Control Systems I

Lecture 10

Nyquist Plot

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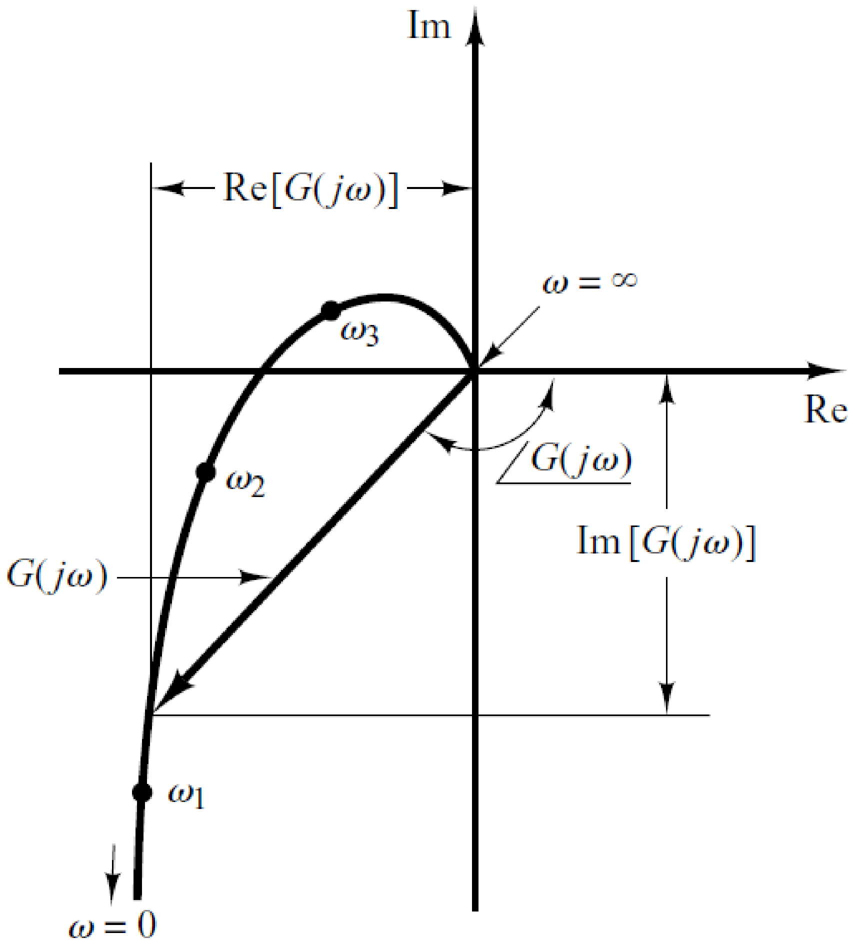
Nyquist Plot (Polar Plot)

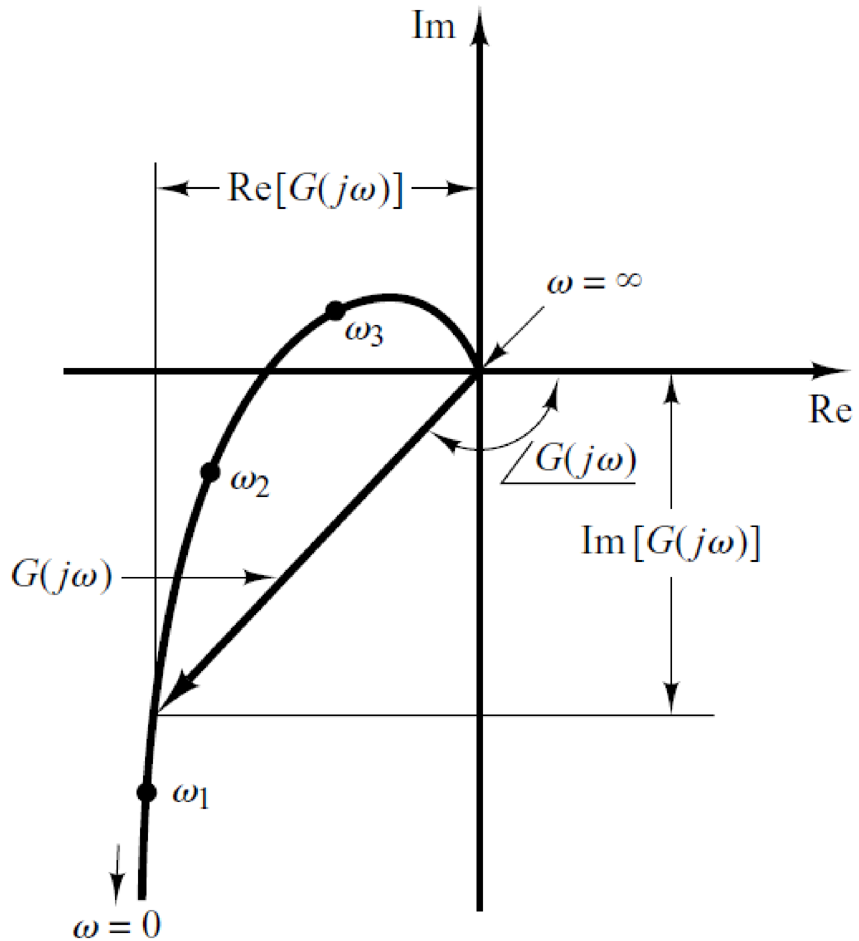
- The polar plot of a sinusoidal transfer function G(jω) is a plot of the magnitude of G(jω) versus the phase angle of G(jω) on polar coordinates as ω is varied from zero to infinity.

