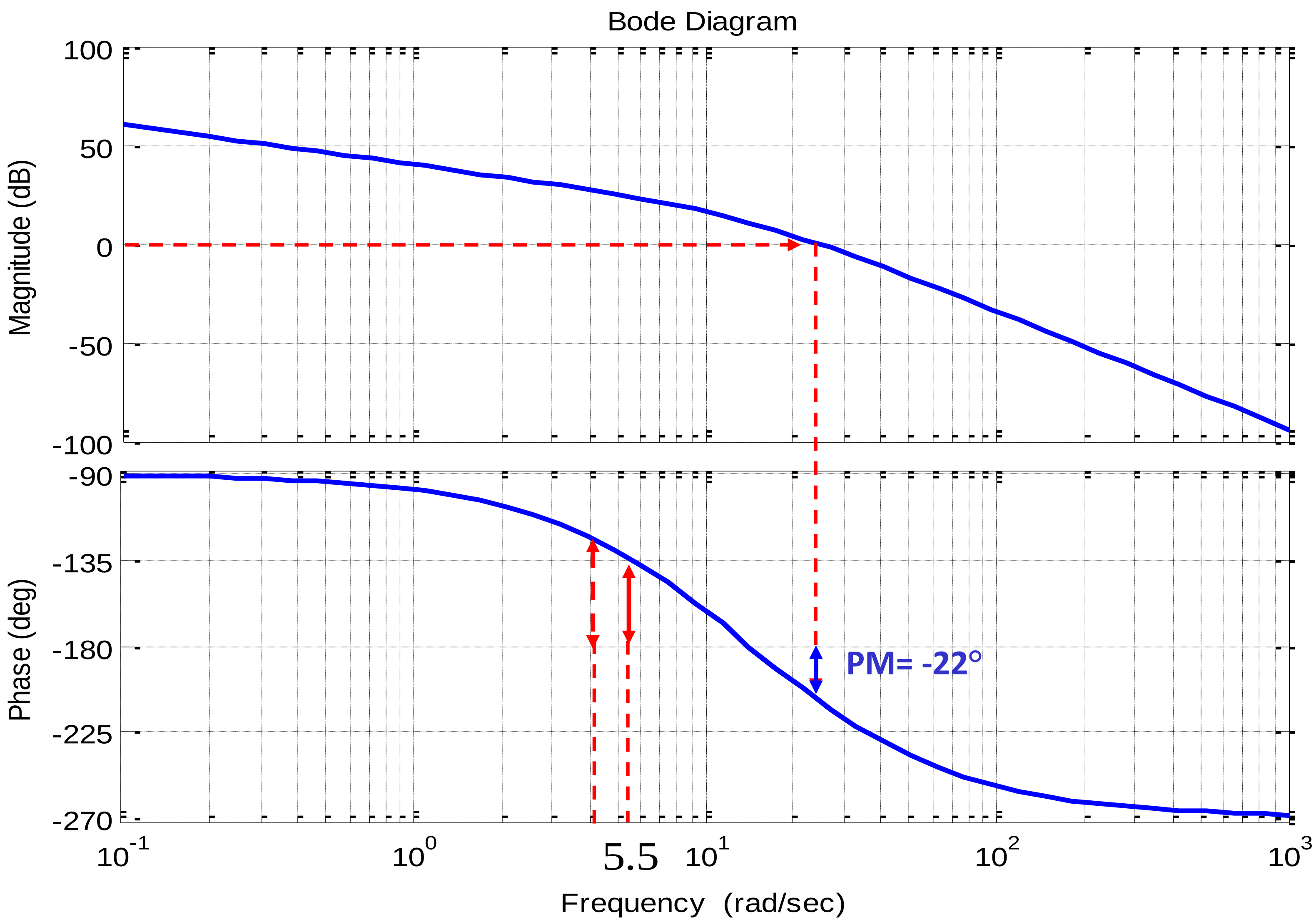
$$k = 20000$$

Step 3: Sketch the Bode plot for G(s).

$$G(s) = \frac{20000}{s(s+10)(s+20)} = \frac{100}{s(s/10+1)(s/20+1)}$$



Bode Diagram
100
50
0
-50
-100
Magnitude (dB)
-90
-135
-180
-225
-270
Phase (deg)
PM= -22°
10^-1
10^0
5.5
10^1
10^2
10^3
Frequency (rad/sec)



Step 4: Find the system PM and if it is not sufficient choose the new gain crossover frequency ω_c^{new} such that the PM is ok <u>(reduce it a little).</u>

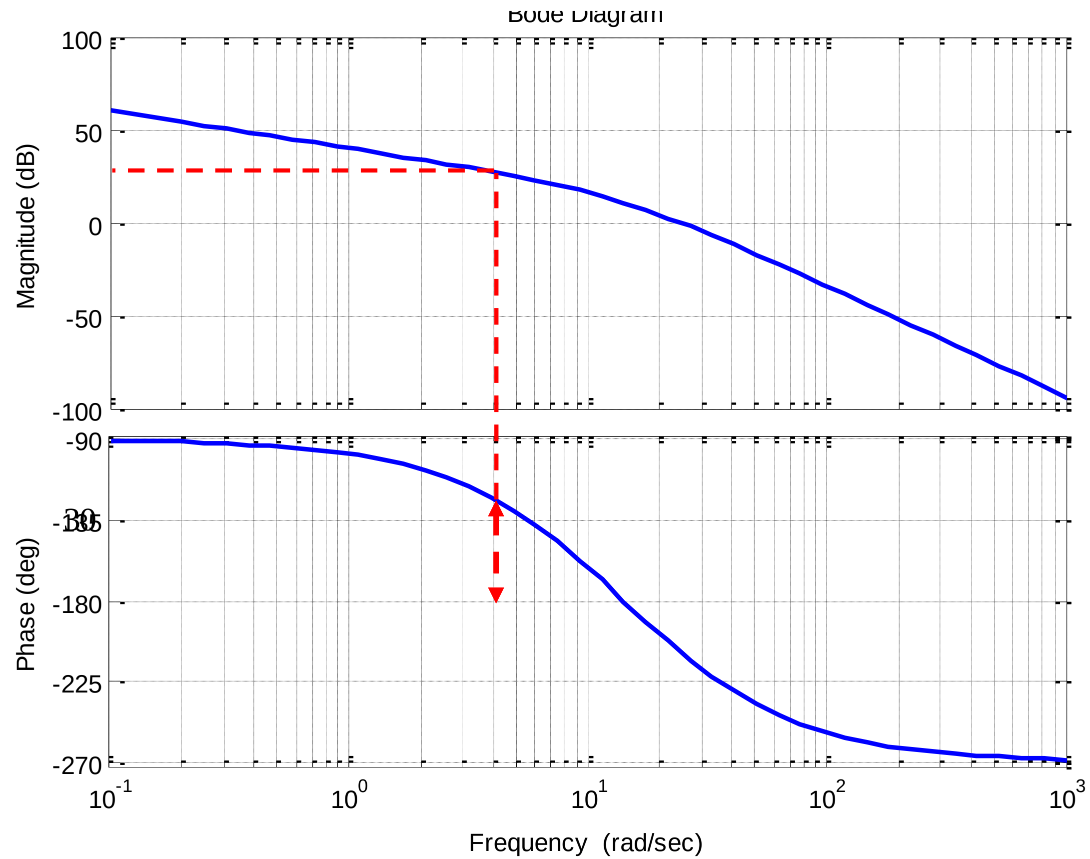
$$\omega_c^{new} = 5.5 \qquad \omega_c^{new} = 4$$

