MOSFET

Drift-Diffusion Current, MOSFET Double Integral, Charge Sheet

Lecture7-8

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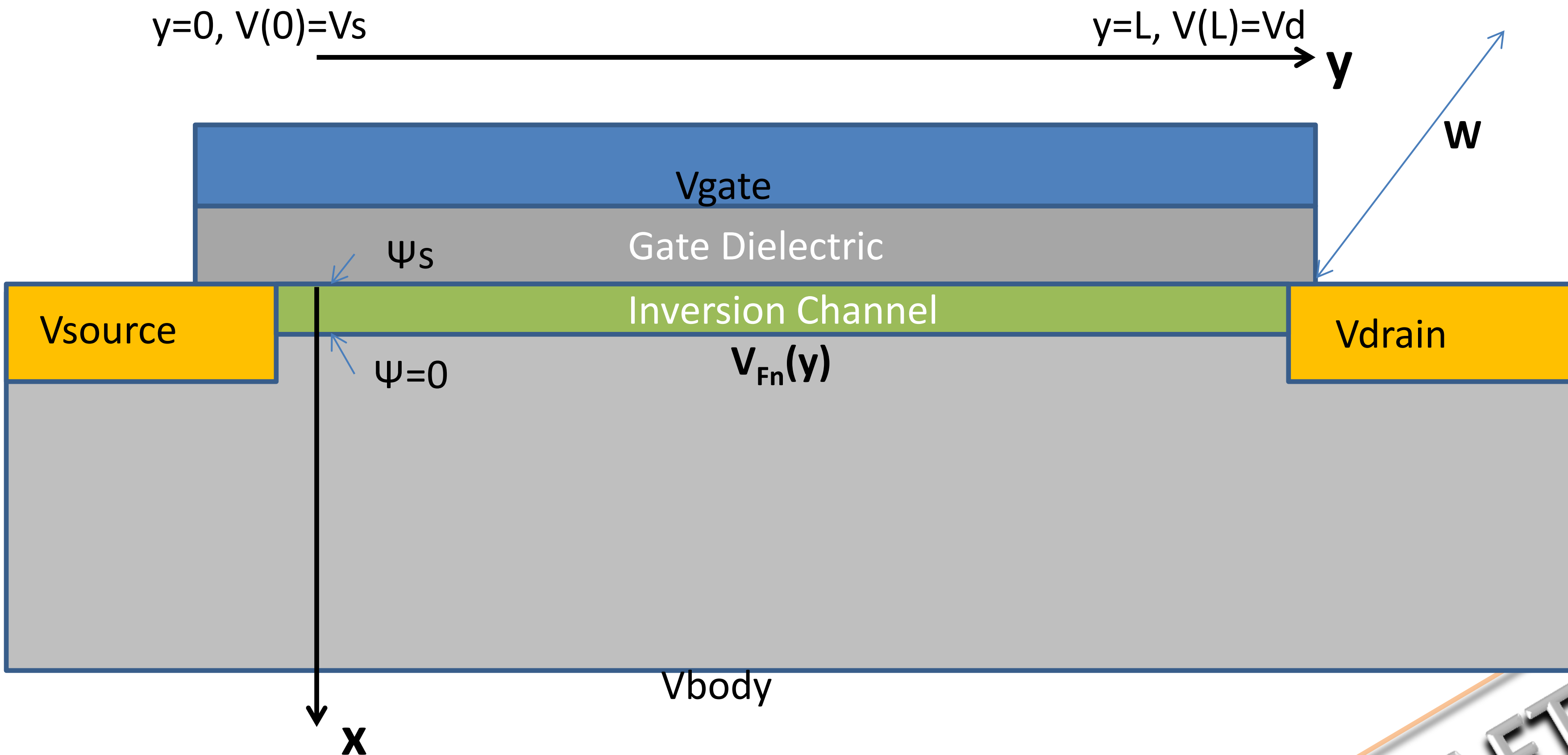
Outline

- Quasi-Fermi Potential and Drift-Diffusion Current
- MOSFET Charge Equation
- MOSFET Current Equation
- Charge Sheet Approximation

MOSFET

$$J_n = -q\, n(x,y)\, \mu_n \frac{d}{dy}[V(y)]$$



Drift-Diffusion Equation Along y-axis (J_n)

$$J_{total} = Jn + Jp$$

For Electrons: $J_n = q\, n(y)\mu_n \mathcal{E} + qD_n \frac{dn(y)}{dy}$

$$= -q\, n(y)\mu_n \frac{dV_i(y)}{dy} + \mu_n K_B T\, \frac{dn(y)}{dy}$$

$$= -q\, n(y)\mu_n \left[\frac{dV_i(y)}{dy} - \frac{K_B T}{q}\frac{1}{n(y)}\frac{dn(y)}{dy}\right]$$

$$= -q\, n(y)\mu_n \left[\frac{dV_i(y)}{dy} - \frac{K_B T}{q}\frac{d\, ln(n(y))}{dy}\right]$$

$$= -q\, n(y)\mu_n \frac{d}{dy}\left[V_i(y) - \frac{K_B T}{q} ln(\, n(y)\,)\right]$$