- Thus, the polar plot is the locus of vectors $|G(j\omega)|\,\underline{/G(j\omega)}$ as ω is varied from zero to infinity.



Nyquist Plot (Polar Plot)

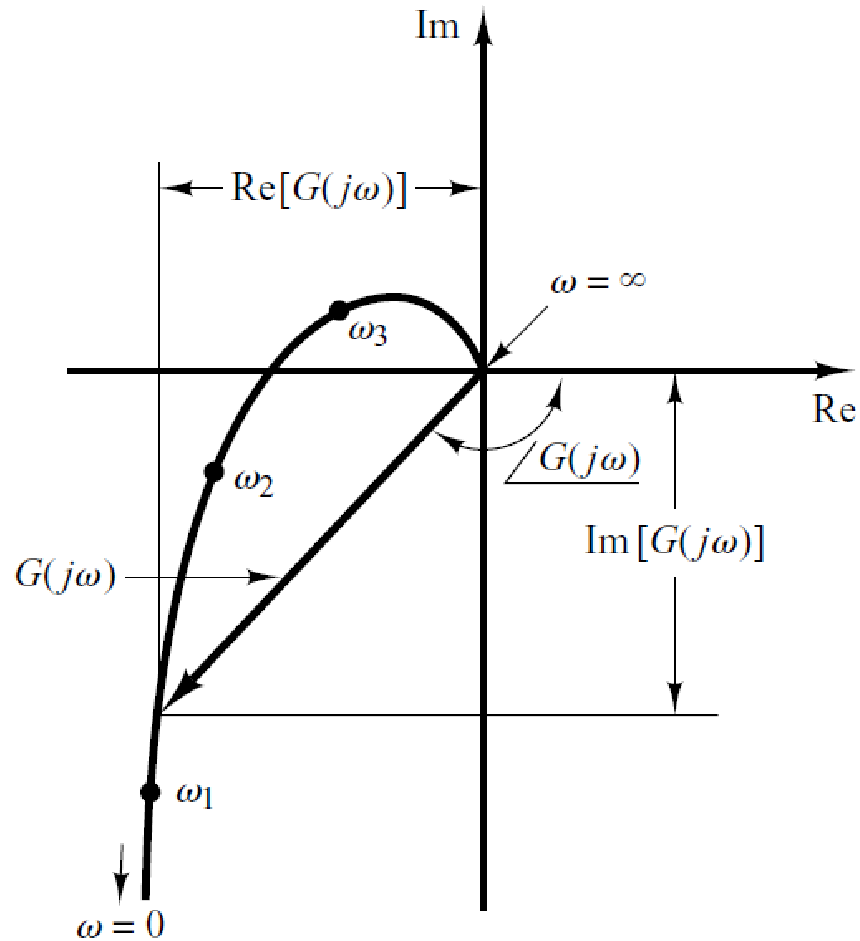
- Each point on the polar plot of G(jω) represents the terminal point of a vector at a particular value of ω.
- The projections of G(jω) on the real and imaginary axes are its real and imaginary components.



