Automatic Control Systems

Lecture-4

Standard Test Signals-

Time Domain Analysis of 1st Order Systems

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Introduction

- It is possible to compute the time response of a system if the nature of input and the mathematical model of the system are known.
- Usually, the input signals to control systems are not known fully ahead of time.
- For example, in a radar tracking system, the position and the speed of the target to be tracked may vary in a random fashion.
- It is therefore difficult to express the actual input signals mathematically by simple equations.

Standard Test Signals

- The characteristics of actual input signals are a sudden shock, a sudden change, a constant velocity, and constant acceleration.

- The dynamic behavior of a system is therefore judged and compared under application of standard test signals – an impulse, a step, a constant velocity, and constant acceleration.

- Another standard signal of great importance is a sinusoidal signal.

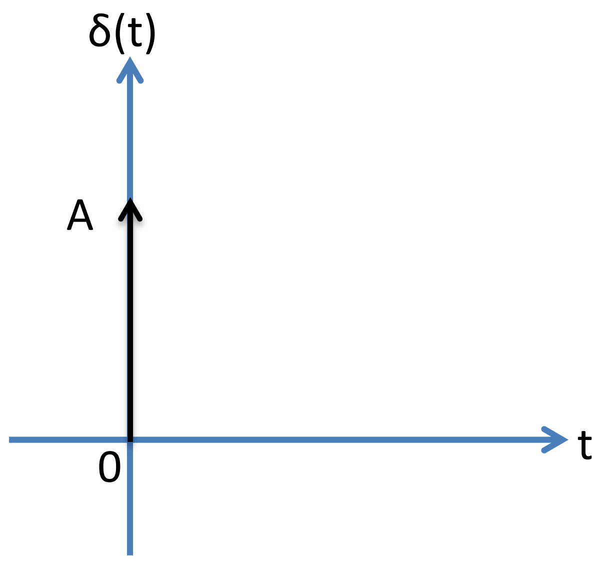
Standard Test Signals

- Impulse signal
  - The impulse signal imitate the sudden shock characteristic of actual input signal.

$$\delta(t) = \begin{cases} A & t = 0 \\ 0 & t \neq 0 \end{cases}$$

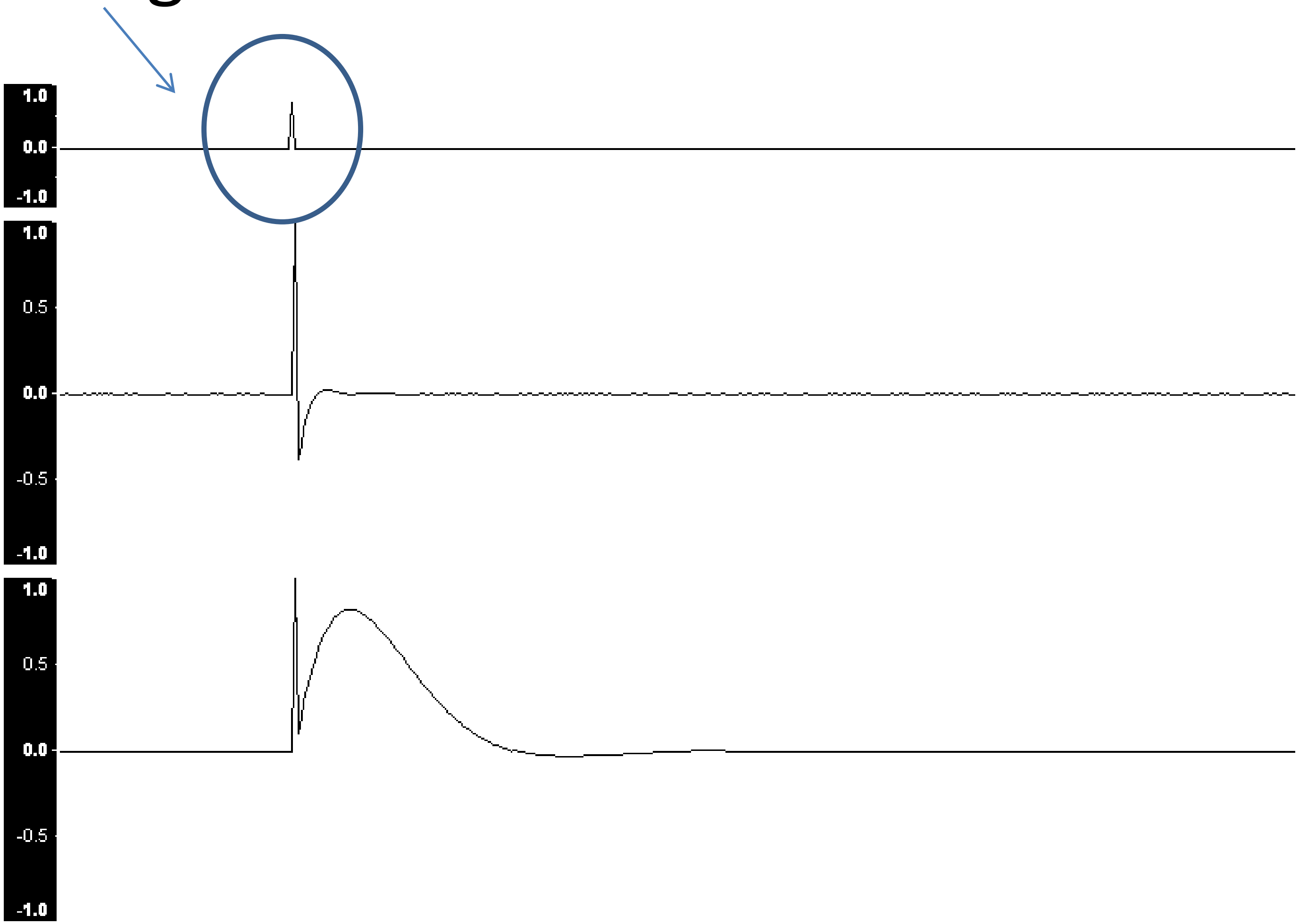
$$L\{\delta(t)\} = \delta(s) = A$$

  - If A=1, the impulse signal is called unit impulse signal.



