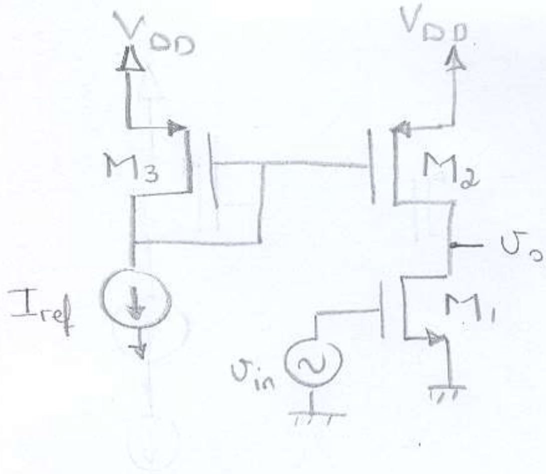
P3 $\mu_n C_{ox} \frac{W}{L} = 20\, \mu A/V^2$

$V_A = 100\ V$

$I_{ref} = 100\, \mu A$

Find: $\frac{v_o}{v_{in}}$, R_{in}, R_{out}



Solution:

DC Analysis

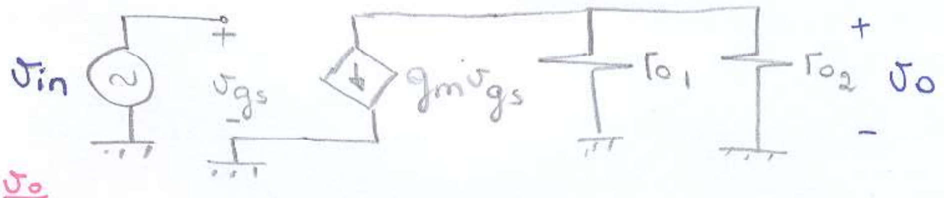
$\because$ M_3 & M_2 are matched transistors

$\therefore$ $I_{ref} = I_{D_3} = I_{D_2} = 100\, \mu A = I_{D_1}$

$g_m = \sqrt{2(\mu_n C_{ox} \frac{W}{L}) I_D} = 6.32 \times 10^{-5}$

$r_o = \frac{V_A}{I_{D_1}} = 1\, M\Omega = r_{o_1} = r_{o_2}$

AC Analysis



To get $\frac{v_o}{v_{in}}$

$v_{in} = v_{gs}$ —— ①

$v_{out} = -g_m v_{gs} (r_{o_1} // r_{o_2})$ —— ②

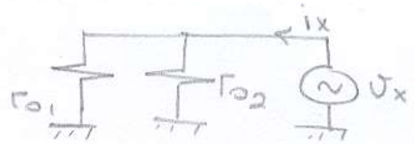
Dividing ② by ①:

$$\frac{v_{out}}{v_{in}} = \frac{-g_m \cancel{v_{gs}} (r_{o_1} // r_{o_2})}{\cancel{v_{gs}}} = -31.6$$

To get Rin

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{v_{in}}{0} = \infty$$

To get Rout



$$R_{out} = \frac{v_x}{i_x}\Big|_{v_{in}=0} = (r_{o_1} // r_{o_2}) = 500\, k\Omega$$

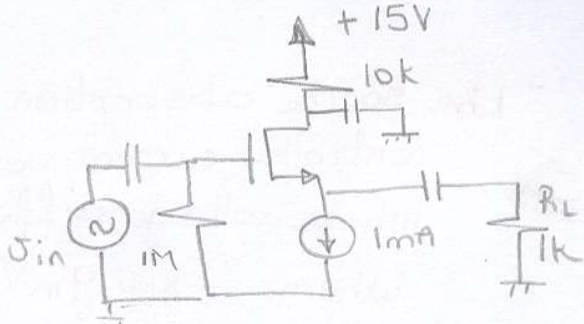
P4 Given: $\mu_n C_{ox} = 0.1\, mA/V^2$

$\frac{W}{L} = 1$

$V_t = 1V$

$\lambda = 0.01\, V^{-1}$

Find: $\frac{v_o}{v_{in}}$, R_{in}, R_{out}



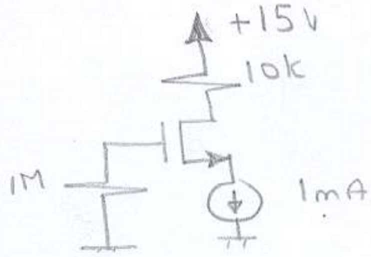
Solution:

