COLLEGE OF ENGINEERING & TECHNOLOGY

Department: Electronics and Communications Engineering

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Course Title: Advanced Engineering Mathematics

Course No.: EC760 **Problem Set #4**

Date: Dec. 9, 2014

Series Solution of Differential Equations

1. Show by inspection that $N!$ is larger than x^N, as *N* tends to ∞ , for any value of *x*.
2. For the function $f(x) = exp(-2x)$:
   a. Verify that a series expansion is possible using Taylor's expansion at x=0
   b. Derive an expression for f(x) as a series expansion.
   c. If only the first three terms of the series are used, calculate the error (in %) for x=1, using direct substitution in series terms.
   d. Repeat **2c**, but use the remainder term to find the error instead of substitution in the series terms
3. For the ordinary differential equation (ODE):
$$\frac{d^2y}{dt^2} + (t-1)\frac{dy}{dt} + (2t-3)y = 0$$
   a. Find the recurrence formula for the series solution of y(t) around t=0
   b. Expand the solution near t=0 for the first powers of t^n, where $n \leq 3$
4. Find a series solution the *Airy* Differential equation using the same steps in Q3:
$$y'' + xy = 0$$
5. Check if t=0 is an ordinary or singular point for the ODE in problems 3, and x=0 for problem 4.
6. Check for the following ODE if they have ordinary, singular, or regular singular points at x=0:
   a. $2x^2y'' + 7x(x+1)y' - 3y = 0$
   b. $x^3y'' + 2x^2y' + y = 0$
7. Apply *Frobenius Method* to find one series solution for the following DE:
$$x^2y'' + (x^2 - 2x)y' + 2y = 0$$
8. Apply *Frobenius Method* to find a solution for the following Bessel's DE:
$$x^2y'' + xy' + x^2y = 0$$
9. Apply *Frobenius Method* to find a solution for the following Bessel's DE:
$$x^2y'' + xy' + (x^2 - 4)y = 0$$
10. Evaluate the solutions for both problems 8 and 9 at x=0, 2,4,6,8,10, then plot them qualitatively (free hand plot) using these points.