Standard Test Signals

- Impulse signal



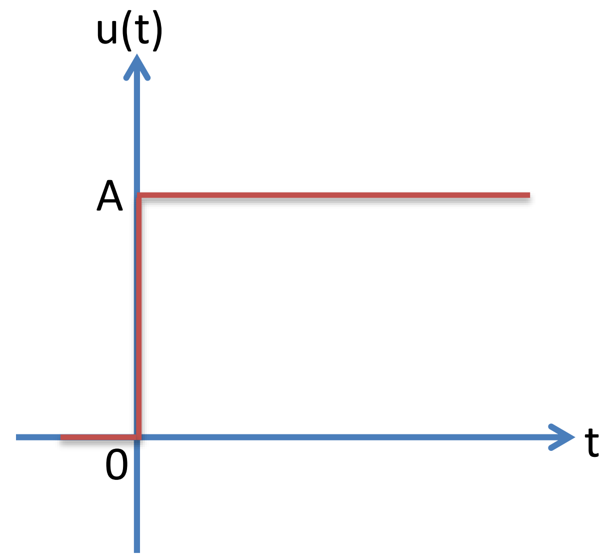
Standard Test Signals

- Step signal
  - The step signal imitate the sudden change characteristic of actual input signal.

$$u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$L\{u(t)\} = U(s) = \frac{A}{S}$$

  - If A=1, the step signal is called unit step signal



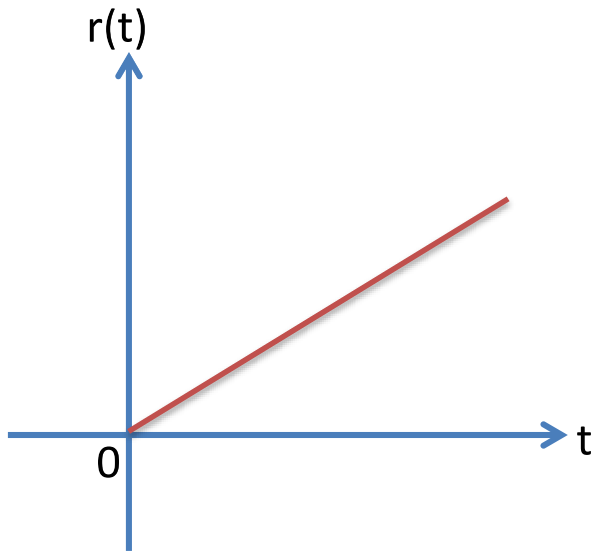
Standard Test Signals

- Ramp signal
  - The ramp signal imitate the constant velocity characteristic of actual input signal.

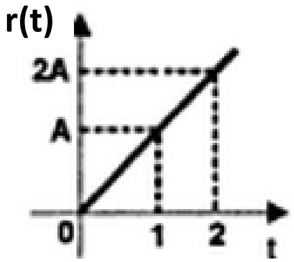
$$r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$L\{r(t)\} = R(s) = \frac{A}{s^2}$$

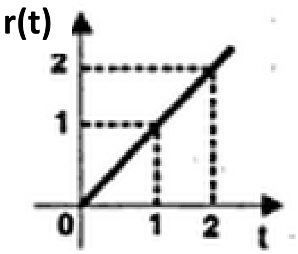
  - If *A=1*, the ramp signal is called unit ramp signal



ramp signal with slope A

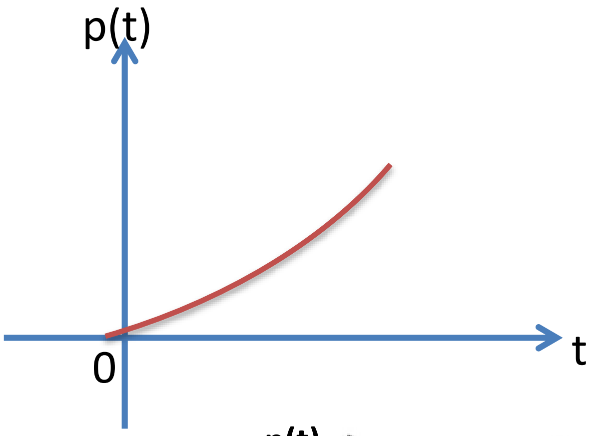


unit ramp signal



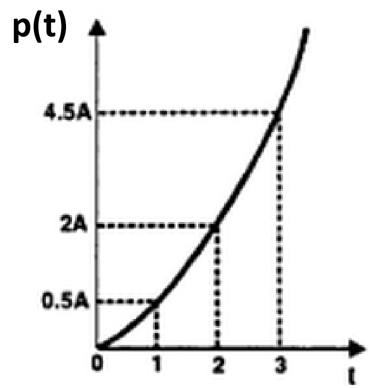
Standard Test Signals

- Parabolic signal
  - The parabolic signal imitate the constant acceleration characteristic of actual input signal.



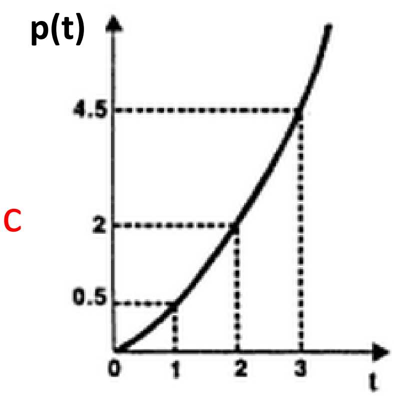
$$p(t) = \begin{cases} \dfrac{At^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$L\{p(t)\} = P(s) = \frac{2A}{S^3}$$



parabolic signal with slope A

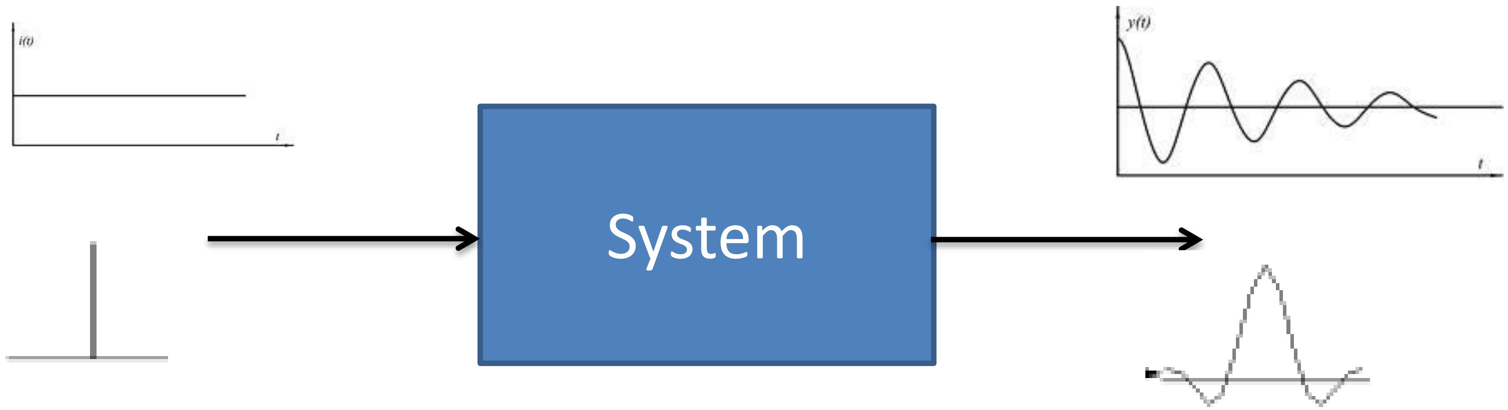
  - If *A=1*, the parabolic signal is called unit parabolic signal.