Nyquist Plot (Polar Plot)

- An advantage in using a polar plot is that it depicts the frequency response characteristics of a system over the entire frequency range in a single plot.

- One disadvantage is that the plot does not clearly indicate the contributions of each individual factor of the open-loop transfer function.



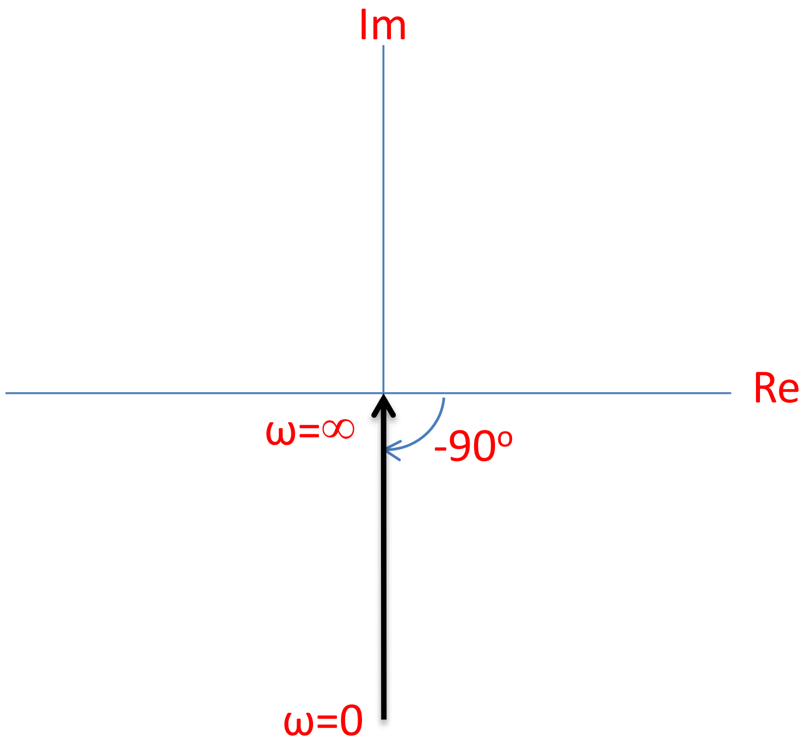
Nyquist Plot of Integral and Derivative Factors

- The polar plot of G(jω)=1/jω is the negative imaginary axis, since

$$G(j\omega) = \frac{1}{j\omega}$$

$$G(j\omega) = \frac{1}{j\omega} \times \frac{-j\omega}{-j\omega} = -j\frac{1}{\omega}$$