Step 5: Find the gain of the system at the new gain crossover frequency and let:

$$20\log|G(j\omega)|_{\omega=\omega_c^{new}} + 20\log(a) = 0$$

$$30 + 20\log(a) = 0$$

$$a = 10^{-\frac{30}{20}} = 0.0316$$

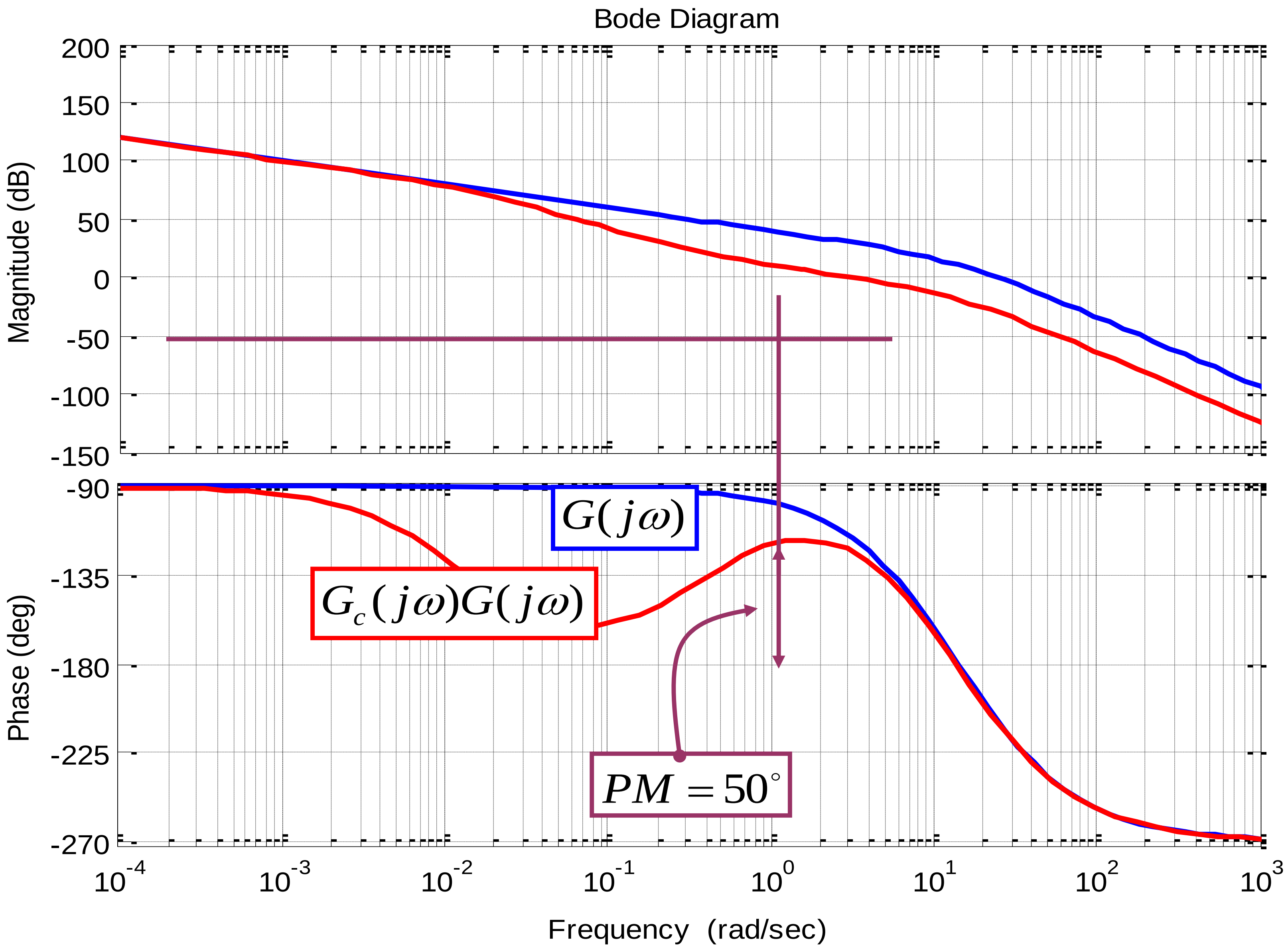


Step 6: Put the right corner of the controller sufficiently far from ω_c^{new}

$$\frac{1}{a\tau} = \frac{\omega_c^{new}}{10} = 0.4 \qquad \tau = 79$$

$$G_c(s) = \frac{a\tau s+1}{\tau s+1} = \frac{2.5s+1}{79s+1}$$

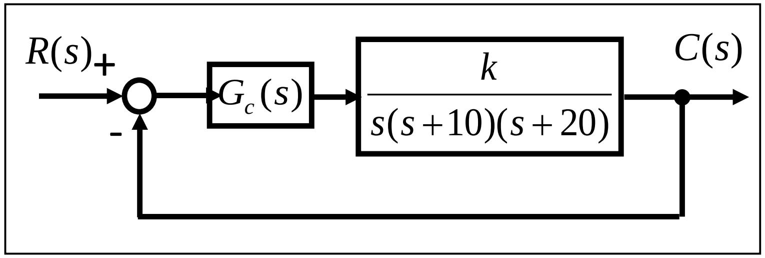
Step 7: Check the designed controller.



End of Lec

Example 6:

Design a lead controller for the following system such that the phase margin be 45° and the ramp error constant be 100.



Step 1: Consider $G_c(s) = k\frac{a\tau s+1}{\tau s+1}$ with $a>1$ as a phase-lead controller.

Note: If the plant has another gain k, let $\Rightarrow$ $G_c(s) = \frac{a\tau s+1}{\tau s+1} \qquad a>1$