Unit parabolic signal

Time Response of Control Systems

- Time response of a dynamic system response to an input expressed as a function of time.



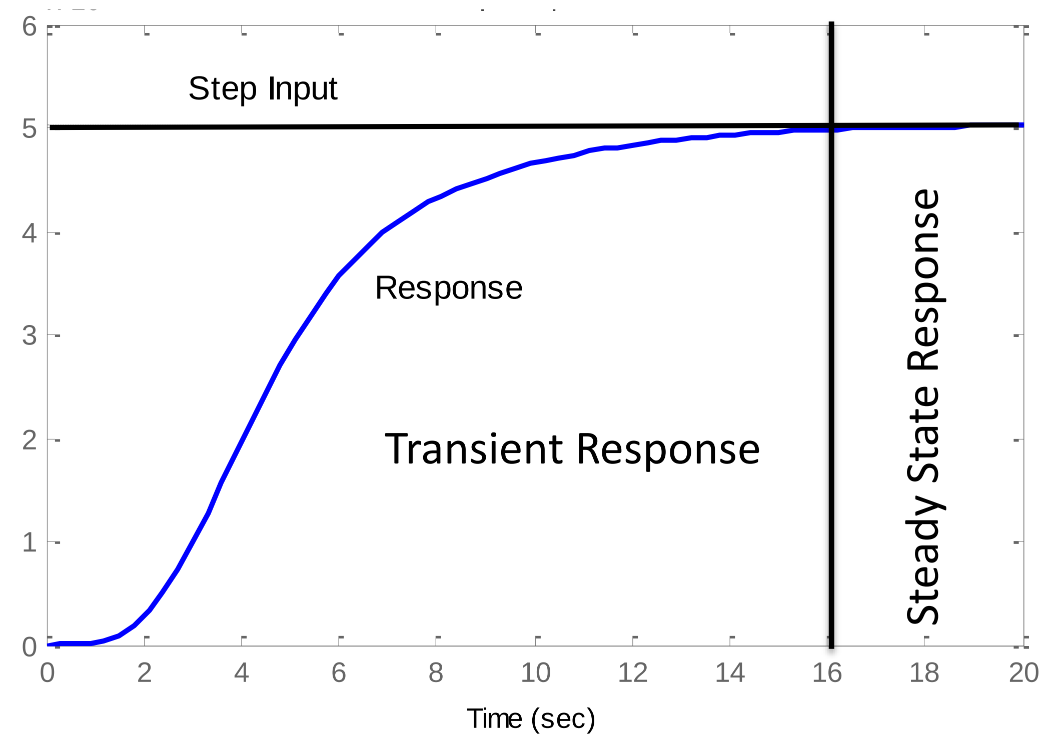
- The time response of any system has two components
  - Transient response
  - Steady-state response.

Time Response of Control Systems

- When the response of the system is changed form rest or equilibrium it takes some time to settle down.

- Transient response is the response of a system from rest or equilibrium to steady state.

- The response of the system after the transient response is called steady state response.

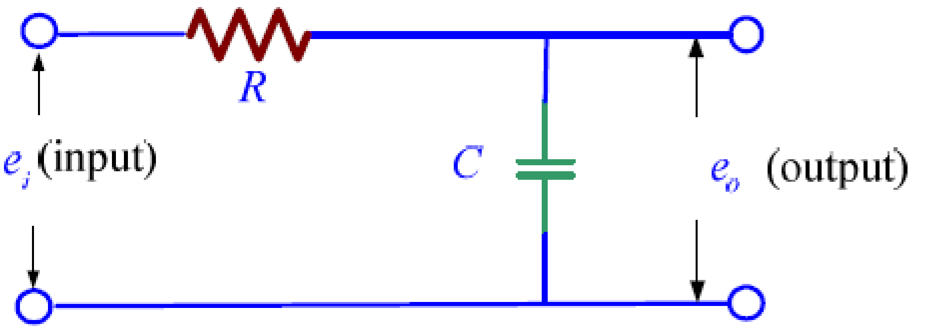


Time Response of Control Systems

- Transient response depend upon the system poles only and not on the type of input.
- It is therefore sufficient to analyze the transient response using a step input.
- The steady-state response depends on system dynamics and the input quantity.
- It is then examined using different test signals by final value theorem.

Examples of First Order Systems

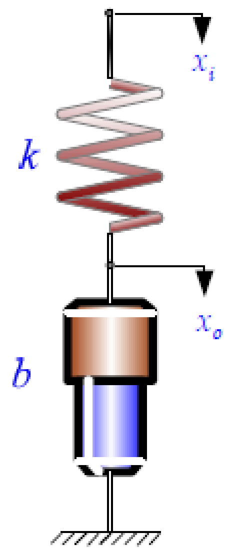
- Electrical System



$$\frac{E_o(s)}{E_i(s)} = \frac{1}{RCs+1}$$

Examples of First Order Systems

- Mechanical System



$$\frac{X_o(s)}{X_i(s)} = \frac{1}{\frac{b}{k}s + 1}$$

First Order Systems

- The first order system has the standard form.

$$\frac{C(s)}{R(s)} = \frac{k}{\tau s + 1}$$

- Where ***K*** is the D.C gain and τ is the time constant of the system.
- Time constant is a measure of how quickly a 1^{st} order system responds to a unit step input.
- D.C Gain of the system is ratio between the input signal and the steady state value of output.

First Order Systems

- The first order system given below.

$$G(s) = \frac{10}{3s+1}$$

- D.C gain is 10 and time constant is 3 seconds.

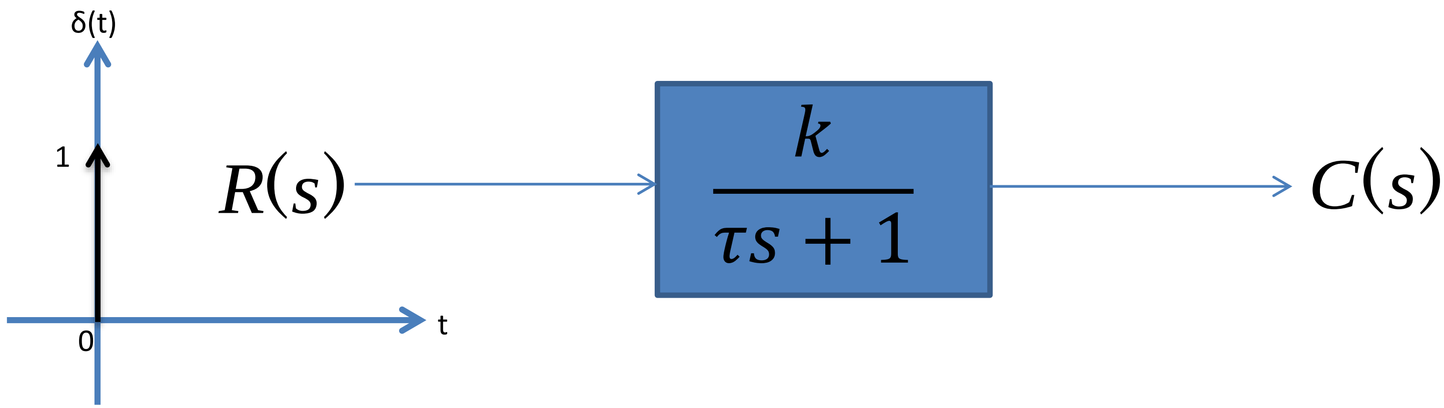
- And for following system

$$G(s) = \frac{3}{s+5} = \frac{3/5}{1/5s+1}$$

- D.C Gain of the system is 3/5 and time constant is 1/5 seconds.

