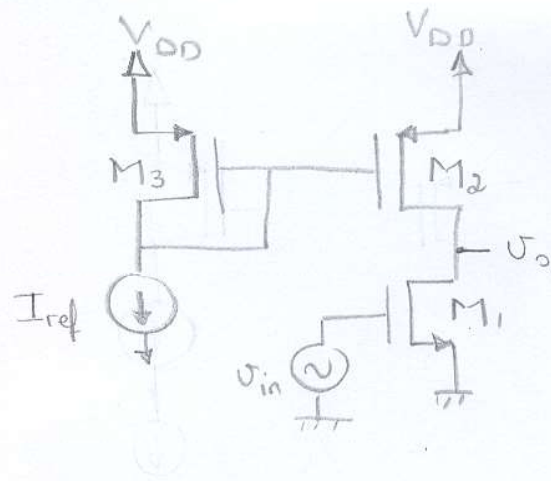


**P3**  $\mu_n C_{ox} \frac{W}{L} = 20 \mu A/V^2$

$V_A = 100 V$

$I_{ref} = 100 \mu A$

Find:  $\frac{v_o}{v_{in}}$ ,  $R_{in}$ ,  $R_{out}$



Solution:

DC Analysis

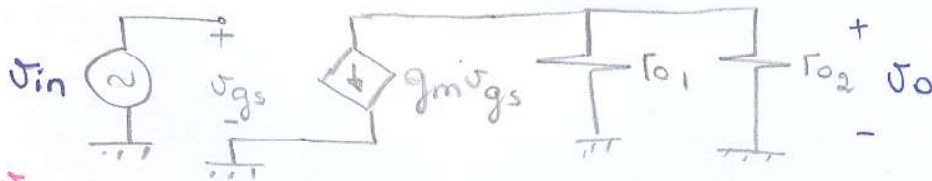
$\because M_3$  &  $M_2$  are matched transistors

$\therefore I_{ref} = I_{D3} = I_{D2} = 100 \mu A = I_{D1}$

$g_m = \sqrt{2(\mu_n C_{ox} \frac{W}{L}) I_D} = 6.32 \times 10^{-5}$

$r_o = \frac{V_A}{I_{D1}} = 1 M\Omega = r_{o1} = r_{o2}$

AC Analysis



To get  $\frac{v_o}{v_{in}}$   
 $v_{in} = v_{gs}$  — (1)

$v_{out} = -g_m v_{gs} (r_{o1} \parallel r_{o2})$  — (2)

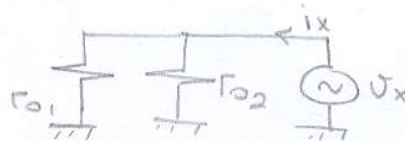
Dividing (2) by (1):

$\frac{v_{out}}{v_{in}} = \frac{-g_m v_{gs} (r_{o1} \parallel r_{o2})}{v_{gs}} = -31.6$

To get  $R_{in}$

$R_{in} = \frac{v_{in}}{i_{in}} = \frac{v_{in}}{0} = \infty$

To get  $R_{out}$



$R_{out} = \frac{v_x}{i_x} \Big|_{v_{in}=0} = (r_{o1} \parallel r_{o2}) = 500 k\Omega$

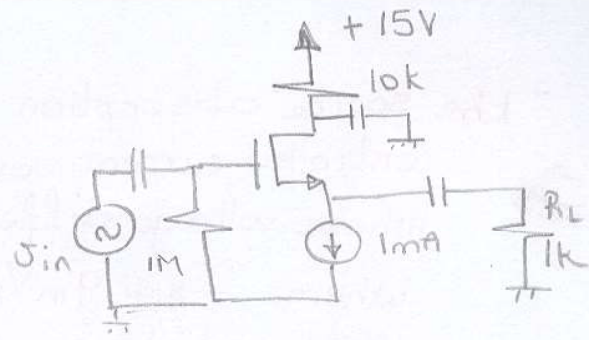
**P4** Given:  $\mu_n C_{ox} = 0.1 \text{ mA/V}^2$

$$\frac{W}{L} = 1$$

$$V_t = 1 \text{ V}$$

$$\lambda = 0.01 \text{ V}^{-1}$$

Find:  $\frac{v_o}{v_{in}}$ ,  $R_{in}$ ,  $R_{out}$



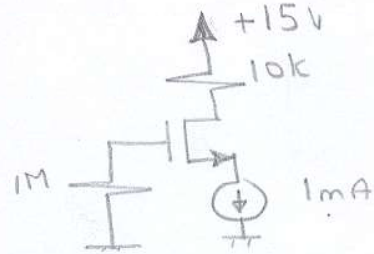
Solution:

