

Robotics

Lecture 9

Velocity Kinematics

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Velocity Kinematics

Velocity Kinematics

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graph TD; A[Velocity Kinematics] --> B[Forward Kinematics]; A --> C[Inverse Kinematics]; B --> B1[Given]; B --> B2[Find]; C --> C1[Given]; C --> C2[Find];
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Forward Kinematics

Given

- The time rate of joint variables

Find

- The Cartesian velocity of end-effector in the global coordinate frame

Inverse Kinematics

- The velocity of end-effector

- The time rate of joint variables

Forward Velocity Kinematics

Forward Velocity Kinematics

- The forward velocity kinematics of a robot solves the problem of relating joint speeds \dot{q} (of an n DOF robot is an $n \times 1$ vector)

$$\dot{q}_{n \times 1} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}_{n \times 1}$$

to the end-effector speeds \dot{X} (generally 6×1 vector).

$$\begin{aligned} \dot{X} &= \begin{bmatrix} \dot{X}_n & \dot{Y}_n & \dot{Z}_n & \omega_{Xn} & \omega_{Yn} & \omega_{Zn} \end{bmatrix}^T \\ &= \begin{bmatrix} {}^0\dot{\mathbf{d}}_n \\ {}_0\omega_n \end{bmatrix} = \begin{bmatrix} {}^0\mathbf{v}_n \\ {}_0\omega_n \end{bmatrix} \end{aligned}$$

Linear Velocity
Angular Velocity

Forward Velocity Kinematics

- The elements of end-effector speed vector \dot{X} are linearly proportional to the elements of joint speed vector \dot{q} ,

$$\dot{X} = J_{6 \times n} \dot{q}_{n \times 1}$$

- $J(q)$: Jacobian matrix

Jacobian

- Suppose a position and orientation vector of a manipulator is a function of 6 joint variables: (from forward kinematics)

$$X = h(q)$$

$$X = \begin{bmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \varphi \end{bmatrix} = h \left(\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} \right)_{6 \times 1} = \begin{bmatrix} h_1(q_1, q_2, \dots, q_6) \\ h_2(q_1, q_2, \dots, q_6) \\ h_3(q_1, q_2, \dots, q_6) \\ h_4(q_1, q_2, \dots, q_6) \\ h_5(q_1, q_2, \dots, q_6) \\ h_6(q_1, q_2, \dots, q_6) \end{bmatrix}_{6 \times 1}$$

Jacobian Matrix

Forward kinematics

$$X_{6 \times 1} = h(q_{n \times 1})$$

$$\dot{X}_{6 \times 1} = \frac{d}{dt} h(q_{n \times 1}) = \frac{dh(q)}{dq} \frac{dq}{dt} = \frac{dh(q)}{dq} \dot{q}$$

Let $J = \frac{dh(q)}{dq}$

$$\dot{X}_{6 \times 1} = J_{6 \times n} \dot{q}_{n \times 1}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \frac{dh(q)}{dq} \end{bmatrix}_{6 \times n} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}_{n \times 1}$$

Jacobian Matrix

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \left[\frac{dh(q)}{dq} \right]_{6 \times n} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}_{n \times 1}$$

$$J = \left(\frac{dh(q)}{dq} \right)_{6 \times n} = \begin{bmatrix} \frac{\partial h_1}{\partial q_1} & \frac{\partial h_1}{\partial q_2} & \dots & \frac{\partial h_1}{\partial q_n} \\ \frac{\partial h_2}{\partial q_1} & \frac{\partial h_2}{\partial q_2} & \dots & \frac{\partial h_2}{\partial q_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial h_6}{\partial q_1} & \frac{\partial h_6}{\partial q_2} & \dots & \frac{\partial h_6}{\partial q_n} \end{bmatrix}_{6 \times n}$$

Jacobian Matrix

The Jacobian Equation

$$\dot{X} = J_{6 \times n} \dot{q}_{n \times 1}$$

$$\dot{X} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} V \\ \omega \end{bmatrix}$$

$$V = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

Linear velocity

$$\omega = \begin{bmatrix} \omega_x = \dot{\phi} \\ \omega_y = \dot{\theta} \\ \omega_z = \dot{\psi} \end{bmatrix}$$

Angular velocity

$$\dot{q}_{n \times 1} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}_{n \times 1}$$