Impulse Response of 1st Order System

- Consider the following 1st order system



$$R(s) = \delta(s) = 1$$

$$C(s) = \frac{k}{\tau s + 1}$$

Impulse Response of 1st Order System

$$C(s) = \frac{k}{\tau s + 1}$$

- Re-arrange following equation as

$$C(s) = \frac{k/\tau}{s + 1/\tau}$$

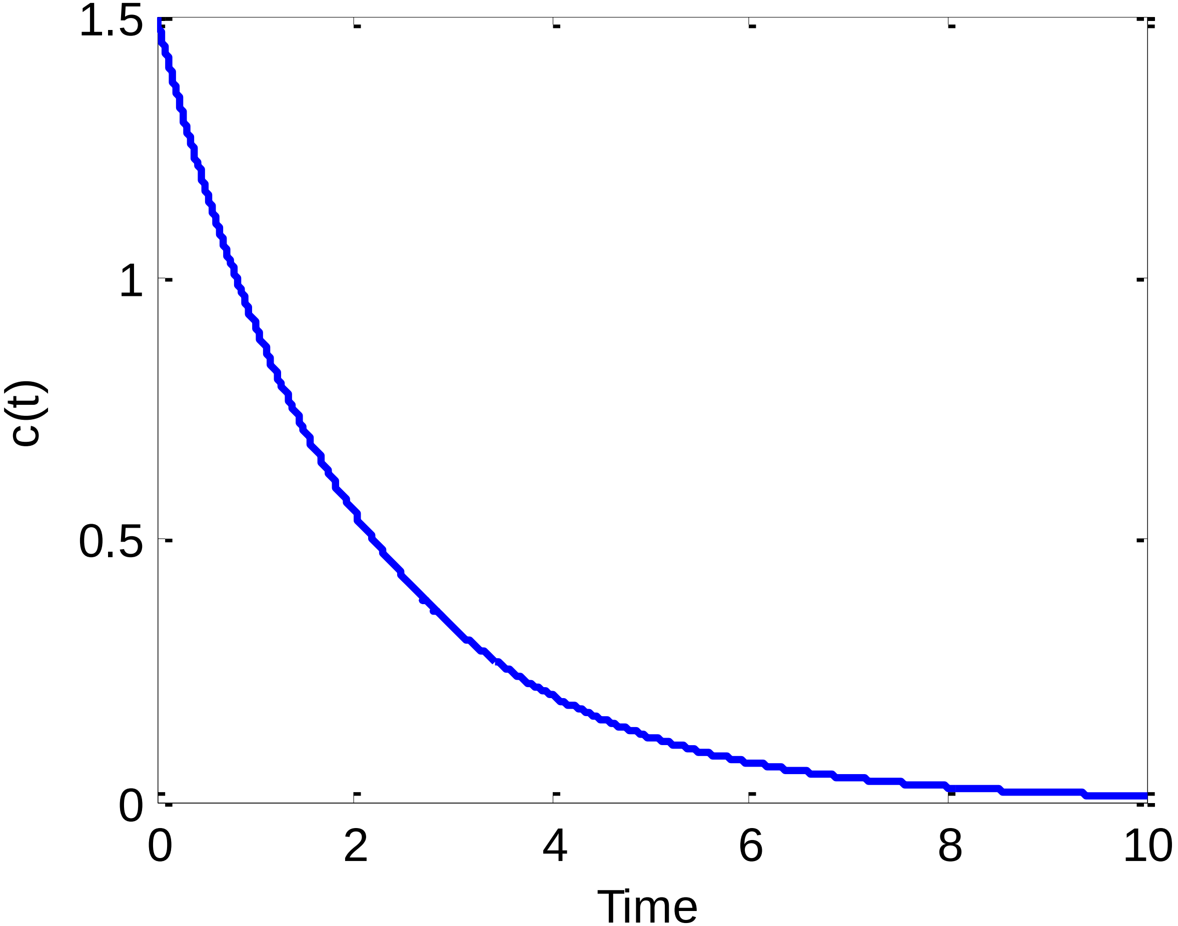
- In order represent the response of the system in time domain we need to compute inverse Laplace transform of the above equation.

$$L^{-1}\left(\frac{C}{s+a}\right) = Ce^{-at}$$

$$c(t) = \frac{k}{\tau} e^{-t/\tau}$$

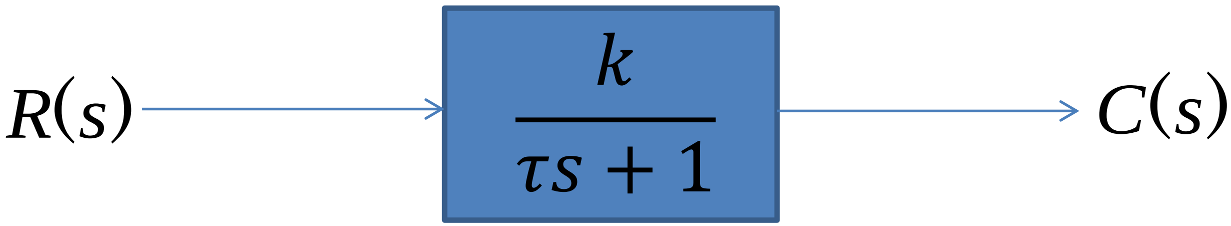
Impulse Response of 1st Order System

- If K=3 and τ =2 sec then $$c(t) = \frac{k}{\tau} e^{-t/\tau}$$



Step Response of 1st Order System

- Consider the following 1st order system



$$R(s) = U(s) = \frac{1}{s}$$

$$C(s) = \frac{k}{s(\tau s+1)}$$

- In order to find out the inverse Laplace of the above equation, we need to break it into partial fraction expansion