In polar form $G(j\omega) = \frac{1}{\omega}\angle -90^\circ$

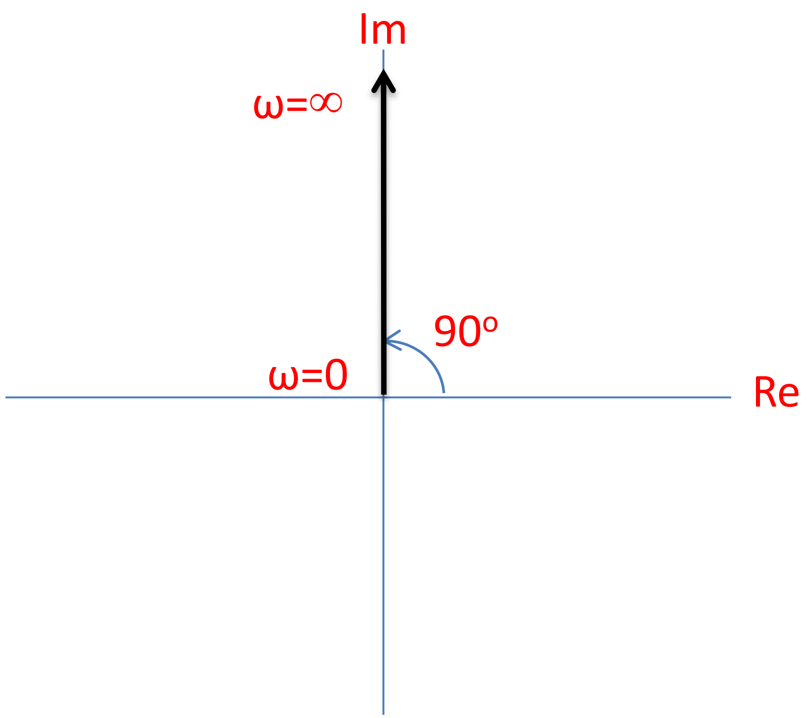


Nyquist Plot of Integral and Derivative Factors

- The polar plot of G(jω)=jω is the positive imaginary axis, since

$$G(j\omega) = j\omega$$

In polar form $G(j\omega) = \omega\angle 90^\circ$

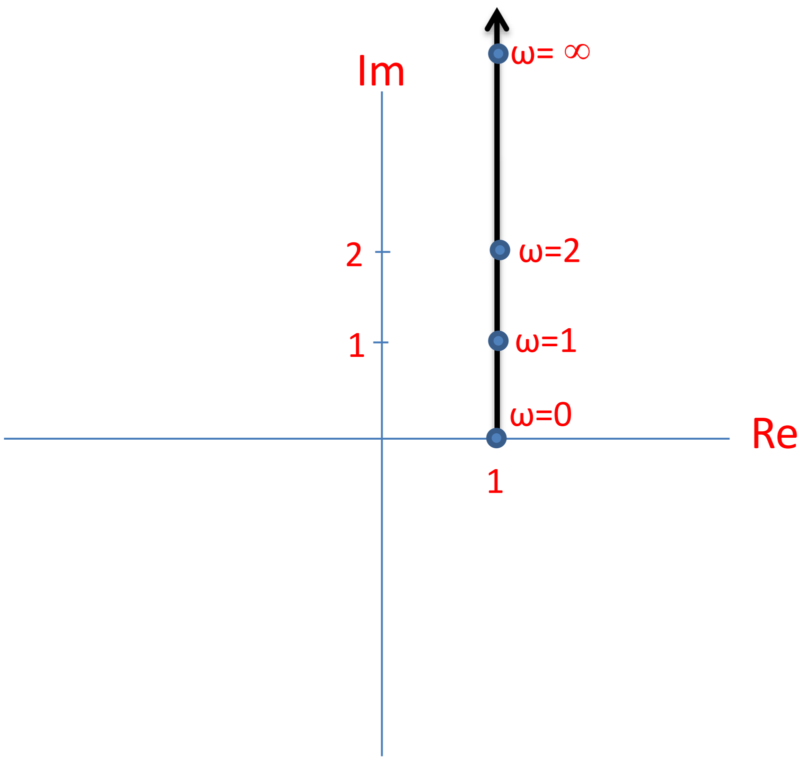


Nyquist Plot of First Order Factors

- The polar plot of first order factor in numerator is

$$G(j\omega) = j\omega + 1$$

ω	Re	Im
0	1	0
1	1	1
2	1	2
∞	1	∞



Nyquist Plot of First Order Factors

- The polar plot of first order factor in denominator is

$$G(j\omega) = \frac{1}{j\omega + 1}$$

$$G(j\omega) = \frac{1}{j\omega + 1} \times \frac{1 - j\omega}{1 - j\omega}$$