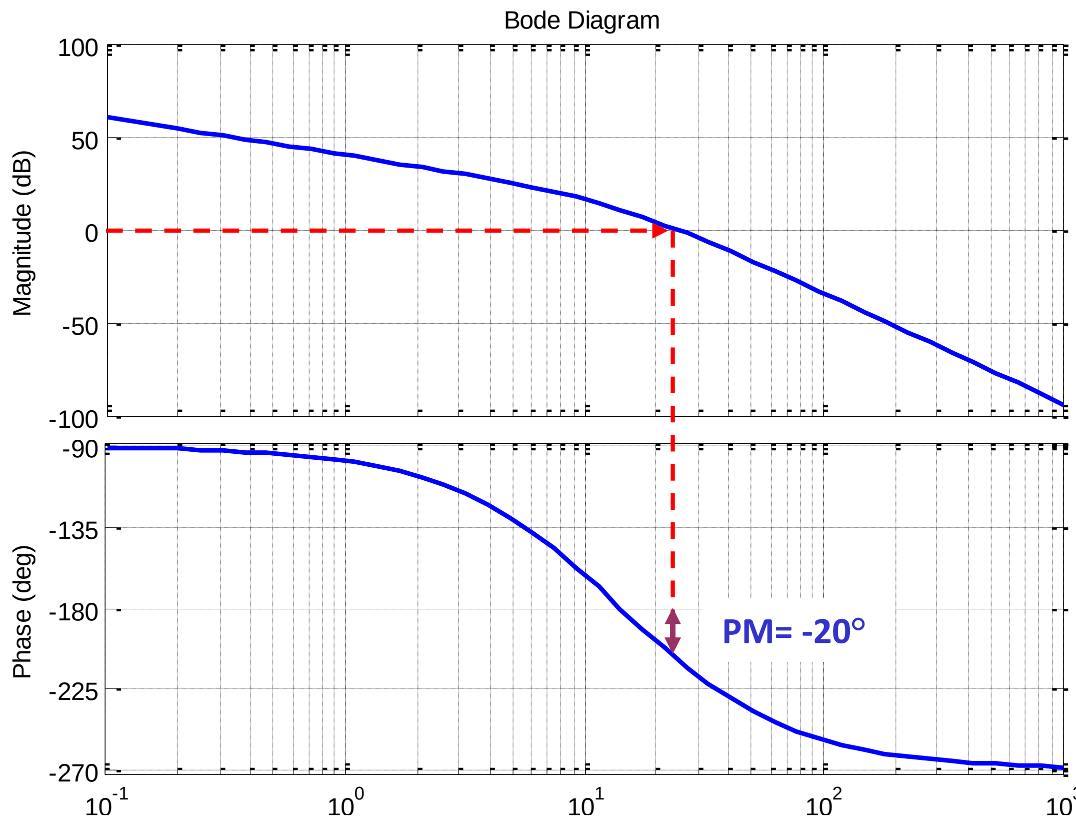
Step 2: Try to fix k according to performance request, otherwise let k=1

$$k_v = \lim_{s\to 0} sG_c(s)\frac{k}{s(s+10)(s+20)} = 100 \quad \Rightarrow \quad k = 20000$$

Step 3: Sketch the Bode plot of system (with the fixed k) without controller.

$$G(s) = \frac{20000}{s(s+10)(s+20)}$$

$$= \frac{100}{s(s/10+1)(s/20+1)}$$



Step 4: Find the system PM and if it is not sufficient choose the required phase by:

$$\varphi_m = \text{Desired PM - Existed PM} + \Delta$$

$$\varphi_m = 45^\circ - (-20) + 5^\circ = 70^\circ \quad \Rightarrow \quad \sin 70 = \frac{a-1}{a+1} \quad \Rightarrow \quad a = 32$$

Step 5: Put the center of controller in the new gain crossover frequency:

$$20\log\left|G(j\omega)\right|_{\omega=\omega_c^{new}} + 10\log(a) = 0$$

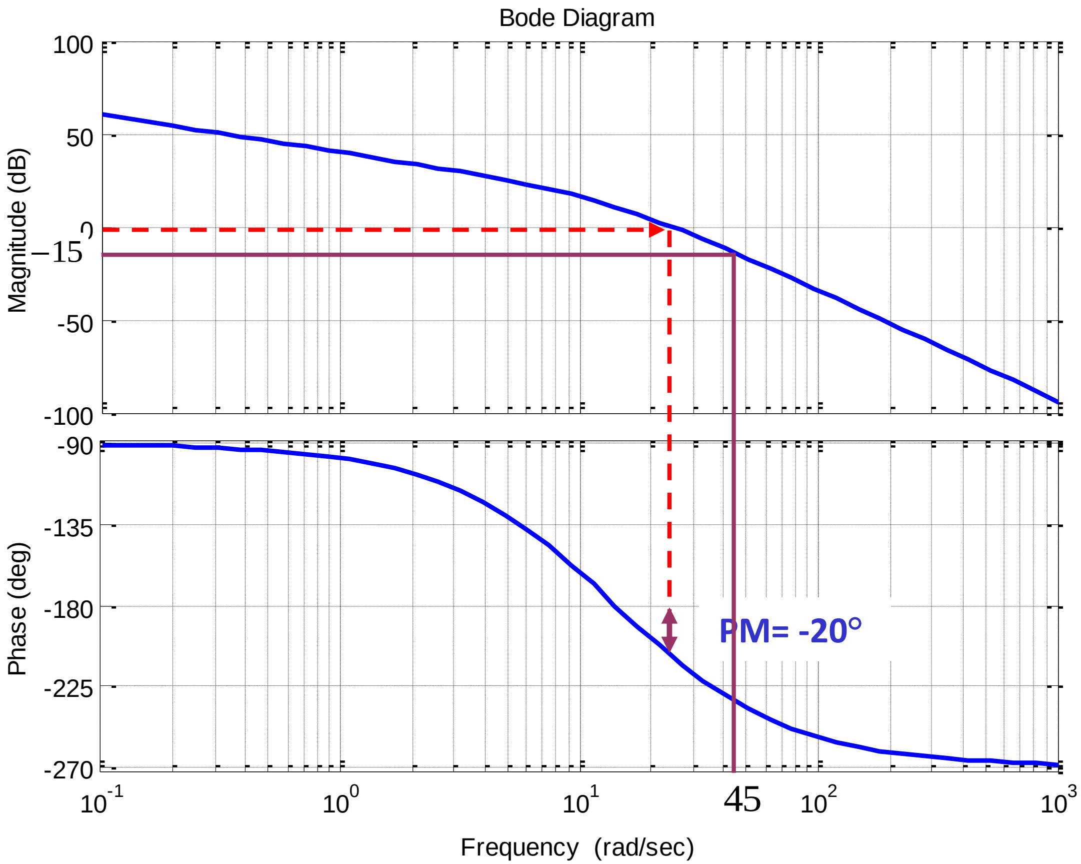
$$20\log\left|G(j\omega)\right|_{\omega=\omega_c^{new}} = -10\log(a) = -15$$

$$\omega_c^{new} = 45$$

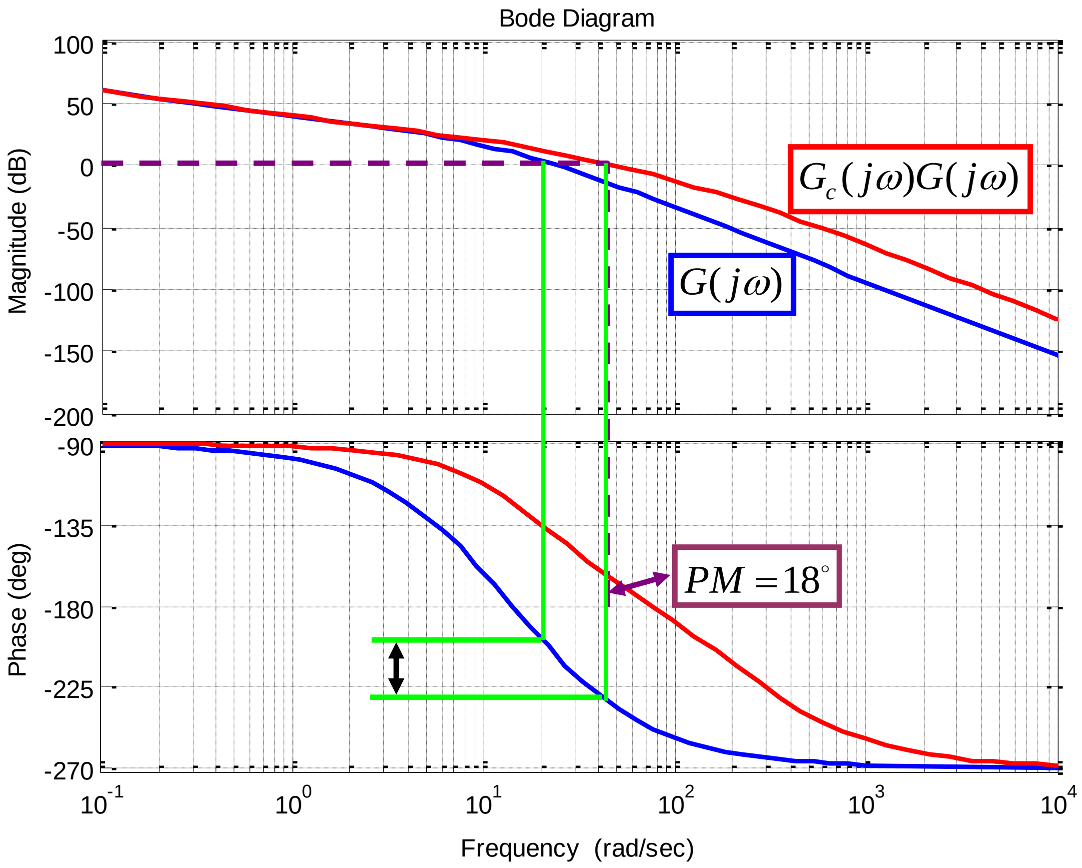
$$\omega_c^{new} = \frac{1}{\tau\sqrt{a}}$$