Forced Response **Natural Response**

$$C(s) = \frac{k}{s} - \frac{k\tau}{\tau s+1}$$

Step Response of 1st Order System

$$C(s) = k\left(\frac{1}{s} - \frac{\tau}{\tau s+1}\right)$$

- Taking Inverse Laplace of above equation

$$c(t) = k(u(t) - e^{-t/\tau})$$

- Where u(t)=1

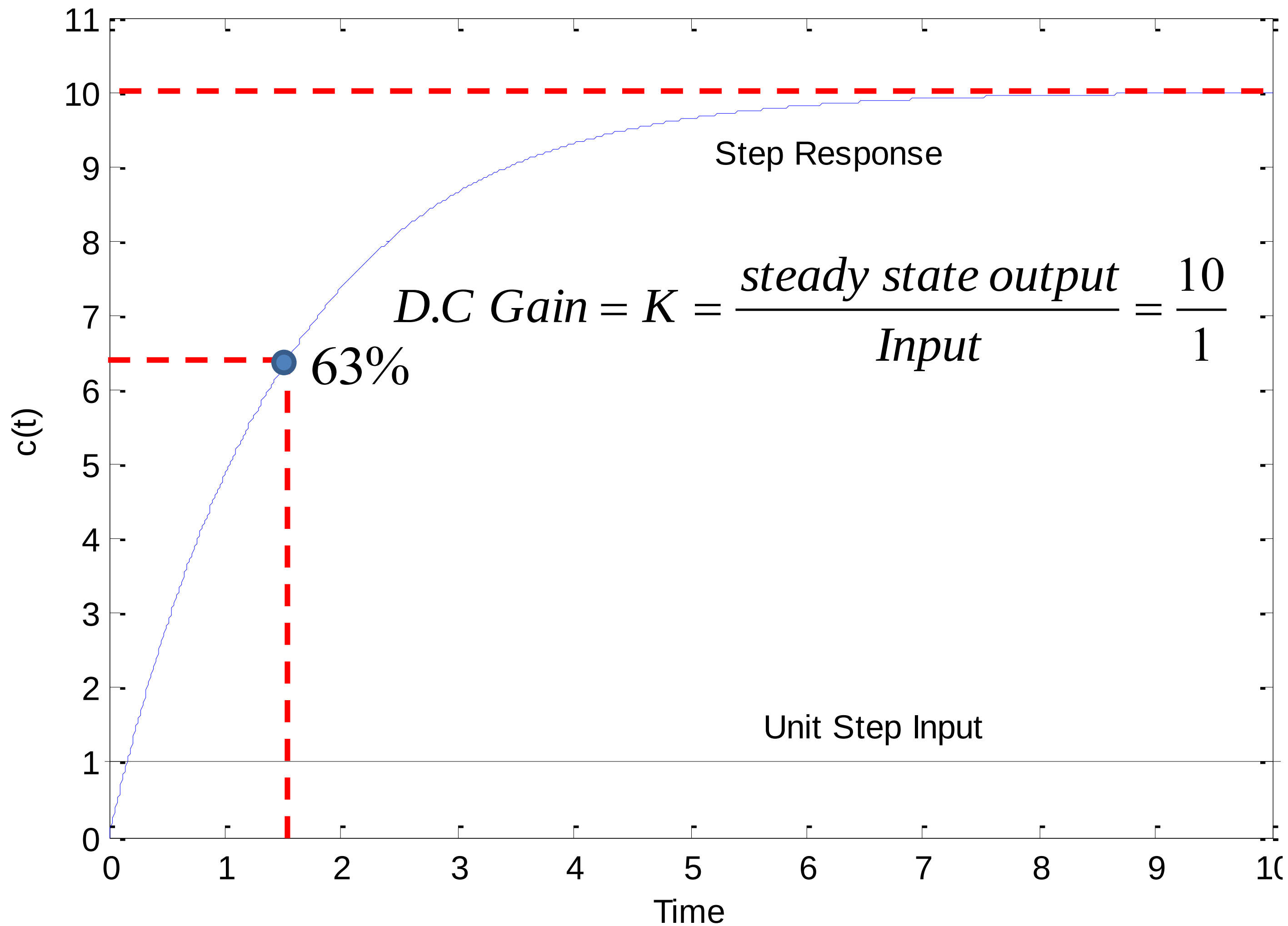
$$c(t) = k(1 - e^{-t/\tau})$$