$$G(j\omega) = \frac{1 - j\omega}{1 + \omega^2}$$

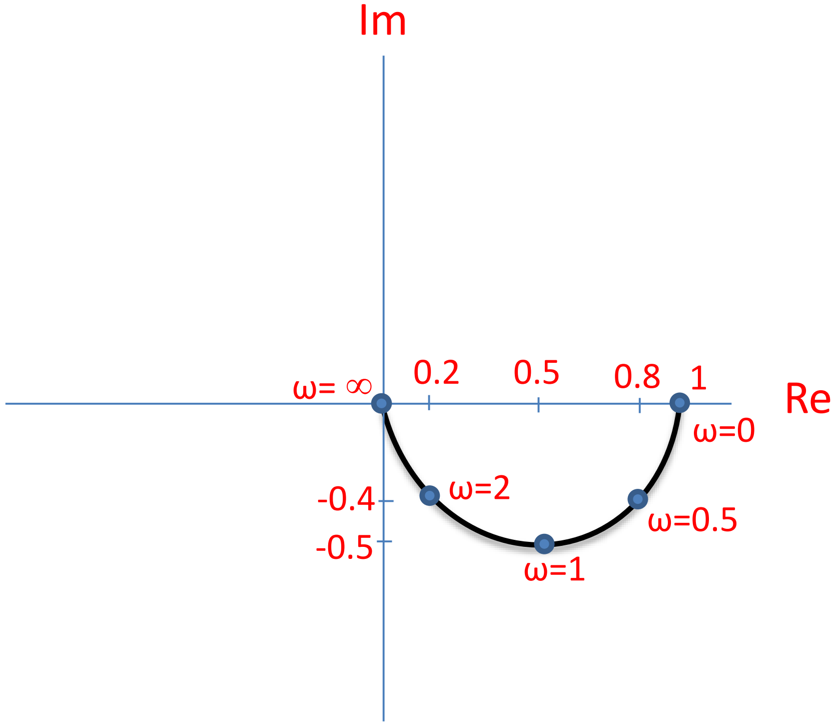
$$G(j\omega) = \frac{1}{1 + \omega^2} - j\frac{\omega}{1 + \omega^2}$$

ω	Re	Im
0	1	0
0.5	0.8	0.4
1	1/2	-1/2
2	1/5	-2/5
∞	0	undefined

Nyquist Plot of First Order Factors

- The polar plot of first order factor in denominator is

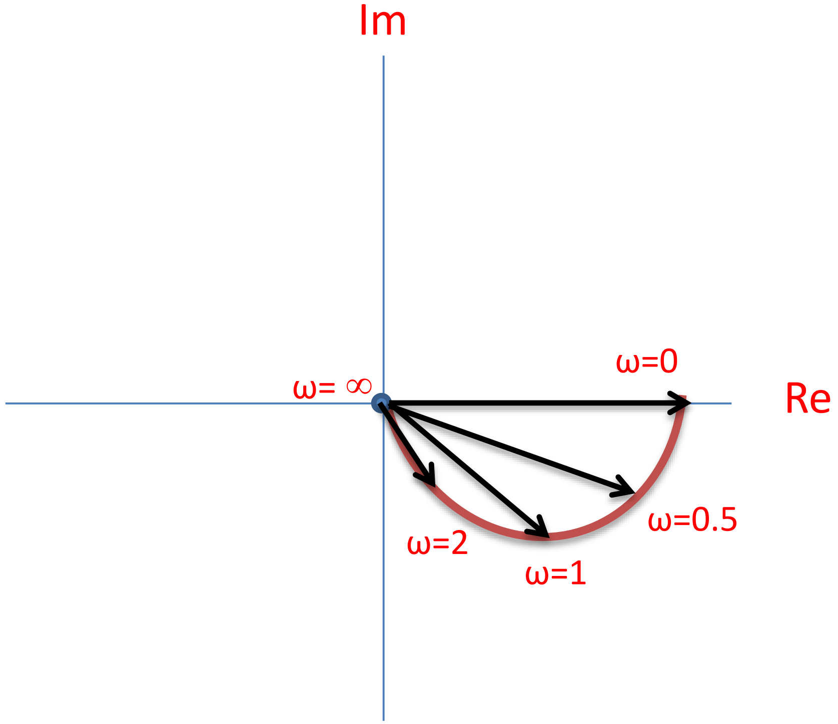
ω	Re	Im
0	1	0
0.5	0.8	-0.4
1	0.5	-0.5
2	0.2	-0.4
∞	0	undefined



Nyquist Plot of First Order Factors

- The polar plot of first order factor in denominator is

ω	Re	Im	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	1	0	1	0°
0.5	0.8	-0.4	0.9	-26°
1	0.5	-0.5	0.7	-45°
2	0.2	-0.4	0.4	-63°
∞	0	0	0	-90



Example#1

- Draw the polar plot of following open loop transfer function.