Example

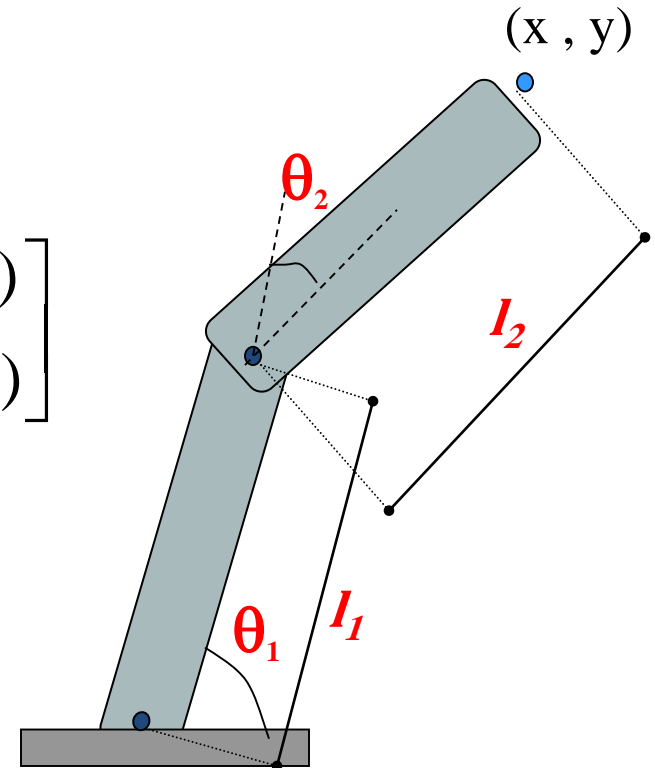
Example

- 2-DOF planar robot arm
 - Given l_1, l_2 , Find: Jacobian

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2) \\ l_1 \sin \theta_1 + l_2 \sin(\theta_1 + \theta_2) \end{bmatrix} = \begin{bmatrix} h_1(\theta_1, \theta_2) \\ h_2(\theta_1, \theta_2) \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\mathbf{J} = \begin{bmatrix} \frac{\partial h_1}{\partial \theta_1} & \frac{\partial h_1}{\partial \theta_2} \\ \frac{\partial h_2}{\partial \theta_1} & \frac{\partial h_2}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} -l_1 \sin \theta_1 - l_2 \sin(\theta_1 + \theta_2) & -l_2 \sin(\theta_1 + \theta_2) \\ l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2) & l_2 \cos(\theta_1 + \theta_2) \end{bmatrix}$$



Remember DH parameter

$$A_i = \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- The transformation matrix T

$$T_0^i = A_1 A_2 \dots A_i$$

Jacobian Matrix

$$J = [J_1 \quad J_2 \quad \dots \quad J_n]$$

where if joint (i) is revolute

$$J_i = \begin{bmatrix} Z_{i-1} \times (O_n - O_{i-1}) \\ Z_{i-1} \end{bmatrix}$$

and if joint (i) is prismatic

$$J_i = \begin{bmatrix} Z_{i-1} \\ 0 \end{bmatrix}$$

Where Z_i is the first three elements in the 3rd column of the T_0^i matrix, and O_i is the first three elements in the 4th column of the T_0^i matrix.

Example

2-DOF planar robot arm. Given l_1, l_2 , Find: Jacobian

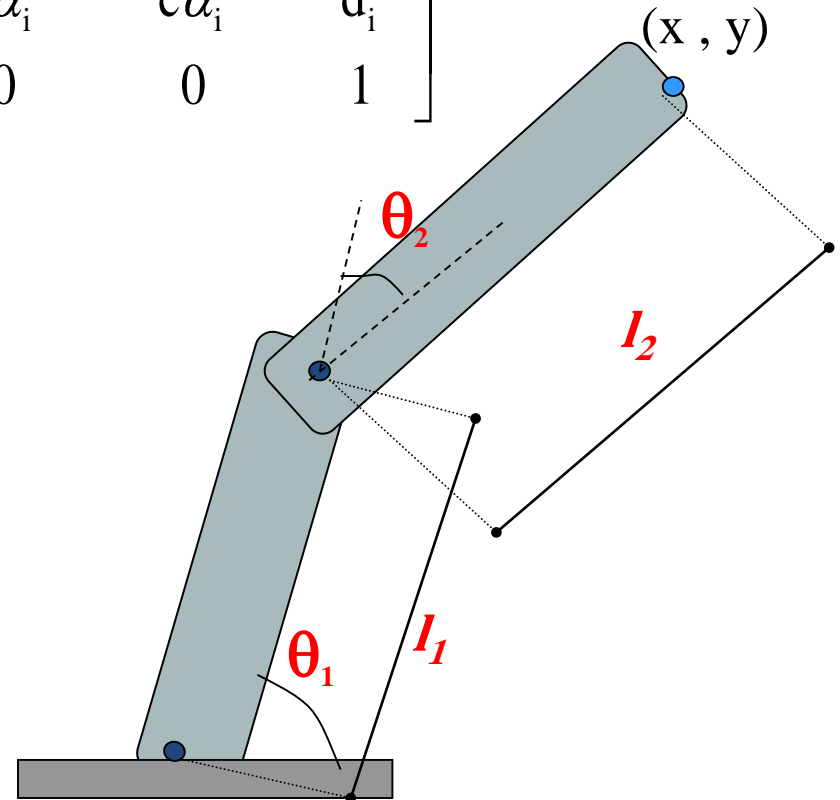
Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1^*
2	a_2	0	0	θ_2^*