- When t=τ

$$c(t) = k(1 - e^{-1}) = 0.632k$$

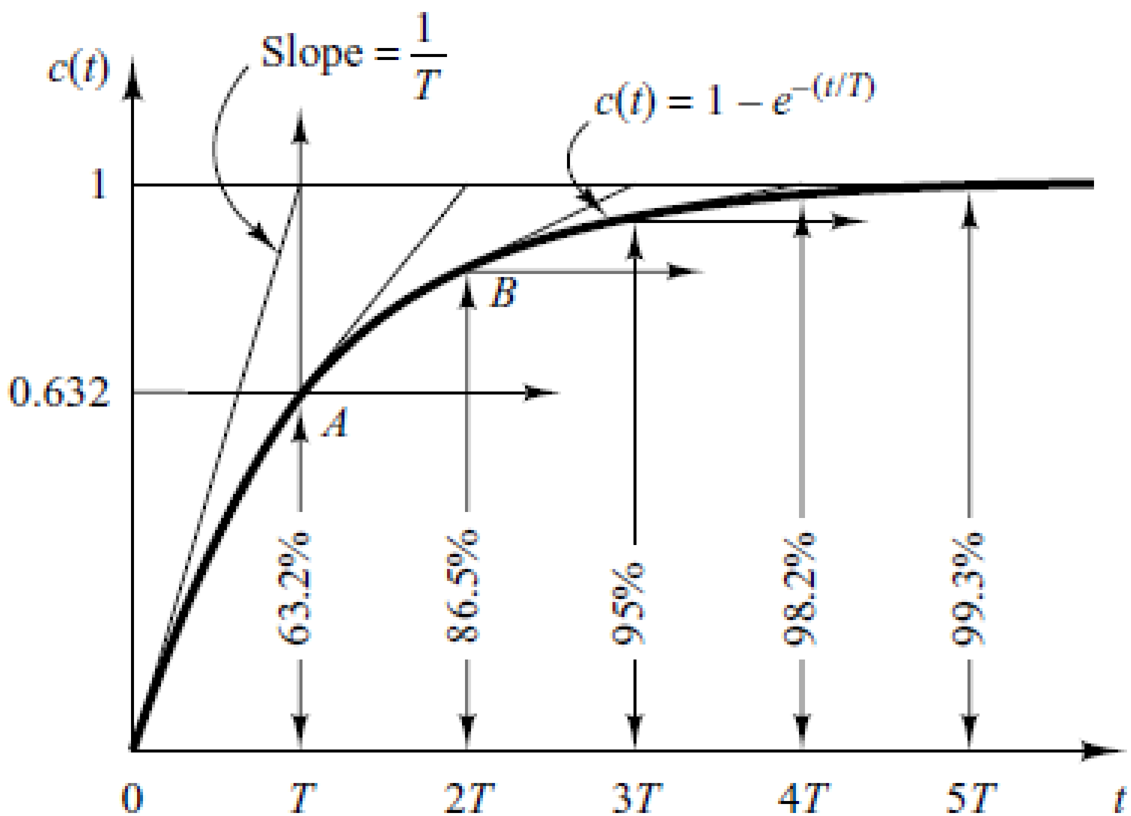
Step Response of 1st Order System

- If K=10 and τ=1.5 s then $$c(t) = k(1 - e^{-t/\tau})$$



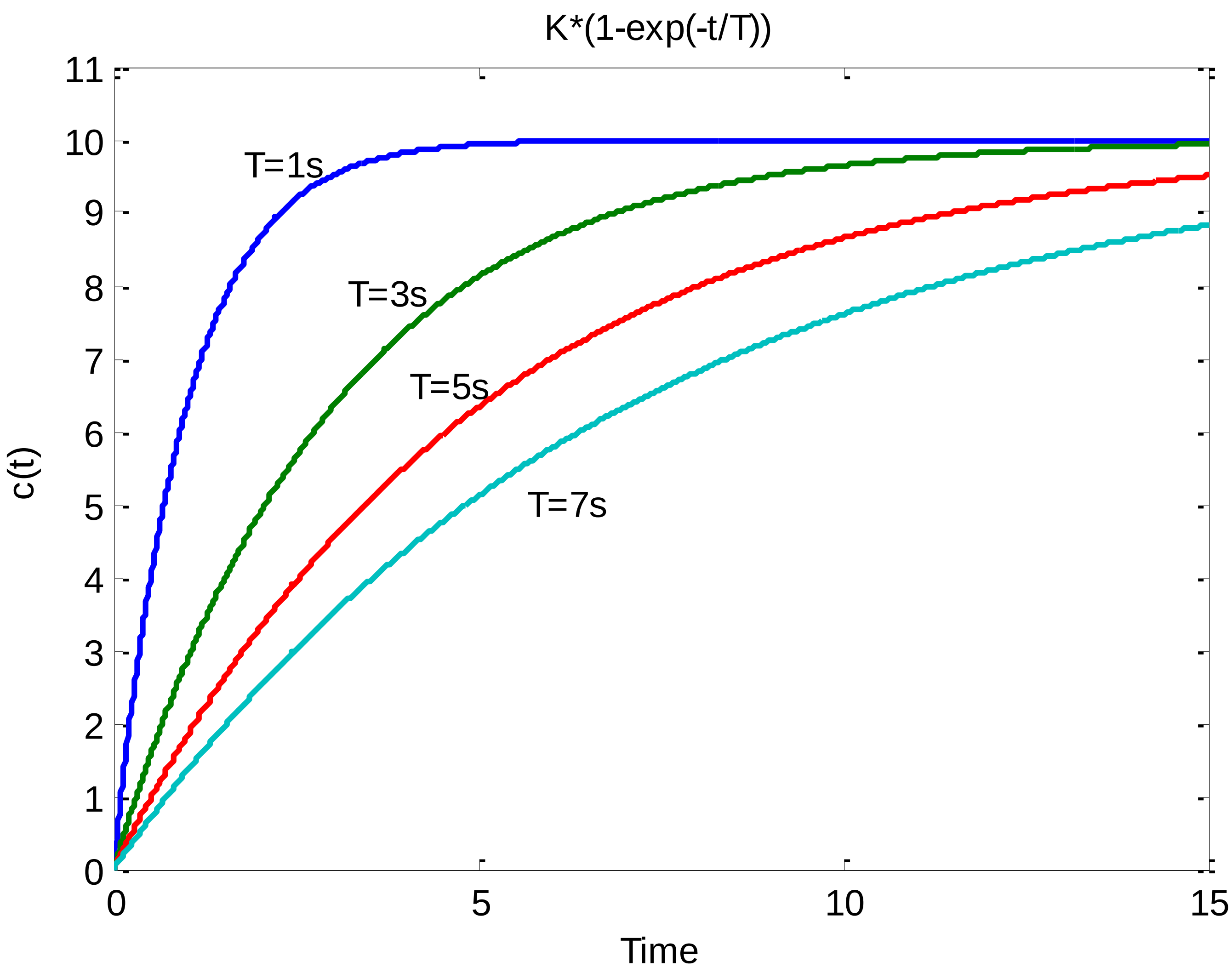
Step Response of 1st order System

- System takes five time constants to reach its final value.