DC Analysis

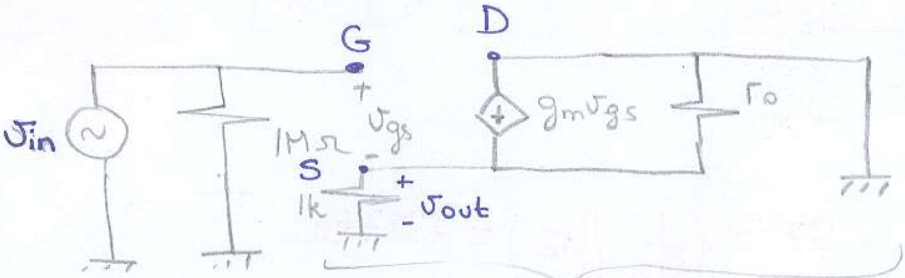
$\because I_D = 1mA$

$g_m = \sqrt{2(\mu_n C_{ox} \frac{W}{L}) I_D} = 4.4 \times 10^{-4}$

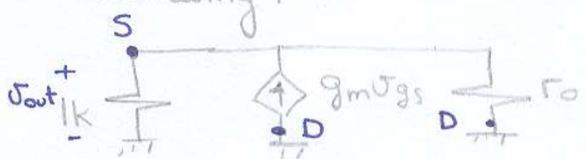
$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.01(1m)} = 100k$



AC Analysis



Redrawing:



To get $\frac{v_o}{v_{in}}$

$v_{in} = v_{gs} + v_{out}$

From the figure:

$v_{out} = g_m v_{gs} (1k \| r_o)$

$v_{out} = g_m (v_{in} - v_{out})(1k \| r_o)$

$v_{out}\left[1 + g_m (1k \| r_o)\right] = v_{in} g_m (1k \| r_o)$

$\frac{v_{out}}{v_{in}} = \frac{g_m (1k \| r_o)}{1 + g_m (1k \| r_o)} = 0.802$

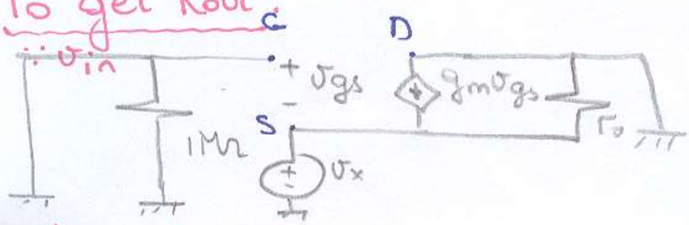
To get R_{in}:

$v_{in} = i_{in} (1M)$

$\therefore R_{in} = \frac{v_{in}}{i_{in}} = 1M$

To get R_{out}:

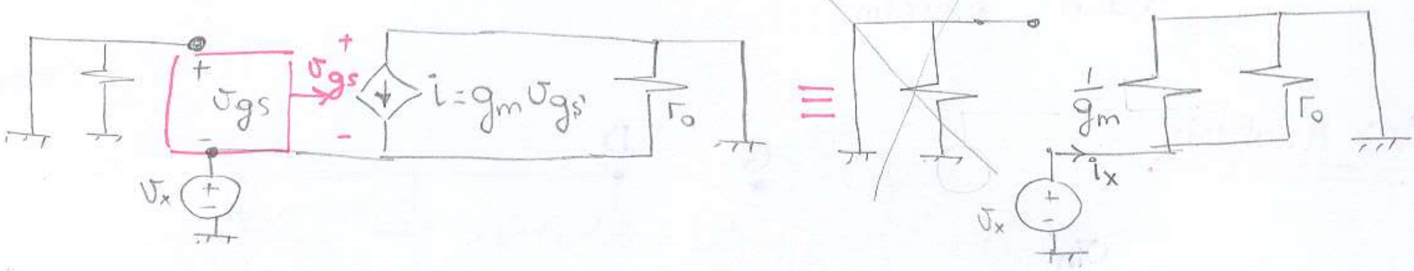
$\because v_{in}$



$v_{in} \underset{\to 0}{} = v_{gs} + v_x$

$v_{gs} \neq 0$ $\therefore$ We must apply source_absorption theorem

The source absorption theorem states that for a voltage-controlled current source appearing between 2 nodes whose voltage difference is the same controlling voltage, where $I_x = g_m V_x$, this dependant source can be replaced by an impedance $Z_x = \frac{V_x}{I_x} = \frac{1}{g_m}$



$\therefore R_{out} = \frac{v_x}{i_x} = \left(\frac{1}{g_m} \,||\, r_o\right) = 2.187\,k\Omega$

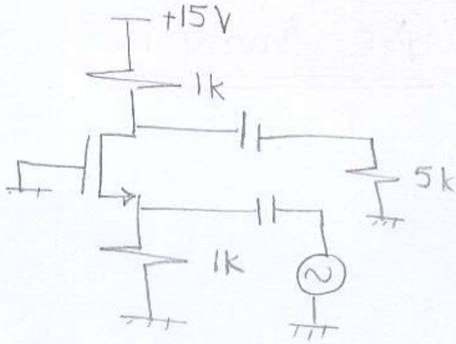
P5 Given: $\mu_n C_{ox} = 0.1 mA/V^2$

$W/L = 8$

$V_t = 1V$

$\lambda = 0.01 V^{-1}$

Find: $\frac{V_o}{V_{in}}$, R_{in} and R_{out}

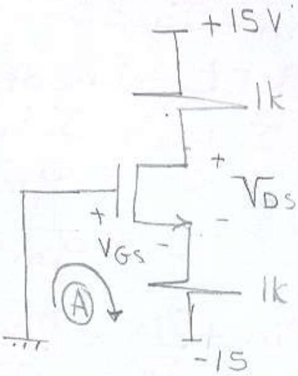


Solution:

DC Analysis

$$I_D = \tfrac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)(V_{GS} - V_t)^2$$

$$I_D = 0.4m (V_{GS} - 1)^2 \quad \text{—— ①}$$



Apply KVL on Ⓐ:

$$V_{GS} + I_D (1k) - 15 = 0 \quad \text{—— ②}$$

Substitute ② in ①:

$$\frac{15 - V_{GS}}{1k} = 0.4m (V_{GS} - 1)^2$$

$$\frac{15}{0.4} = V_{GS}^2 - 2V_{GS} - \frac{V_{GS}}{0.4} + 1$$

$V_{GS} = 5.79$ $\qquad$ $V_{GS} = -6.29$

$V_{GS} > V_t$ ✓ $\qquad$ $V_{GS} < V_t$ ✗

Accepted $\qquad$ Rejected

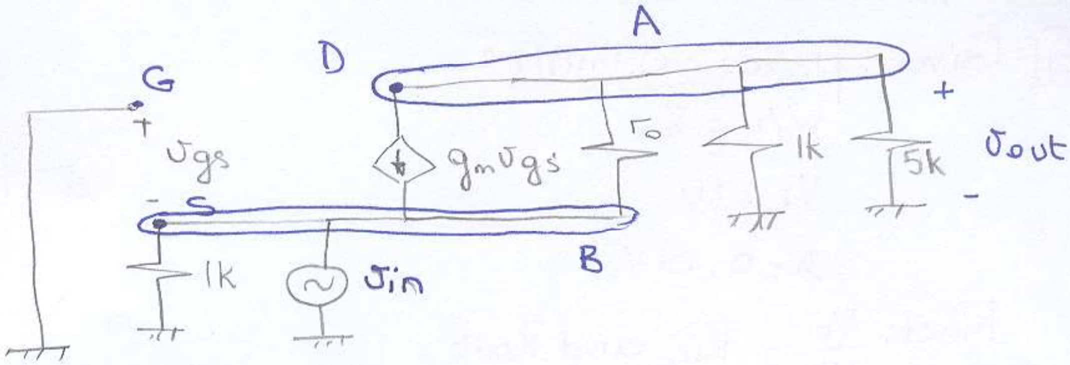
Sustitute with V_{GS} back in ①:

$$I_D = 0.4m (5.79 - 1)^2 = 9.2 mA$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.01 (9.2mA)} = 10.87 k\Omega$$

$$g_m = \sqrt{2 k I_{DS}} = \sqrt{2 (0.8m)(9.2m)} = 3.83 \, m\Omega^{-1}$$

AC Analysis



To get $\frac{v_{out}}{v_{in}}$

$v_{in} + v_{gs} = 0 \quad \therefore v_{in} = -v_{gs}$

Apply KCL at node A :

$$\Sigma i_{in} = \Sigma i_{out}$$

$$0 = g_m v_{gs} + \frac{V_D - V_S}{r_o} + \frac{v_{out}}{1k // 5k}$$

$$0 = g_m v_{gs} + \frac{v_{out} - v_{in}}{r_o} + \frac{v_{out}}{1k // 5k}$$

$$g_m v_{in} + \frac{v_{in}}{r_o} = \frac{v_{out}}{r_o} + \frac{v_{out}}{1k // 5k}$$

$$v_{in}\left[g_m + \frac{1}{r_o}\right] = v_{out}\left[1/r_o + 1/_{1k//5k}\right]$$

$$\frac{v_{out}}{v_{in}} = \frac{g_m + 1/r_o}{1/r_o + \frac{1}{1k // 5k}} = 3.03$$

To get Rin

Apply KCL at node B :

$$\Sigma i_{in} = \Sigma i_{out}$$

$$i_{in} + g_m v_{gs} + \frac{v_{out} - v_{in}}{r_o} = \frac{v_{in}}{1k}$$

$$i_{in} = g_m v_{in} + \frac{v_{in}}{r_o} + \frac{v_{in}}{1k} - \frac{v_{out}}{r_o}$$

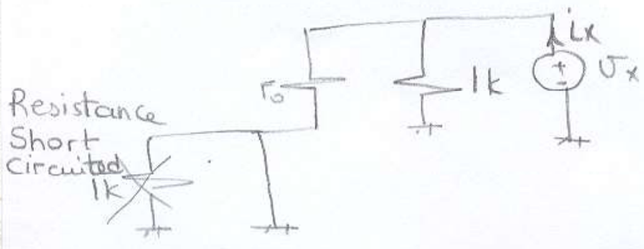
$$i_{in} = v_{in}\left[g_m + \frac{1}{r_o} + \frac{1}{1k}\right] - \frac{v_{out}}{r_o} \qquad \left(\times \frac{1}{v_{in}}\right)$$

$$\frac{i_{in}}{v_{in}} = g_m + 1/r_o + 1/_{1k} - \frac{v_{out}/v_{in}}{r_o}$$

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{g_m + 1/r_o + 1/_{1k} - \frac{3.03}{r_o}} = 215.36\,\Omega$$

To get Rout

$\because v_{in} = 0 \quad \therefore v_{gs} \quad \therefore g_m v_{gs} = 0$



$$\therefore R_{out} = \left.\frac{v_x}{i_x}\right|_{v_{in}=0} = 1k // r_o = 915.75\,\Omega$$