COLLEGE OF ENGINEERING & TECHNOLOGY

Department: Electronics and Communications Engineering

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Course Title: Advanced Engineering Mathematics

Course No.: EC738 **Problem Set #2**

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Eigenvalue Problems (Analytical)

1. Find the eigenvalues and eigenvectors for:

a) $A = \begin{bmatrix} 3 & 4 \\ -5 & -5 \end{bmatrix}$

b) $A = \begin{bmatrix} 5 & 2 & 2 \\ 3 & 6 & 3 \\ 6 & 6 & 9 \end{bmatrix}$

   i. Show that this problem is an example of an eigenvector with multiplicity more than 1.
   ii. Use the LU factorization to solve for the eigenvectors (after finding the corresponding eigenvalue).

c) In problem 1b, prove that the sum of eigenvalues equals the trace of the matrix (the trace of a matrix is the sum of the diagonal components)

Eigenvalue Problems (Iterative: Polynomial Method)

2. For the problem in lecture 3 (Schrodinger Equation for a 1D infinite potential well, use four internal nodes:
   a. Write down the new matrix equation
   b. Use the polynomial method the approximate the 4 eigenvalues
   c. Show the % error over 2-node & 3-node solutions given in lecture.

Eigenvalue Problems (Iterative: Power Method)

3. For problem 1b, use the power method (3 iterations only) to find the eigenvalue with the largest absolute value, and the corresponding eigenvector. Compare with analytical results.

4. For problem 1b, use the inverse power method (3 iterations only) to find the eigenvalue with the smallest absolute value, and the corresponding eigenvector. Compare with analytical results. Note that the LU factorization needs to be calculated only <u>once</u>, since only the right hand side *(RHS)* vector is changing after each iteration.

5. Use the shifted power method to calculate the second highest (in absolute value) eigenvalue/eigenvector for:

$$A = \begin{bmatrix} 5 & -1 & 7 \\ -1 & -1 & 1 \\ 7 & 1 & 5 \end{bmatrix}$$