$$\tau = 0.0039$$

$$G_c(s) = \frac{a\tau s+1}{\tau s+1} = \frac{0.1248s+1}{0.0039s+1}$$



Step 6: Check the controller.

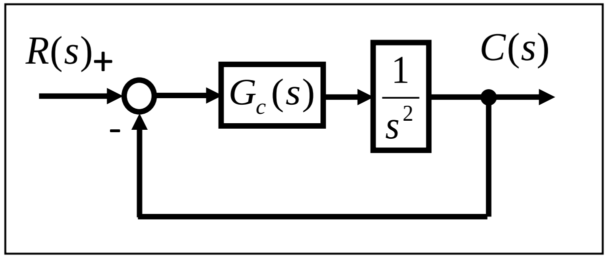


Note: Design is not possible!

Why?

Example 7:

Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec



Step 1: Consider $G_c(s) = k\dfrac{a\tau s+1}{\tau s+1}$ with $a>1$ as a phase-lag controller.

Note: If the plant has another gain k, let $\Rightarrow$ $G_c(s) = k\dfrac{a\tau s+1}{\tau s+1}$ $\quad a>1$

Step 2: Try to fix k according to performance request, otherwise let k=1

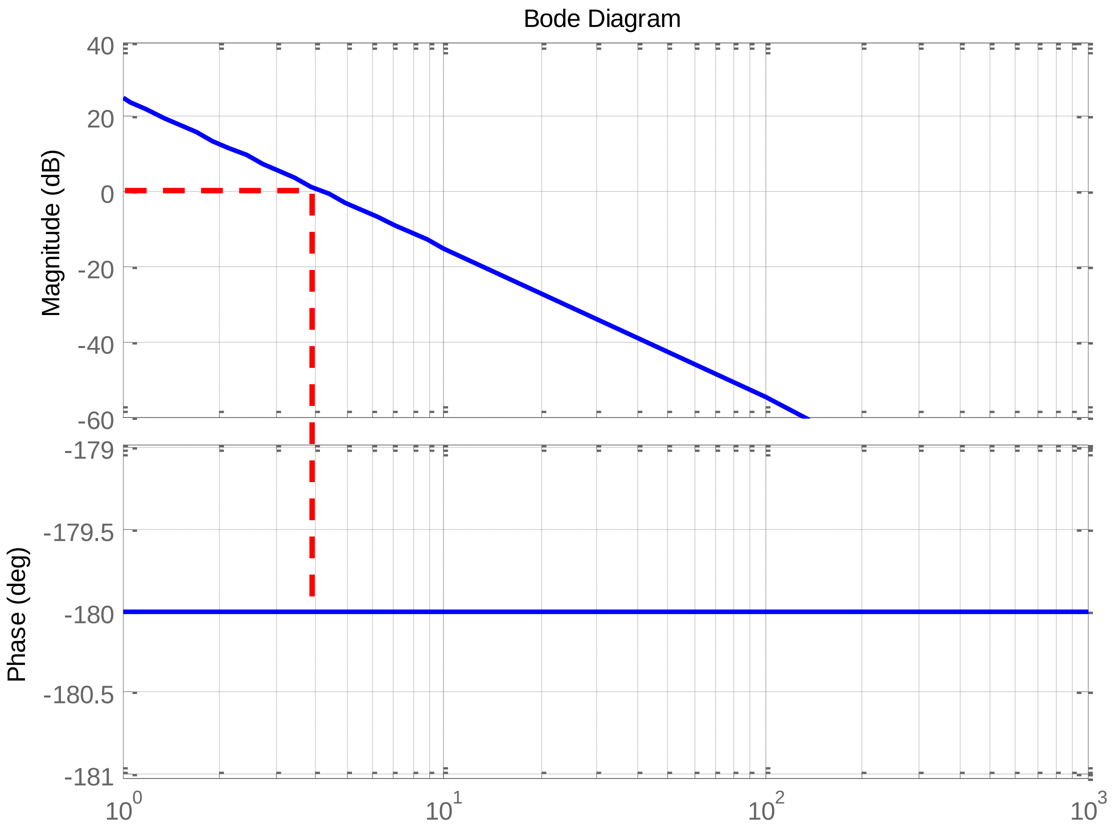
$$20\log\left|\frac{k}{\omega^2}\right|_{\omega=5} = -3$$

Why?

$\Rightarrow$ $k = 17.7$

Step 3: Sketch the Bode plot of system (with the fixed k) without controller.

$$G(s) = \frac{17.7}{s^2}$$



Step 4: Find the system PM and if it is not sufficient choose the required phase by:

$$\varphi_m = \text{Desired PM - Existed PM} + \Delta$$

$$\varphi_m = 45^\circ - (0) + 0^\circ = 45^\circ \quad \Rightarrow \quad \sin 45^\circ = \frac{a-1}{a+1} \quad \Rightarrow \quad a = 5.8$$