DC Analysis

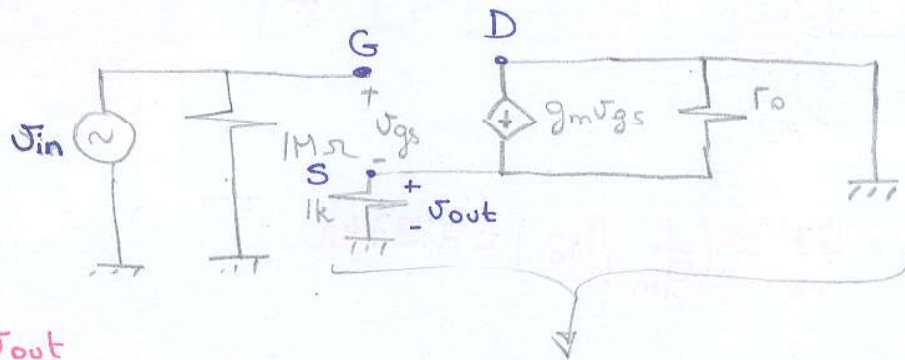
$$\therefore I_D = 1 \text{ mA}$$

$$g_m = \sqrt{2(\mu_n C_{ox} \frac{W}{L}) I_D} = 4.4 \times 10^{-4}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.01(1\text{m})} = 100 \text{ k}$$



AC Analysis



To get  $\frac{v_o}{v_{in}}$

$$v_{in} = v_{gs} + v_{out}$$

From the figure:

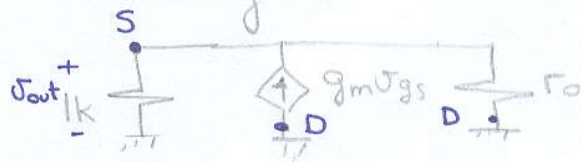
$$v_{out} = g_m v_{gs} (1 \text{ k} \parallel r_o)$$

$$v_{out} = g_m (v_{in} - v_{out}) (1 \text{ k} \parallel r_o)$$

$$v_{out} [1 + g_m (1 \text{ k} \parallel r_o)] = v_{in} g_m (1 \text{ k} \parallel r_o)$$

$$\frac{v_{out}}{v_{in}} = \frac{g_m (1 \text{ k} \parallel r_o)}{1 + g_m (1 \text{ k} \parallel r_o)} = 0.802$$

Redrawing:

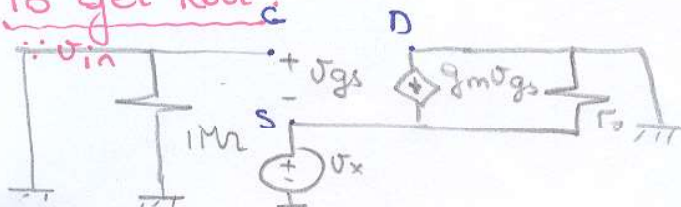


To get  $R_{in}$ :

$$v_{in} = i_{in} (1 \text{ M})$$

$$\therefore R_{in} = \frac{v_{in}}{i_{in}} = 1 \text{ M}$$

To get  $R_{out}$ :