Step Response of 1st Order System

- If K=10 and τ =1, 3, 5, 7 $$c(t) = k(1 - e^{-t/\tau})$$

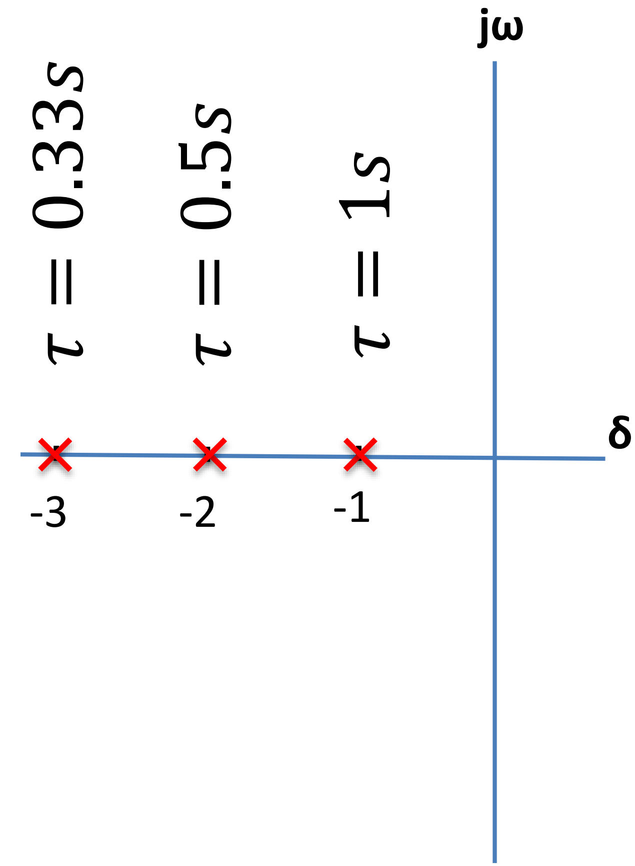


PZ-map and Step Response

$$\frac{C(s)}{R(s)} = \frac{10}{s+1}$$

$$\frac{C(s)}{R(s)} = \frac{10}{s+2} = \frac{5}{0.5s+1}$$

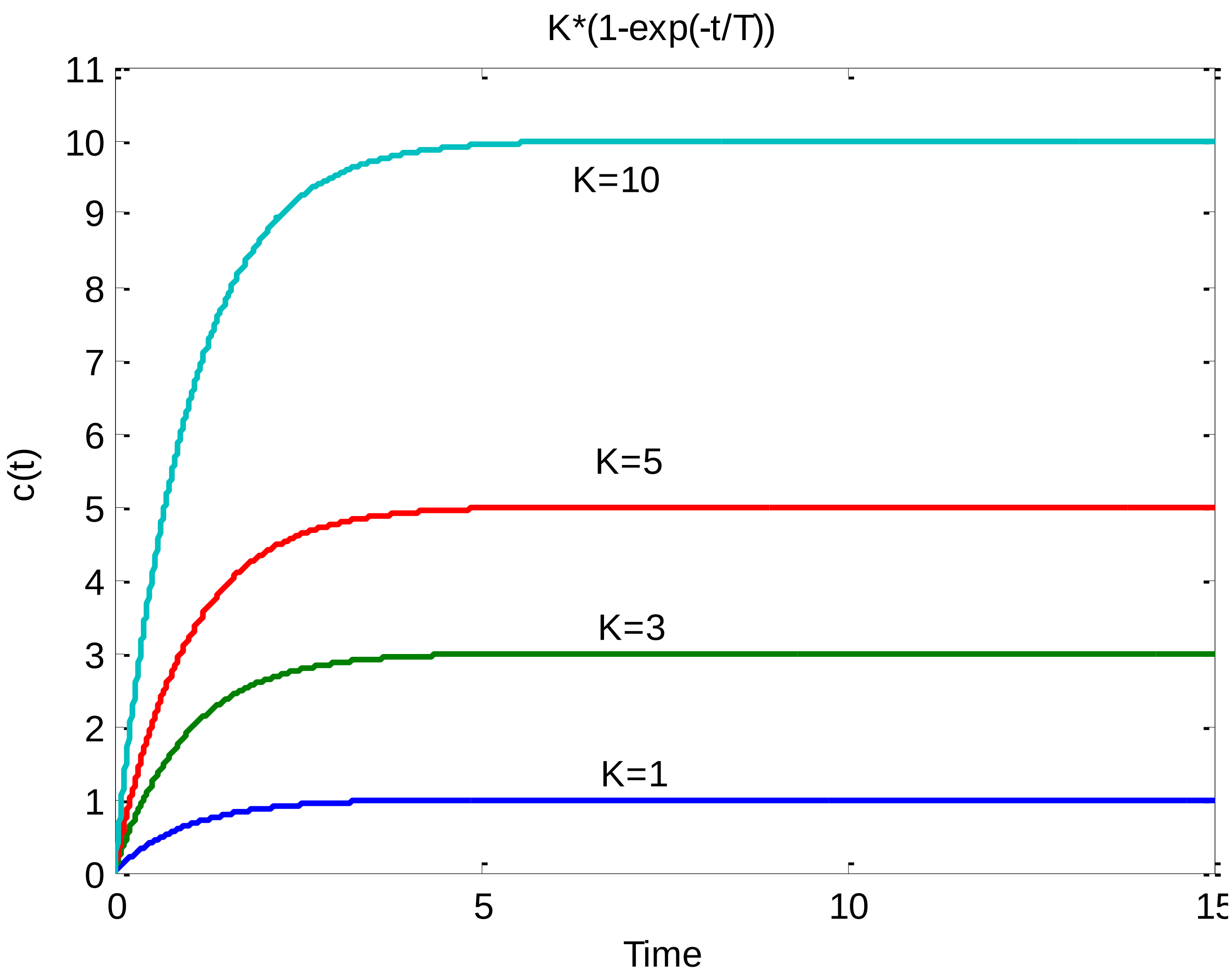
$$\frac{C(s)}{R(s)} = \frac{10}{s+3} = \frac{3.3}{0.33s+1}$$