Step 5: Put the center of controller in the new gain crossover frequency:

$$20\log\left|G(j\omega)\right|_{\omega=\omega_c^{new}} + 10\log(a) = 0$$

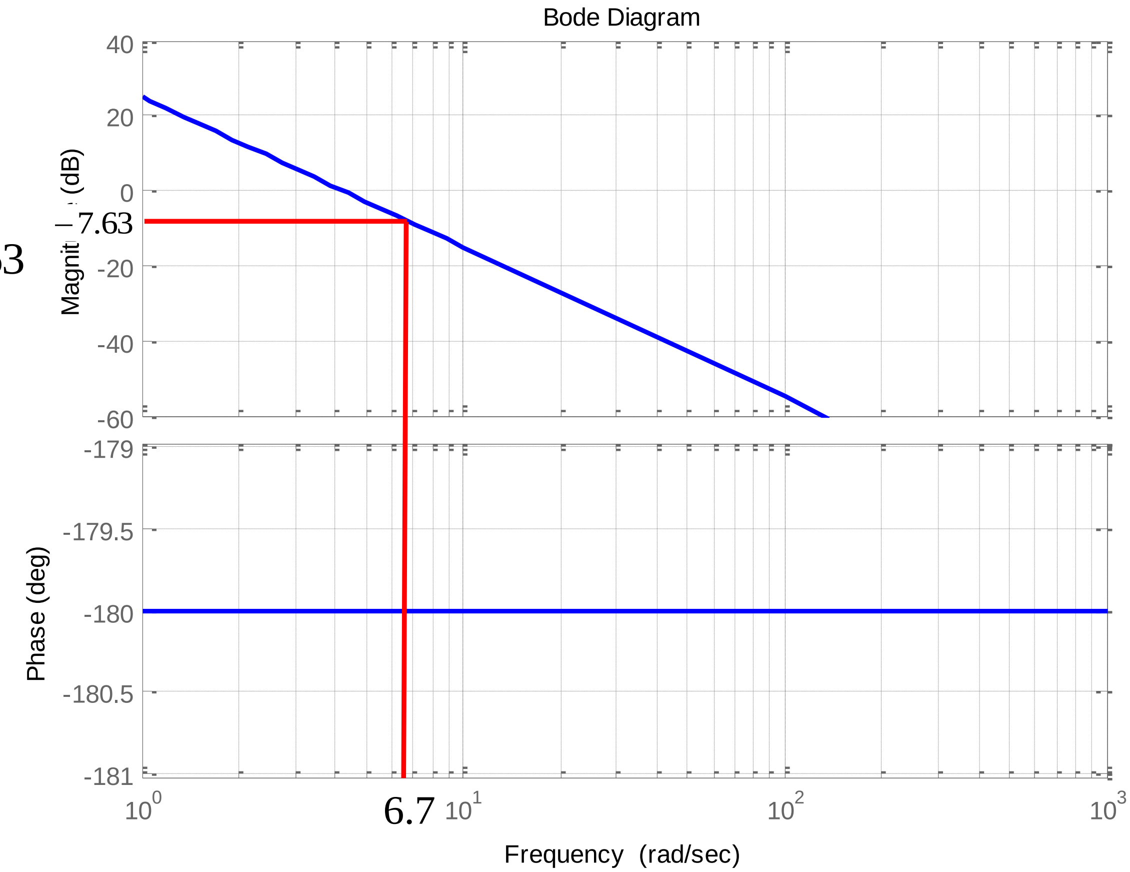
$$20\log\left|G(j\omega)\right|_{\omega=\omega_c^{new}} = -10\log(a) = -7.63$$

$$\omega_c^{new} = 6.7$$

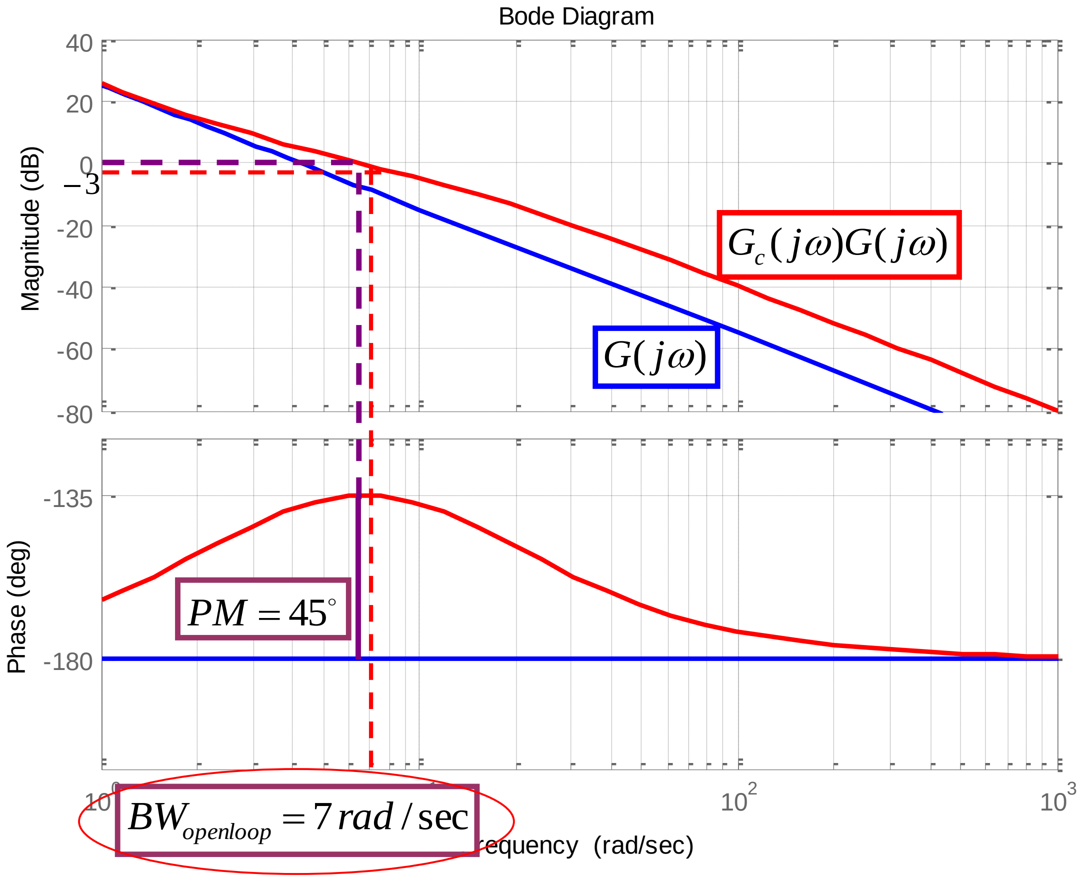
$$\omega_c^{new} = \frac{1}{\tau\sqrt{a}}$$

$$\tau = 0.062$$

$$G_c(s) = \frac{a\tau s+1}{\tau s+1} = \frac{0.3596s+1}{0.062s+1}$$



Step 6: Check the controller.

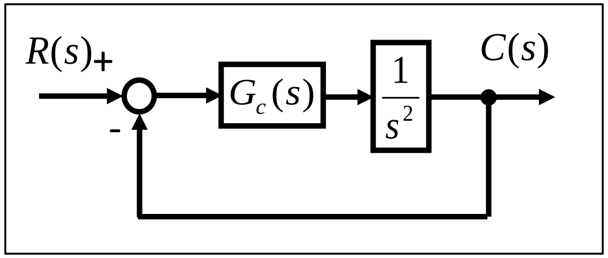


Controller is not ok

Try again

Example 7(Continue):

Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec



Step 2: Try to fix k according to performance request, otherwise let k=1

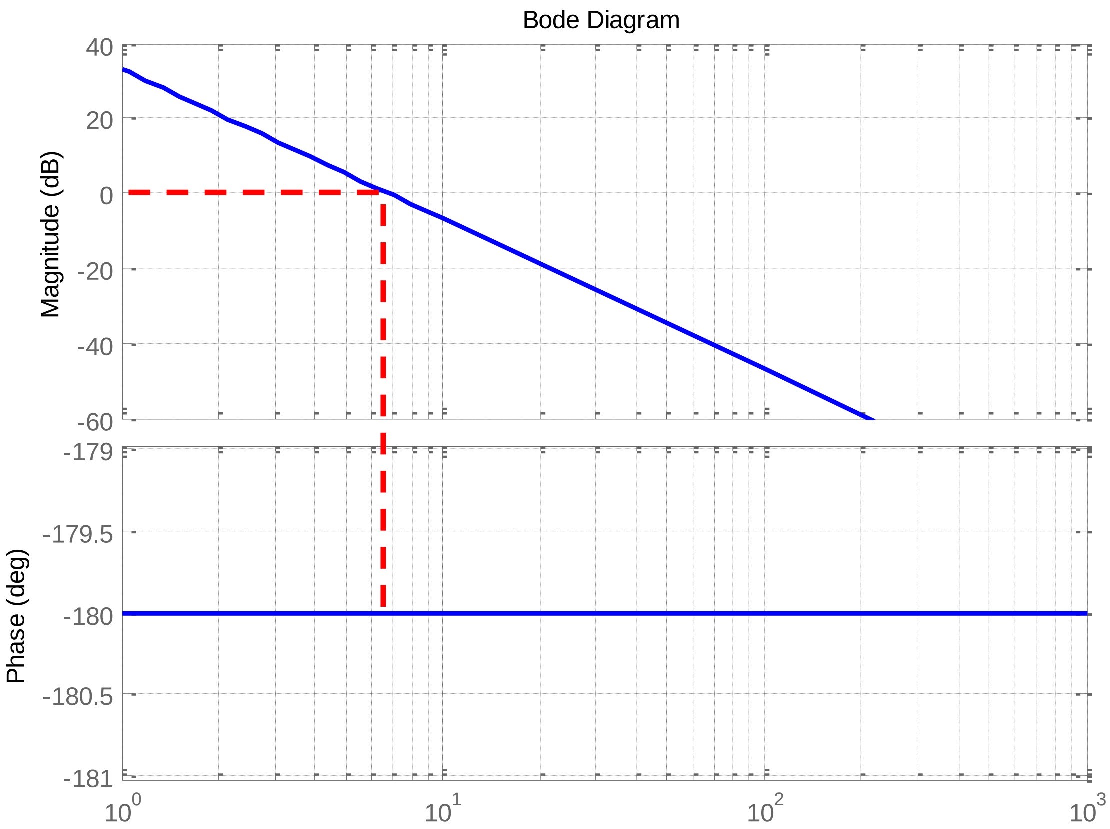
Open loop bandwidth is near to gain crossover frequency so:

$$20\log\left|\frac{k}{\omega^2}\right|_{\omega=5} = -3 \quad\Rightarrow\quad k = 17.7$$

$$20\log\left|\frac{k}{\omega^2}\right|_{\omega=7} = -3 \quad\Rightarrow\quad k = 35.7$$

Step 3: Sketch the Bode plot of system (with the fixed k) without controller.

$$G(s) = \frac{35.7}{s^2}$$



Step 4: Find the system PM and if it is not sufficient choose the required phase by:

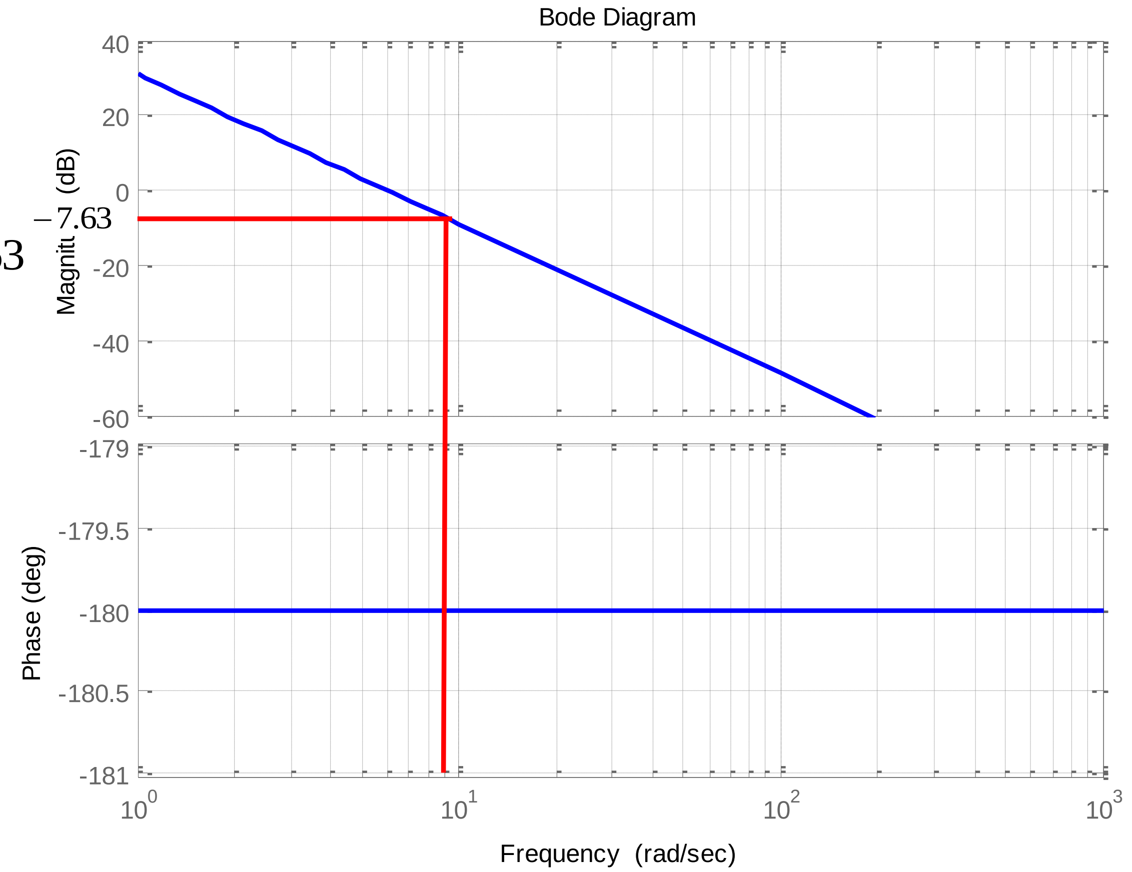
$$\varphi_m = \text{Desired PM} - \text{Existed PM} + \Delta$$

$$\varphi_m = 45^\circ - (0) + 0^\circ = 45^\circ \quad \Rightarrow \quad \sin 45^\circ = \frac{a-1}{a+1} \quad \Rightarrow \quad a = 5.8$$