$$G(s) = \frac{1}{s(s+1)}$$

Solution

$$Put \quad s = j\omega$$

$$G(j\omega) = \frac{1}{j\omega(j\omega+1)}$$

$$G(j\omega) = \frac{1}{-\omega^2 + j\omega}$$

$$G(j\omega) = \frac{1}{-\omega^2 + j\omega} \times \frac{-\omega^2 - j\omega}{-\omega^2 - j\omega}$$

$$G(j\omega) = \frac{-\omega^2 - j\omega}{\omega^4 + \omega^2}$$

Example#1

$$G(j\omega) = \frac{-\omega^2 - j\omega}{\omega^4 + \omega^2}$$

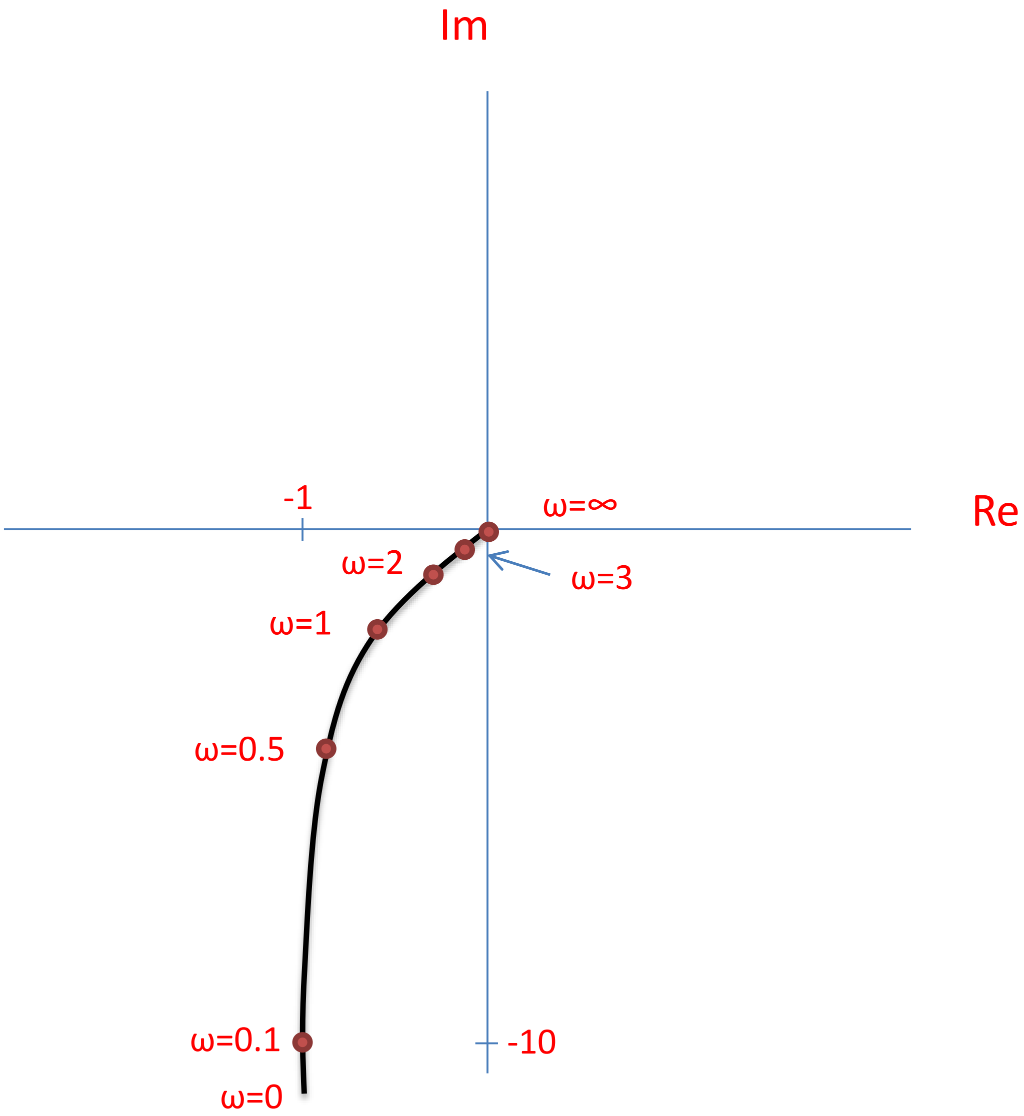
$$G(j\omega) = \frac{-\omega^2}{\omega^4 + \omega^2} - j\frac{\omega}{\omega^4 + \omega^2}$$

$$G(j\omega) = \frac{-1}{\omega^2 + 1} - j\frac{1}{\omega(\omega^2 + 1)}$$

ω	Re	Im
0	-1	∞
0.1	-1	-10
0.5	-0.8	-1.6
1	-0.5	-0.5
2	-0.2	-0.1
3	-0.1	-0.03
∞	0	0