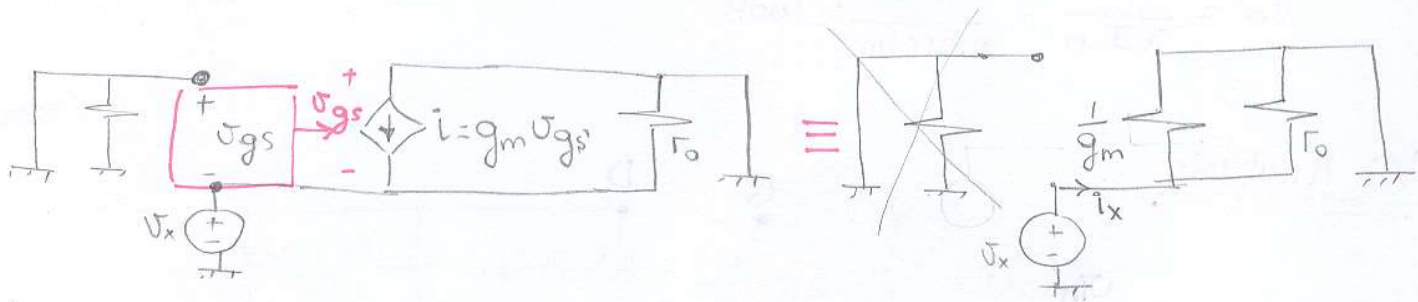


$$v_{in} = v_{gs} + v_x$$

$$i_{in} \neq 0$$

$\therefore$  We must apply source-absorption theorem

The source absorption theorem states that for a voltage-controlled current source appearing between 2 nodes whose voltage difference is the same controlling voltage, where  $I_x = g_m V_x$ , this dependent source can be replaced by an impedance  $Z_x = \frac{V_x}{I_x} = \frac{1}{g_m}$



Apply

$$\therefore R_{out} = \frac{v_x}{i_x} = \left( \frac{1}{g_m} \parallel r_o \right) = 2.187 \text{ k}\Omega$$

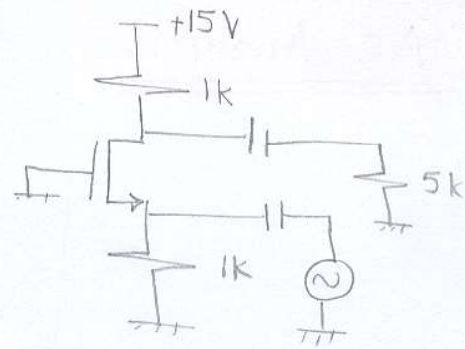
**P5** Given:  $\mu_n C_{ox} = 0.1 \text{ mA/V}^2$

$$W/L = 8$$

$$V_t = 1 \text{ V}$$

$$\lambda = 0.01 \text{ V}^{-1}$$

Find:  $\frac{V_o}{V_{in}}$ ,  $R_{in}$  and  $R_{out}$



Solution:

DC Analysis

$$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_t)^2$$

$$I_D = 0.4 \text{ m} (V_{GS} - 1)^2 \quad \text{--- (1)}$$

Apply KVL on (A):

$$V_{GS} + I_D (1\text{k}) - 15 = 0 \quad \text{--- (2)}$$

Substitute (2) in (1):

$$\frac{15 - V_{GS}}{1\text{k}} = 0.4 \text{ m} (V_{GS} - 1)^2$$

$$\frac{15}{0.4} = V_{GS}^2 - 2V_{GS} - \frac{V_{GS}}{0.4} + 1$$

$$V_{GS} = 5.79$$

$$V_{GS} = -6.29$$

$$V_{GS} > V_t$$

$$V_{GS} < V_t$$

Accepted

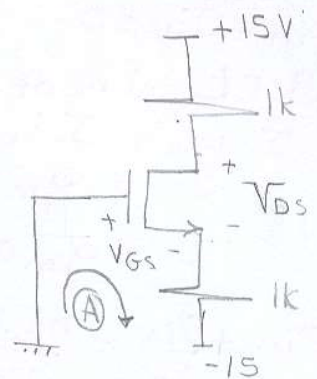
Rejected

Substitute with  $V_{GS}$  back in (1):