Step 5: Put the center of controller in the new gain crossover frequency:

$$20\log\left|G(j\omega)\right|_{\omega=\omega_c^{new}} + 10\log(a) = 0$$

$$20\log\left|G(j\omega)\right|_{\omega=\omega_c^{new}} = -10\log(a) = -7.63$$



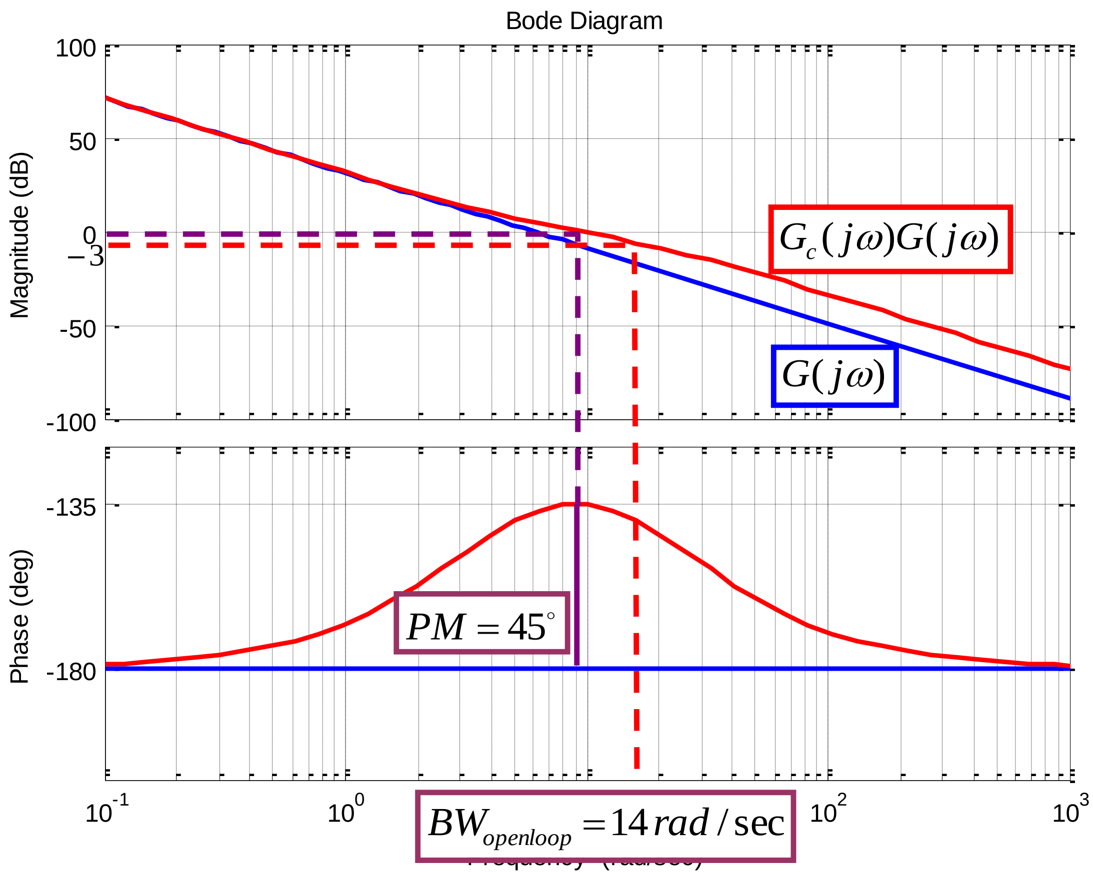
$$\omega_c^{new} = 9$$

$$\omega_c^{new} = \frac{1}{\tau\sqrt{a}}$$

$$\tau = 0.0461$$

$$G_c(s) = \frac{a\tau s+1}{\tau s+1} = \frac{0.2674s+1}{0.0461s+1}$$

Step 6: Check the controller.

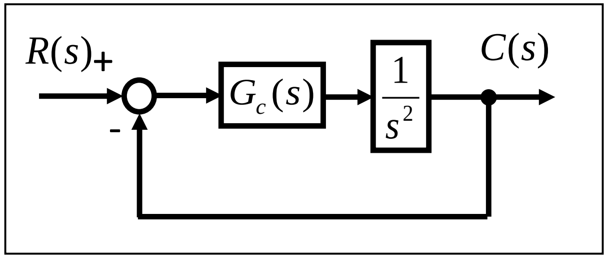


Controller is not ok

Try again

Example 7(Continue):

Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec



Step 2: Try to fix k according to performance request, otherwise let k=1

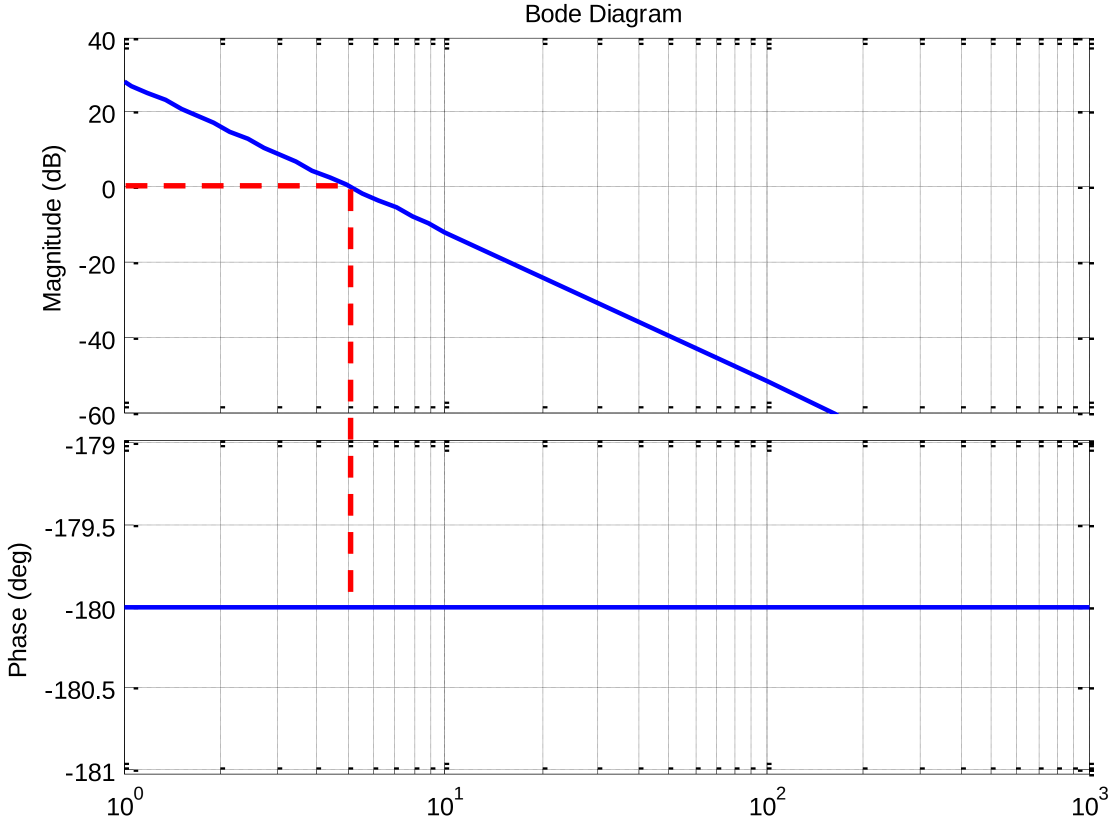
Open loop bandwidth is near to gain crossover frequency so:

$$20\log\left|\frac{k}{\omega^2}\right|_{\omega=7} = -3 \quad \Rightarrow \quad k = 35.7$$

$$20\log\left|\frac{k}{\omega^2}\right|_{\omega=6} = -3 \quad \Rightarrow \quad k = 25.5$$

Step 3: Sketch the Bode plot of system (with the fixed k) without controller.