Example#1

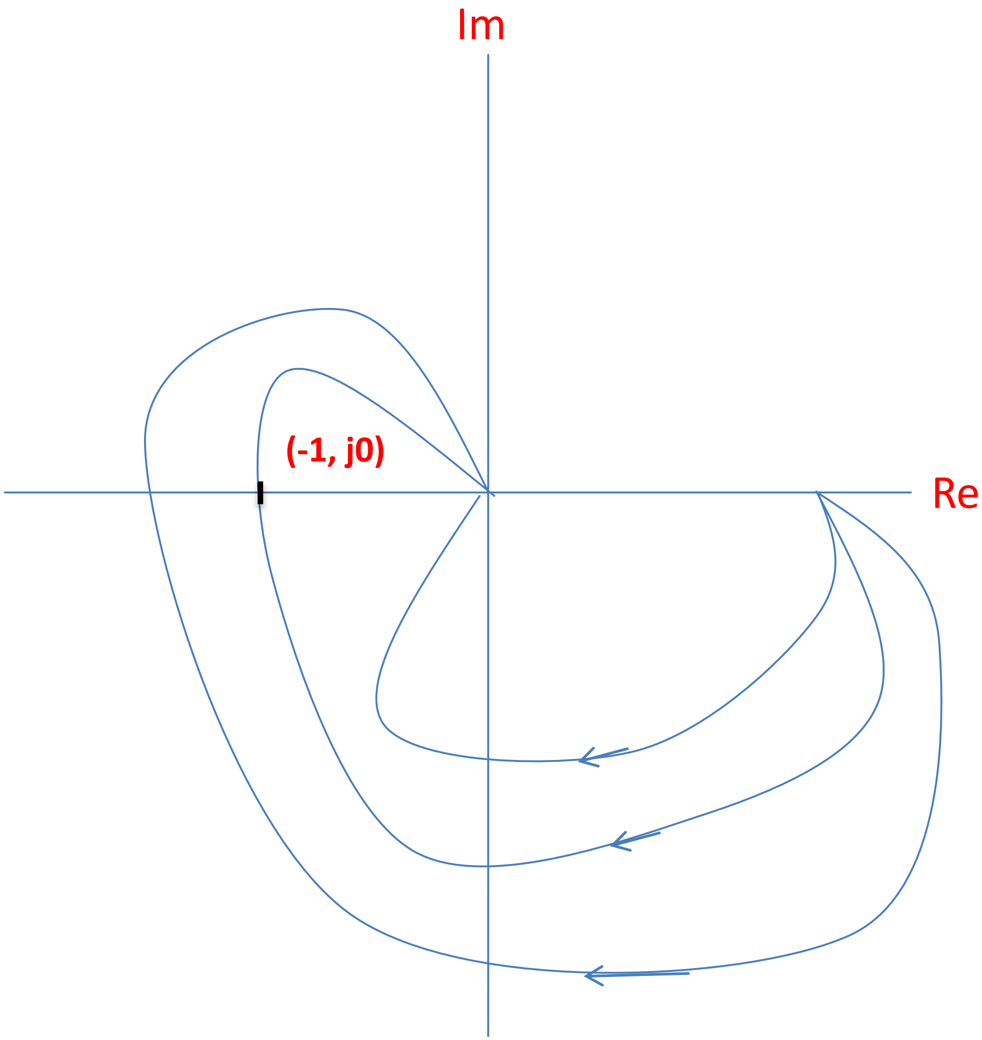
ω	Re	Im
0	-1	∞
0.1	-1	-10
0.5	-0.8	-1.6
1	-0.5	-0.5
2	-0.2	-0.1
3	-0.1	-0.03
∞	0	0

