Computer Control of Dynamic Systems

Lecture-3
Pole Placement

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Pole Placement (Assignment) Technique

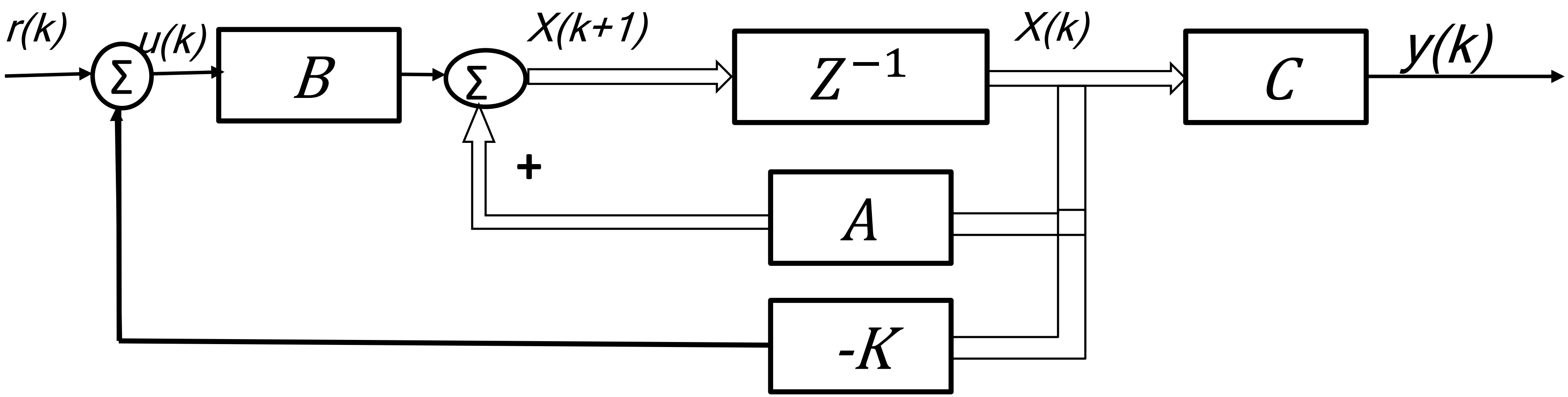
- We assume that all state variables are measurable and are available for feedback.

- If the system is completely state ***controllable***, then poles of the closed-loop system may be placed at any desired locations by means of state feedback through an appropriate state feedback gain matrix.

- This method is applicable only to linear time-invariant systems.

Pole Placement

$$x\,(k+1) = A\,x\,(k) + B\,u(k)$$
$$y(k) = C\,x\,(k)$$



$$u(k) = -k\,x\,(k) + r(k)$$

Pole Placement

$$x\,(k+1) = A\,x\,(k) + B\,(-k\,x\,(k) + r(k)\,)$$

$Let \quad r(k) = 0$

$$x\,(k+1) = A\,x\,(k) - Bk\,x\,(k)$$

$$x\,(k+1) = (A - Bk)\,x\,(k)$$

The system matrix changed to be $(A - Bk)$

Let $A_f = (A - Bk)$ *and at input* $r(k)$

Pole Placement

$$x\,(k+1) = A_f\,x\,(k) + B\,r(k)$$

$$y(k) = C\,x\,(k)$$

Example

$$x(k+1) = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} x(k) + \begin{bmatrix} 0.00484 \\ 0.00952 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)$$

- *Consider* $u(k) = -k\, x(k)$
- *Design a controller given matrix K such that the time constant of the system will be 0.5 sec, ξ=0.5.*

Example

$$S_{1,2} = -\xi\ \omega_n \pm j\ \omega_n \sqrt{1 - \xi^2}$$

$t = \frac{1}{real\ part} = \frac{1}{\xi\ \omega_n} = 0.5$

$\xi = 0.5, \quad \omega_n = 4$

$$S_{1,2} = -2 \pm j3.464$$

$$Z = e^{sT}$$

Let T=0.1 sec.

$$Z = e^{0.1s}$$

Example

$$Z_{1,2} = e^{0.1(-2 \pm j3.464)}$$

$$Z_1 = e^{-0.2 \; + j0.3464} = e^{-0.2} \, e^{\, j0.3464}$$

$$Z_1 = a\angle\theta = 0.8187\angle 0.3464$$

$$Z_1 = 0.8187\angle 19.85^{o}$$

$$Z_{1,2} = 0.77 \pm j\, 0.278$$

Example

- *Design of K by Equating the Coefficients*
- *The system must be completely state controllable.*
- *I) Desired c/cs eq. is:*

$$(Z-Z_1)(Z-Z_2)=0$$

$$(Z-0.77+j\,0.278)(Z-0.77-j\,0.278)=0$$

$$Z^2-1.54Z+0.64=0 \quad \ldots\ldots..(1)$$

Example

- *II) For the system:*

system matrix:

$$|ZI - A_f| = 0$$

$$A_f = (A - Bk)$$

$$|ZI - A + Bk\,| = 0$$

$$K = [k_1\; k_2]$$

$$Bk = \begin{bmatrix} 0.00484 \\ 0.00952 \end{bmatrix} [k_1 \quad k_2] = \begin{bmatrix} 0.00484k_1 & 0.00484k_2 \\ 0.00952k_1 & 0.00952k_2 \end{bmatrix}$$

Example

$$A - Bk = \begin{bmatrix} 1 - 0.00484k_1 & 0.0952 - 0.00484k_2 \\ -0.00952k_1 & 0.905 - 0.00952k_2 \end{bmatrix}$$

$$|ZI - A + Bk\,| = |\begin{bmatrix} Z - 1 + 0.00484k_1 & -0.0952 + 0.00484k_2 \\ 0.00952k_1 & Z - 0.905 + 0.00952k_2 \end{bmatrix}|$$

$$(Z - 1 + 0.00484k_1)(Z - 0.905 + 0.00952k_2) + 0.00952k_1(-0.0952 + 0.00484k_2) = 0 \quad \text{..........(2)}$$

Equating the coefficient in(1)&(2)

End Of Lecture