Step Response of 1st Order System

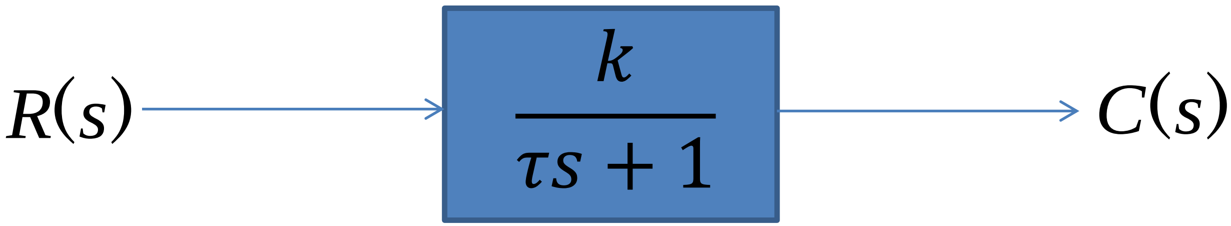
- If K=1, 3, 5, 10 and τ =1

$$c(t) = k(1 - e^{-t/\tau})$$



Ramp Response of 1st Order System

- Consider the following 1st order system



$$R(s) = \frac{1}{s^2}$$

$$C(s) = \frac{k}{s^2(\tau s + 1)}$$

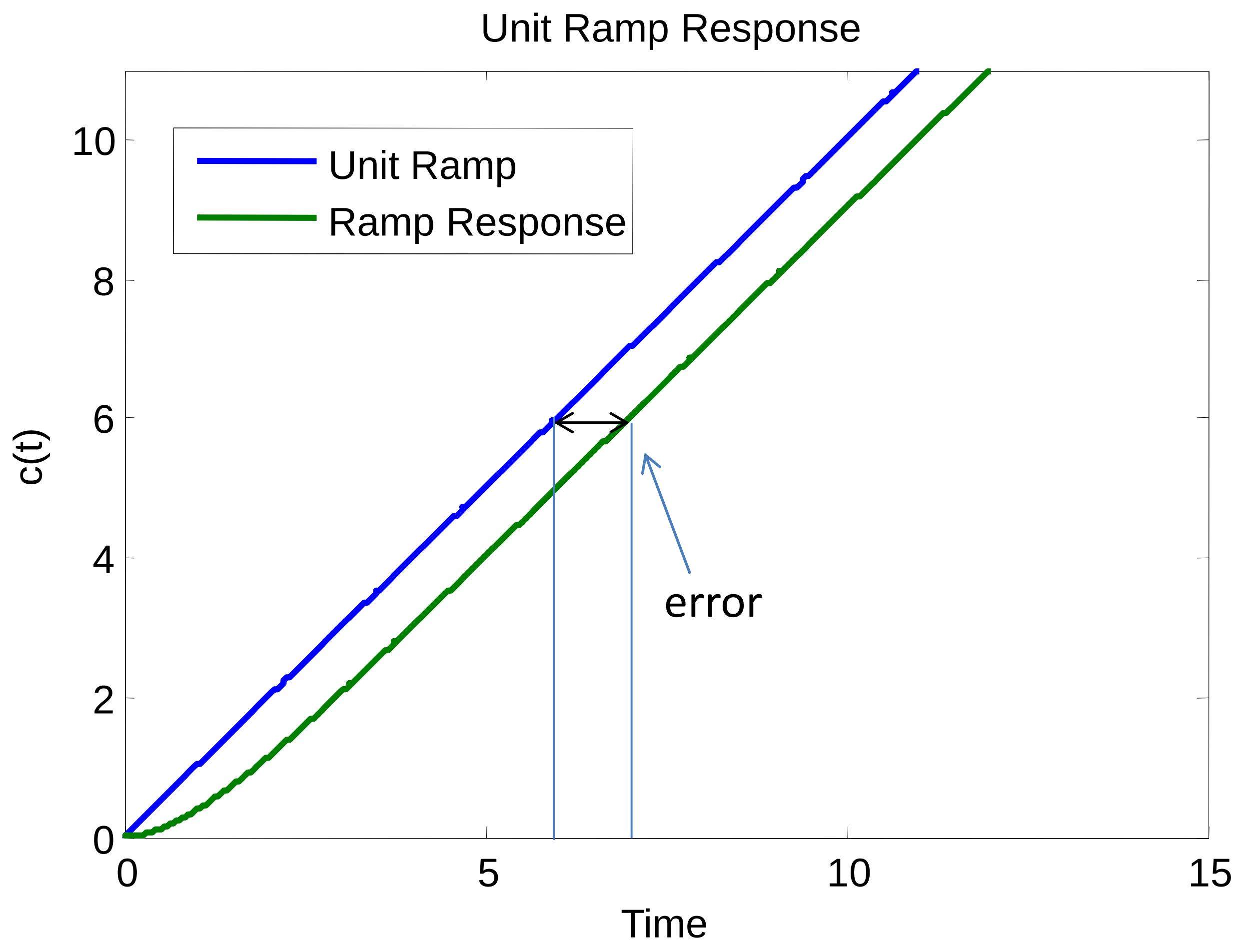
- The ramp response is given as

$$c(t) = k(t - \tau + \tau e^{-t/\tau})$$

Ramp Response of 1st Order System

- If K=1 and τ =1

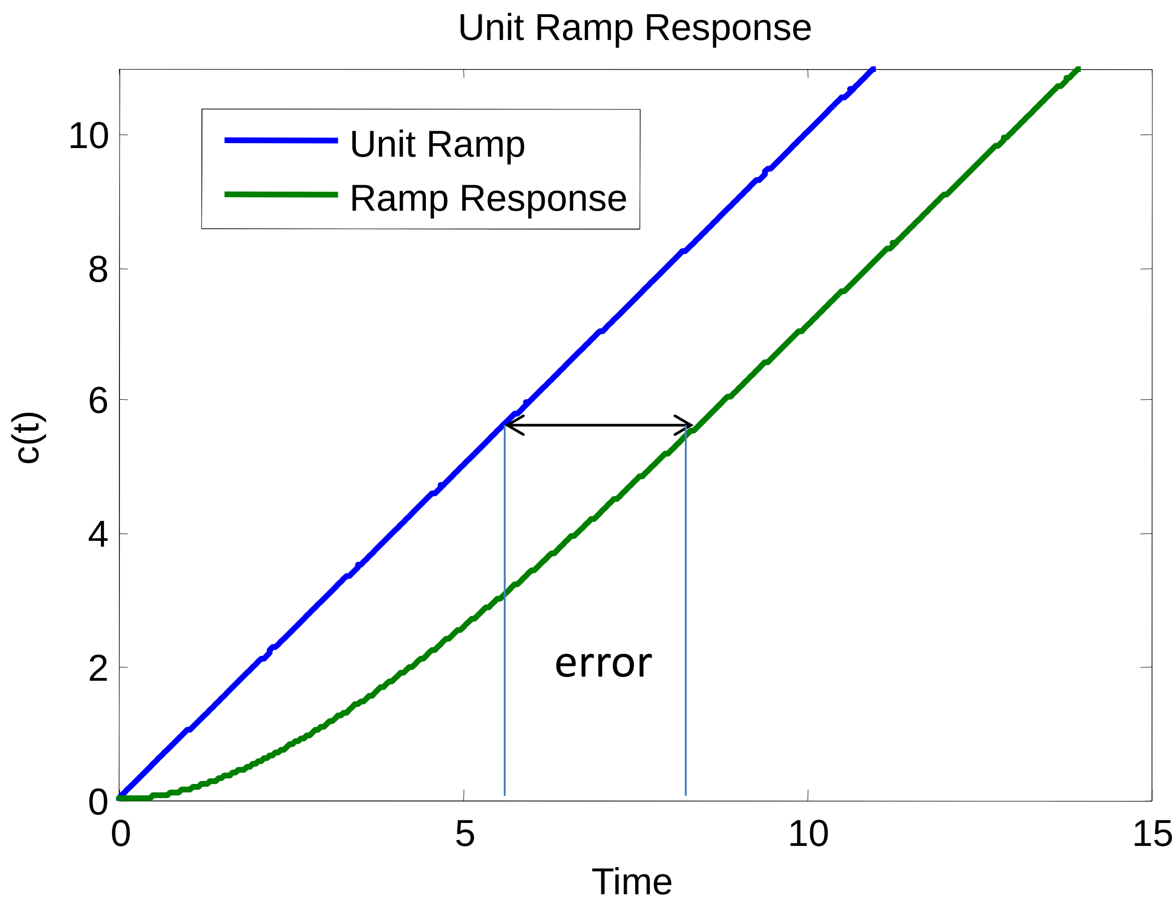
$$c(t) = k(t - \tau + \tau e^{-t/\tau})$$



Ramp Response of 1st Order System

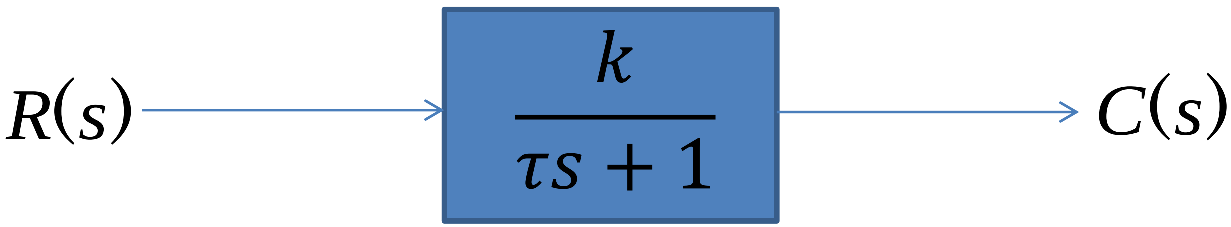
- If $K=1$ and $\tau=3$

$$c(t) = k(t - \tau + \tau e^{-t/\tau})$$



Parabolic Response of 1st Order System

- Consider the following 1st order system



$$R(s) = \frac{1}{s^3} \quad \text{Therefore,} \quad C(s) = \frac{k}{s^3(\tau s + 1)}$$

- Do it yourself

End of Lec.