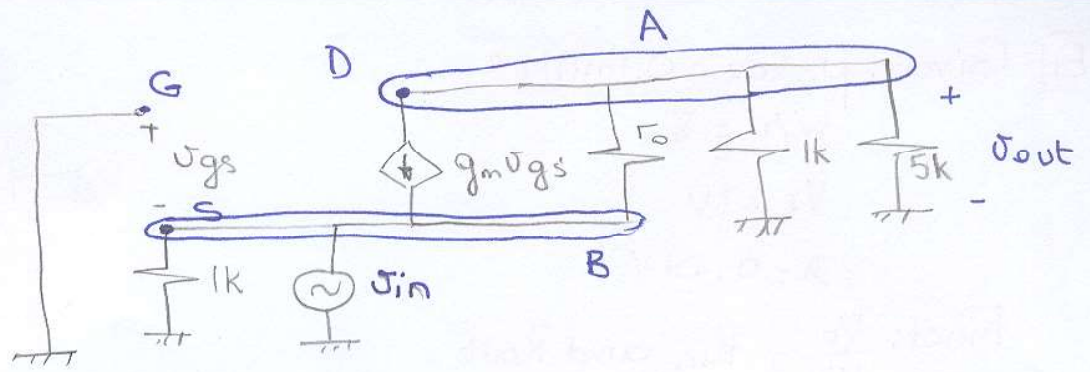
$$I_D = 0.4 \text{ m} (5.79 - 1)^2 = 9.2 \text{ mA}$$

$$r_c = \frac{1}{\lambda I_D} = \frac{1}{0.01 (9.2 \text{ mA})} = 10.87 \text{ k}\Omega$$

$$g_m = \sqrt{2 k I_{DS}} = \sqrt{2 (0.8 \text{ m}) (9.2 \text{ m})} = 3.83 \text{ mS}^{-1}$$



# AC Analysis



To get  $\frac{v_{out}}{v_{in}}$

$$v_{in} + v_{gs} = 0 \quad \therefore v_{in} = -v_{gs}$$

Apply KCL at node A:

$$\sum i_{in} = \sum i_{out}$$

$$0 = g_m v_{gs} + \frac{v_D - v_S}{r_o} + \frac{v_{out}}{1k // 5k}$$

$$0 = g_m v_{gs} + \frac{v_{out} - v_{in}}{r_o} + \frac{v_{out}}{1k // 5k}$$

$$g_m v_{in} + \frac{v_{in}}{r_o} = \frac{v_{out}}{r_o} + \frac{v_{out}}{1k // 5k}$$

$$v_{in} \left[ g_m + \frac{1}{r_o} \right] = v_{out} \left[ \frac{1}{r_o} + \frac{1}{1k // 5k} \right]$$

$$\frac{v_{out}}{v_{in}} = \frac{g_m + 1/r_o}{1/r_o + 1/(1k // 5k)} = 3.03$$

To get  $R_{in}$

Apply KCL at node B:

$$\sum i_{in} = \sum i_{out}$$

$$i_{in} + g_m v_{gs} + \frac{v_{out} - v_{in}}{r_o} = \frac{v_{in}}{1k}$$

$$i_{in} = g_m v_{in} + \frac{v_{in}}{r_o} + \frac{v_{in}}{1k} - \frac{v_{out}}{r_o}$$

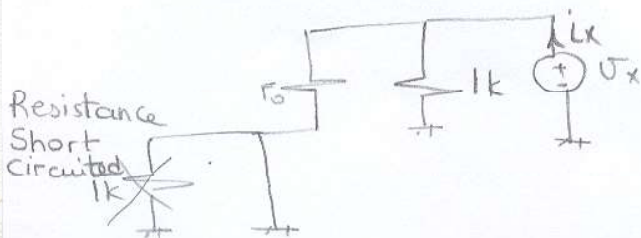
$$i_{in} = v_{in} \left[ g_m + \frac{1}{r_o} + \frac{1}{1k} \right] - \frac{v_{out}}{r_o} \quad \left( \times \frac{1}{v_{in}} \right)$$

$$\frac{i_{in}}{v_{in}} = g_m + \frac{1}{r_o} + \frac{1}{1k} - \frac{v_{out}/v_{in}}{r_o}$$

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{g_m + 1/r_o + 1/1k - \frac{3.03}{r_o}} = 215.36 \Omega$$

To get  $R_{out}$

$$\therefore v_{in} = 0 \quad \therefore v_{gs} = 0 \quad \therefore g_m v_{gs} = 0$$



$$\therefore R_{out} = \frac{v_x}{i_x} \Big|_{v_{in}=0} = 1k // r_o = 915.75 \Omega$$