$$G(s) = \frac{25.5}{s^2}$$



Step 4: Find the system PM and if it is not sufficient choose the required phase by:

$$\varphi_m = \text{Desired PM} - \text{Existed PM} + \Delta$$

$$\varphi_m = 45^\circ - (0) + 0^\circ = 45^\circ \quad \Rightarrow \quad \sin 45^\circ = \frac{a-1}{a+1} \quad \Rightarrow \quad a = 5.8$$

Step 5: Put the center of controller in the new gain crossover frequency:

$$20\log\left|G(j\omega)\right|_{\omega=\omega_c^{new}} + 10\log(a) = 0$$

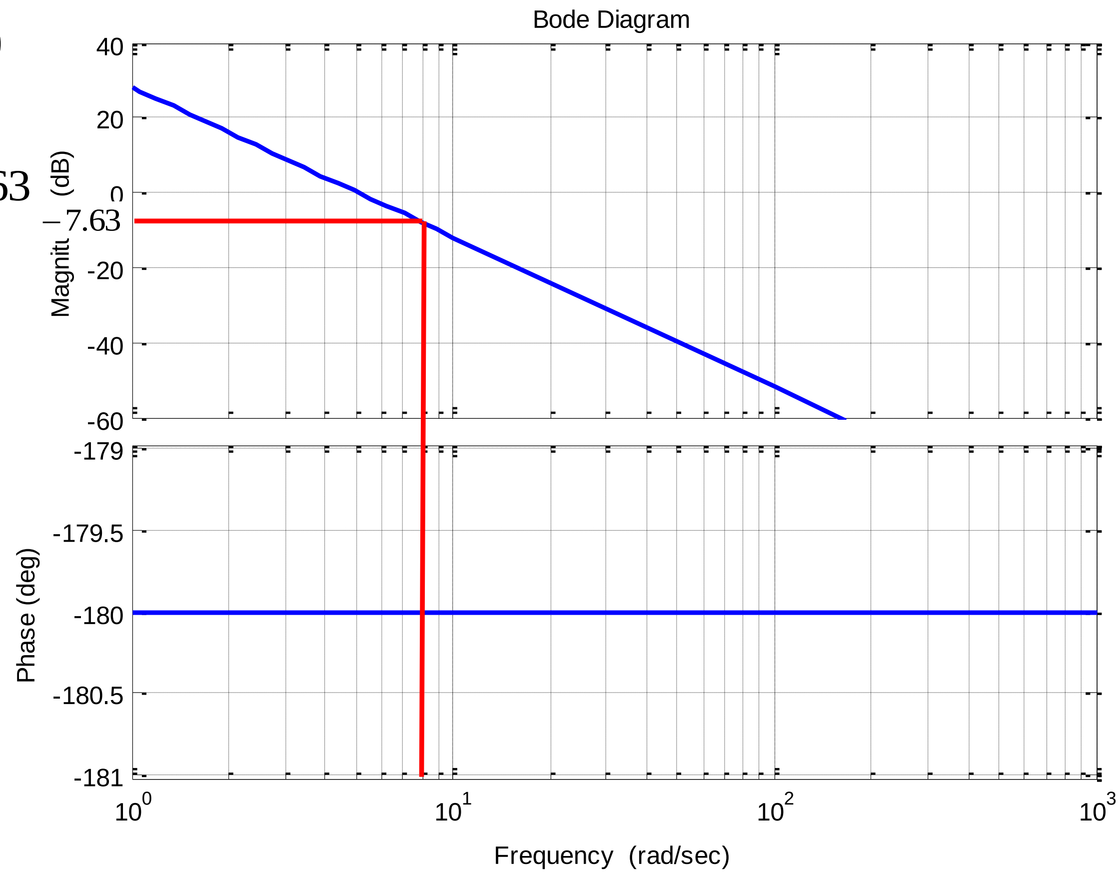
$$20\log\left|G(j\omega)\right|_{\omega=\omega_c^{new}} = -10\log(a) = -7.63$$

$$\omega_c^{new} = 8$$

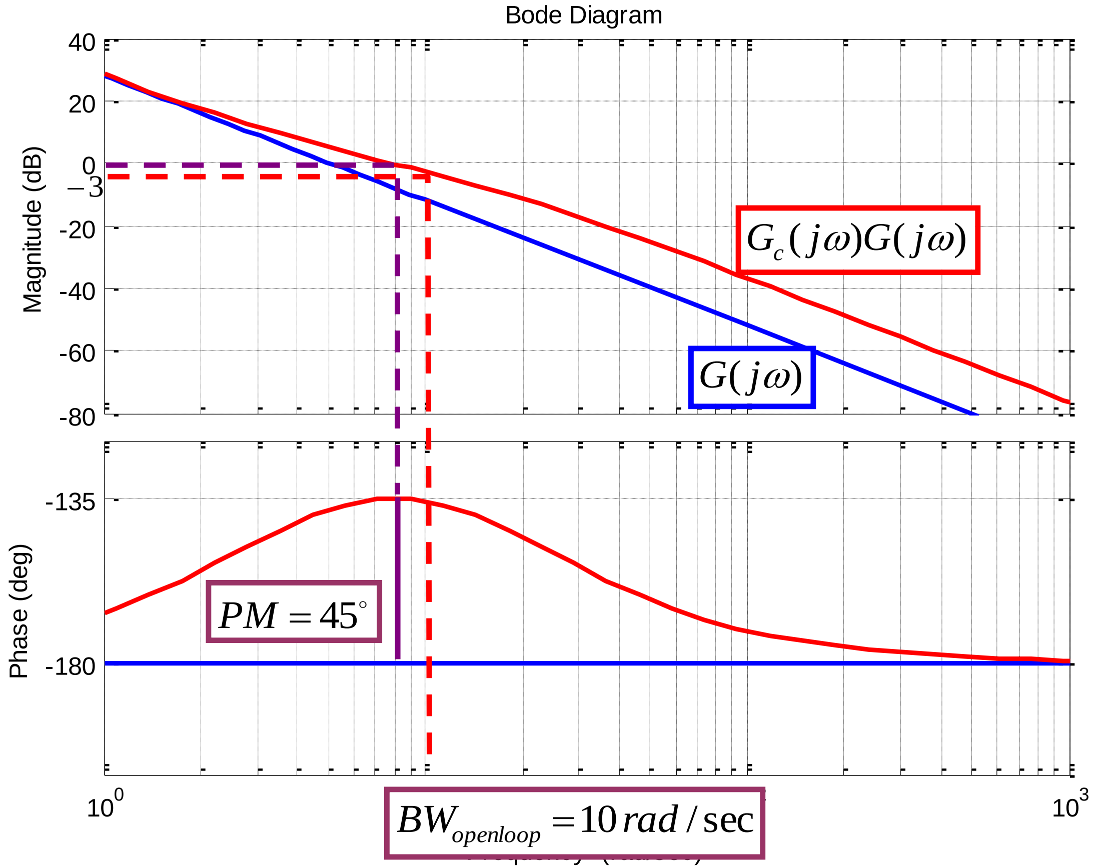
$$\omega_c^{new} = \frac{1}{\tau\sqrt{a}}$$

$$\tau = 0.0519$$

$$G_c(s) = \frac{a\tau s+1}{\tau s+1} = \frac{0.301s+1}{0.0519s+1}$$



Step 6: Check the controller.



<u>Controller is ok</u>