$$A_i = \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* variable

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$T_1^0 = A_1.$$

$$T_2^0 = A_1 A_2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1 c_1 + a_2 c_{12} \\ s_{12} & c_{12} & 0 & a_1 s_1 + a_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where $(\theta_1 + \theta_2)$ denoted by θ_{12} and $\cos(\theta_1 + \theta_2)$ by c_{12}

$$Z_0 = Z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, O_1 = \begin{bmatrix} a_1 \cos \theta_1 \\ a_1 \sin \theta_1 \\ 0 \end{bmatrix}, O_2 = \begin{bmatrix} a_1 \cos \theta_1 + a_2 \cos(\theta_1 + \theta_2) \\ a_1 \sin \theta_1 + a_2 \sin(\theta_1 + \theta_2) \\ 0 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} z_0 \times (o_2 - o_0) \\ z_0 \end{bmatrix}, J_2 = \begin{bmatrix} z_1 \times (o_2 - o_1) \\ z_1 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} z_0 \times (o_2 - o_0) \\ z_0 \end{bmatrix}$$

$$Z_0 \times (o_2 - o_0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_1 \cos \theta_1 + a_2 \cos(\theta_1 + \theta_2) \\ a_1 \sin \theta_1 + a_2 \sin(\theta_1 + \theta_2) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} i & j & k \\ 0 & 0 & 1 \\ a_1 \cos \theta_1 + a_2 \cos(\theta_1 + \theta_2) & a_1 \sin \theta_1 + a_2 \sin(\theta_1 + \theta_2) & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -a_1 \sin \theta_1 - a_2 \sin(\theta_1 + \theta_2) \\ a_1 \cos \theta_1 + a_2 \cos(\theta_1 + \theta_2) \\ 0 \end{bmatrix}$$

Note: Cross Products

- If $A = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}, B = \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$

- Then the cross product

$$A \times B = \begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{bmatrix} a_y b_z - a_z b_y \\ -(a_x b_z - a_z b_x) \\ a_x b_y - a_y b_x \end{bmatrix}$$

$$J_2 = \begin{bmatrix} z_1 \times (o_2 - o_1) \\ z_1 \end{bmatrix}$$

$$z_1 \times (o_2 - o_1) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_2 \cos(\theta_1 + \theta_2) \\ a_2 \sin(\theta_1 + \theta_2) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} i & j & k \\ 0 & 0 & 1 \\ a_2 \cos(\theta_1 + \theta_2) & a_2 \sin(\theta_1 + \theta_2) & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -a_2 \sin(\theta_1 + \theta_2) \\ a_2 \cos(\theta_1 + \theta_2) \\ 0 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} -a_1 \sin \theta_1 - a_2 \sin(\theta_1 + \theta_2) \\ a_1 \cos \theta_1 + a_2 \cos(\theta_1 + \theta_2) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} -a_2 \sin(\theta_1 + \theta_2) \\ a_2 \cos(\theta_1 + \theta_2) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The required Jacobian matrix J

$$J = [J_1 \quad J_2]$$

Example

Stanford Manipulator

The DH parameters are:

Link	d_i	a_i	α_i	θ_i
1	0	0	-90	*
2	d_2	0	+90	*
3	*	0	0	0
4	0	0	-90	*
5	0	0	+90	*
6	d_6	0	0	*

$$A_i = \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* joint variable

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^1 = A_1 \quad \begin{matrix} \boxed{Z_1} & \boxed{O_1} \end{matrix}$$

$$T_0^2 = A_1 A_2 = \begin{bmatrix} c_1 c_2 & -s_1 & c_1 s_2 & -d_2 s_1 \\ s_1 c_2 & c_1 & s_1 s_2 & d_2 c_1 \\ -s_2 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_0^3 = A_1 A_2 A_3 = \begin{bmatrix} c_1 c_2 & -s_1 & c_1 s_2 & d_3 c_1 s_2 - d_2 s_1 \\ s_1 c_2 & c_1 & s_1 s_2 & d_3 s_1 s_2 + d_2 c_1 \\ -s_2 & 0 & c_2 & d_3 c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{Z_2} \quad \boxed{O_2}$$

$$\boxed{Z_3} \quad \boxed{O_3}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad z_1 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix} \quad z_2 = \begin{bmatrix} c_1 s_2 \\ s_1 s_2 \\ c_2 \end{bmatrix} \quad z_3 = \begin{bmatrix} c_1 s_2 \\ s_1 s_2 \\ c_2 \end{bmatrix} \quad O_0 = O_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad O_2 = \begin{bmatrix} -d_2 s_1 \\ d_2 c_1 \\ 0 \end{bmatrix} \quad O_3 = \begin{bmatrix} d_3 c_1 s_2 - d_2 s_1 \\ d_3 s_1 s_2 + d_2 c_1 \\ d_3 c_2 \end{bmatrix}$$

$$T_0^4 = A_1 A_2 A_3 A_4$$

$$T_0^5 = A_1 A_2 A_3 A_4 A_5$$

$$T_0^6 = A_1 A_2 A_3 A_4 A_5 A_6$$

$$T_0^4 = \begin{bmatrix} c_1 c_2 c_4 - s_1 s_4 & -c_1 s_2 & -c_1 c_2 s_4 - s_1 c_4 & c_1 s_2 d_3 - s_1 d_2 \\ s_1 c_2 c_4 + c_1 s_4 & -s_1 s_2 & -s_1 c_2 s_4 + c_1 c_4 & s_1 s_2 d_3 + c_1 d_2 \\ -s_2 c_4 & -c_2 & s_2 s_4 & c_2 d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$z_4 = \begin{bmatrix} -c_1 c_2 s_4 - s_1 c_4 \\ -s_1 c_2 s_4 + c_1 c_4 \\ s_2 s_4 \end{bmatrix}$$

$$O_4 = \begin{bmatrix} d_3 c_1 s_2 - d_2 s_1 \\ d_3 s_1 s_2 + d_2 c_1 \\ d_3 c_2 \end{bmatrix}$$

$$T_0^5 = \begin{bmatrix} (c_1c_2c_4-s_1s_4)c_5-c_1s_2s_5 & c_1c_2s_4+s_1c_4 & (c_1c_2c_4-s_1s_4)s_5+c_1s_2c_5 \\ (s_1c_2c_4+c_1s_4)c_5-s_1s_2s_5 & s_1c_2s_4-c_1c_4 & (s_1c_2c_4+c_1s_4)s_5+s_1s_2c_5 \\ -s_2c_4c_5-c_2s_5 & -s_2s_4 & -s_2c_4s_5+c_2c_5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$z_5 = \begin{bmatrix} c_1c_2c_4s_5 - s_1s_4s_5 + c_1s_2c_5 \\ s_1c_2c_4s_5 + c_1s_4s_5 + s_1s_2c_5 \\ -s_2c_4s_5 + c_2c_5 \end{bmatrix}$$

$$z_5 = \begin{bmatrix} c_1c_2c_4s_5 - s_1s_4s_5 + c_1s_2c_5 \\ s_1c_2c_4s_5 + c_1s_4s_5 + s_1s_2c_5 \\ -s_2c_4s_5 + c_2c_5 \end{bmatrix}$$