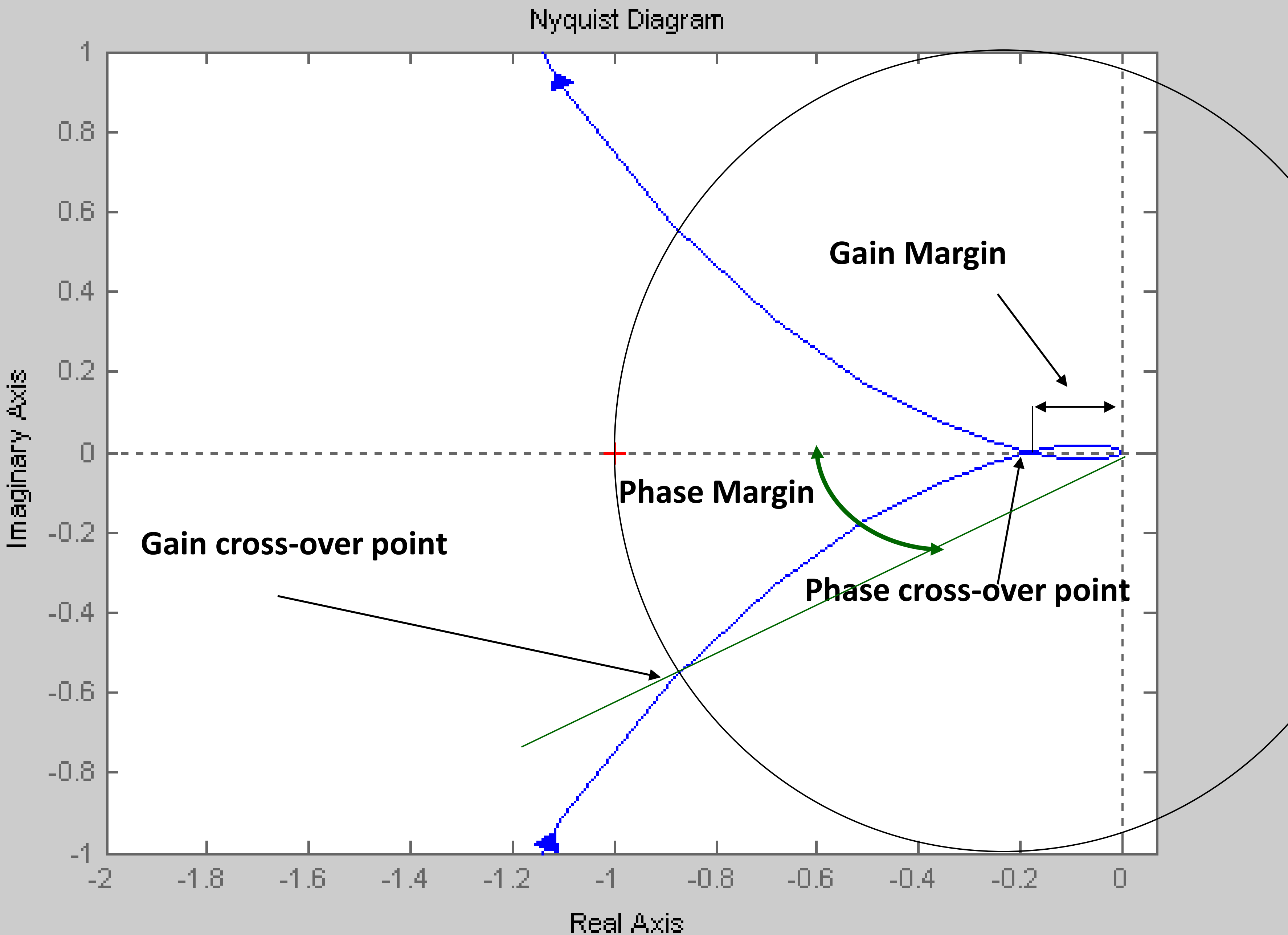


Nyquist Stability Criterion

- The Nyquist stability criterion determines the stability of a closed-loop system from its open-loop frequency response and open-loop poles.

- *A minimum phase closed loop system will be stable if the Nyquist plot of open loop transfer function does not encircle (-1, j0) point.*




Nyquist Diagram
Gain Margin
Phase Margin
Gain cross-over point
Phase cross-over point
Imaginary Axis
Real Axis
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-2
-1.8
-1.6
-1.4
-1.2
-1
-0.8
-0.6
-0.4
-0.2
0

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