$$T_0^5 = \begin{bmatrix} (c_1c_2c_4-s_1s_4)c_5-c_1s_2s_5 & c_1c_2s_4+s_1c_4 & (c_1c_2c_4-s_1s_4)s_5+c_1s_2c_5 & c_1s_2d_3-s_1d_2 \\ (s_1c_2c_4+c_1s_4)c_5-s_1s_2s_5 & s_1c_2s_4-c_1c_4 & (s_1c_2c_4+c_1s_4)s_5+s_1s_2c_5 & s_1s_2d_3+c_1d_2 \\ -s_2c_4c_5-c_2s_5 & -s_2s_4 & -s_2c_4s_5+c_2c_5 & c_2d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$z_5 = \begin{bmatrix} c_1c_2c_4s_5 - s_1s_4s_5 + c_1s_2c_5 \\ s_1c_2c_4s_5 + c_1s_4s_5 + s_1s_2c_5 \\ -s_2c_4s_5 + c_2c_5 \end{bmatrix}$$

$$O_5 = \begin{bmatrix} d_3c_1s_2 - d_2s_1 \\ d_3s_1s_2 + d_2c_1 \\ d_3c_2 \end{bmatrix}$$

$$T_0^6 = \begin{bmatrix} (c6c5c1c2c4-c6c5s1s4-c6c1s2s5+s6c1c2s4+s6s1c4) & -c5c1c2c4+s6c5s1s4+s6c1s2s5+c6c1c2s4+c6s1c4 & s5c1c2c4-s5s1s4+c1s2c5 & d6s5c1c2c4-d6s5s1s4+d6c1s2c5+c1s2d3-s1d2 \\ c6c5s1c2c4+c6c5c1s4-c6s1s2s5+s6s1c2s4-s6c1c4 & s6c5s1c2c4-s6c5c1s4+s6s1s2s5+c6s1c2s4-c6c1c4 & s5s1c2c4+s5c1s4+s1s2c5 & d6s5s1c2c4+d6s5c1s4+d6s1s2c5+s1s2d3+c1d2 \\ -c6s2c4c5-c6c2s5-s2s4s6 & s6s2c4c5+s6c2s5-s2s4c6 & -s2c4s5+c2c5 & -d6s2c4s5+d6c2c5+c2d3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} T_6 = & [c6c5c1c2c4-c6c5s1s4-c6c1s2s5+s6c1c2s4+s6s1c4, - \\ & c5c1c2c4+s6c5s1s4+s6c1s2s5+c6c1c2s4+c6s1c4, s5c1c2c4-s5s1s4+c1s2c5, \\ & d6s5c1c2c4-d6s5s1s4+d6c1s2c5+c1s2d3-s1d2] \\ & [c6c5s1c2c4+c6c5c1s4-c6s1s2s5+s6s1c2s4-s6c1c4, -s6c5s1c2c4- \\ & s6c5c1s4+s6s1s2s5+c6s1c2s4-c6c1c4, s5s1c2c4+s5c1s4+s1s2c5, \\ & d6s5s1c2c4+d6s5c1s4+d6s1s2c5+s1s2d3+c1d2] \\ & [-c6s2c4c5-c6c2s5-s2s4s6, s6s2c4c5+s6c2s5-s2s4c6, -s2c4s5+c2c5, - \\ & d6s2c4s5+d6c2c5+c2d3] \\ & [0, 0, 0, 1] \end{aligned}$$

$$Z_6 = \begin{bmatrix} s5c1c2c4-s5s1s4+c1s2c5 \\ s5s1c2c4+s5c1s4+s1s2c5 \\ -s2c4s5+c2c5 \end{bmatrix}$$

$$O_6 = \begin{bmatrix} d6s5c1c2c4-d6s5s1s4+d6c1s2c5+c1s2d3-s1d2 \\ d6s5s1c2c4+d6s5c1s4+d6s1s2c5+s1s2d3+c1d2 \\ -d6s2c4s5+d6c2c5+c2d3 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} z_0 \times (o_6 - o_0) \\ z_0 \end{bmatrix}, J_2 = \begin{bmatrix} z_1 \times (o_6 - o_1) \\ z_1 \end{bmatrix} \quad \text{Joints 1,2 are revolute}$$

$$J_3 = \begin{bmatrix} z_2 \\ 0 \end{bmatrix} \quad \text{Joint 3 is prismatic}$$

$$J_4 = \begin{bmatrix} z_3 \times (o_6 - o_3) \\ z_3 \end{bmatrix}, J_5 = \begin{bmatrix} z_4 \times (o_6 - o_4) \\ z_4 \end{bmatrix}, J_6 = \begin{bmatrix} z_5 \times (o_6 - o_5) \\ z_5 \end{bmatrix}$$

The required Jacobian matrix J

$$J = [J_1 \quad J_2 \quad J_3 \quad J_4 \quad J_5 \quad J_6]$$

End of Lec

End of Lec