Setting Quasi-Fermi Potential $V_{QF}(y)$=V (y):

$$J_n = -q\, n(y)\, \mu_n \frac{d}{dy}[V(y)]$$

Drift-Diffusion Equation Along y-axis (J_p)

For Holes:

$$J_p = q\, p(y)\mu_p \boldsymbol{\mathcal{E}} - qD_p \frac{dp(y)}{dy}$$

$$= -q\, p(y)\, \mu_p \frac{dVi(y)}{dy} + \mu_p K_B T \frac{dp(y)}{dy}$$

$$J_p = -q\, p(y)\, \mu_p \left[\frac{dV_i(y)}{dy} + \frac{K_B T}{qp(y)} \frac{dp(y)}{dy}\right]$$

$$= -q\, p(y)\, \mu_p \left[\frac{dV_i(y)}{dy} + \frac{K_B T}{q} \frac{d\, ln(p(y))}{dy}\right]$$

$$= -q\, p(y)\, \mu_p \frac{d}{dy}\left[V_i(y) + \frac{K_B T}{q} ln(\, p(y)\,)\right]$$

Setting Quasi-Fermi Potential $V_{QF}(y)$=V(y):

$$J_p = -q\, p(y)\, \mu_p \frac{d}{dy}[V(y)]$$

Current Equation

$$J_n = -q\, n(x,y)\, \mu_n \frac{d}{dy}[\boldsymbol{V_{Fn}(y)}]$$

Current dI flowing in a small cross section of thickness dx (from drain to source, i.e. in direction of –ve y-axis (opposite to Jn):

$$dI = -J_n * \ Cross\ section\ area\ of\ channel = -J_n\ [d_x\, W]$$

$$I_{ds} = \int_0^{xi(y)} dI = \int_0^{xi(y)} -J_n\, W\, dx = \int_0^{xi(y)} q\, n(x,y)\, \mu_n \frac{d}{dy}[V(y)]W dx$$

$$= \mu_{neff}\, [W \int_0^{xi(y)} q\, n(x,y)\, dx] \frac{d}{dy}[V(y)]$$

$$I_{ds} = \mu_{neff}\, W[-Q_i(y)] \frac{d}{dy}[V(y)] = \boldsymbol{constant}$$

$$I_{ds} \int_0^L dy = \mu_{neff}\, W \int_{Vs}^{Vd} [-Q_i(y)] d[V(y)]$$

Current Equation (2): Double Integral

$$I_{ds} = \mu_{neff} \frac{W}{L} \int_{Vs}^{Vd} -Q_i(y)\, dV(y)$$

$$I_{ds} = \mu_{neff} \frac{W}{L} \int_{Vs}^{Vd} \left[\int_{0}^{x_i(y)} n(x,y)\, dx \right] dV(y)$$

Where $x_i(y)$,the inversion channel thickness at y is defined as:

$$x_i(y) = x \; at \; which \; inversion \; ends \rightarrow x \; at \; \psi = \psi_B$$

Current Equation (2):
Pao-Sah Double Integral

$$n(x,y) = n(\Psi, V) = \frac{n_i^{\,2}}{N_A}\, e^{q(\Psi - V)/kT}$$

$$Q_{i(V)} = -q \int_{\Psi s}^{\Psi B} n(x,y)\, \frac{dx}{d\Psi}\, d\Psi$$

$$= -q \int_{\Psi B}^{\Psi s} \frac{n_i^{\,2}}{N_A}\, e^{q(\Psi - V)/kT}\, \frac{-1}{\mathcal{E}_x(\Psi, V)}\, d\Psi$$

$$I_{ds} = q\, \mu_{neff}\, \frac{W}{L} \int_{Vs}^{Vd} \left[\int_{\Psi B}^{\Psi s} \frac{n_i^{\,2}}{N_A}\, e^{q(\Psi - V)/kT}\, \frac{-1}{\mathcal{E}_x(\Psi, V)}\, d\Psi \right] dV(y)$$

Charge Sheet Approximation: Strong Inversion

In the presence of quasi-Fermi potential V(y):

$$\Psi_s \rightarrow 2\Psi_B + V(y)$$

$$Q_i(V) = C_{ox}[V_{gs} - (V_{FB} + \frac{\sqrt{2\varepsilon_{si} q N_A (2\Psi_{Bulk} + V(y))}}{C_{ox}} + 2\Psi_{Bulk} + V(y))]$$

Using Vs=0 as reference:

$$I_{ds} = \mu_{neff} \frac{W}{L} \int_0^{Vds} [-Q_i(y)\ ]dV(y)$$

Charge Sheet Approximation: Strong Inversion

$$Q_i(V) = -C_{ox}[V_{gs} - (V_{FB} + 2\Psi_B + V(y))] + \sqrt{2\varepsilon_{si} q N_A (2\Psi_{Bulk} + V(y))}$$

$$I_{ds} = \mu_{neff} \frac{W}{L} C_{ox}[\left(V_{gs} - VF_B - 2\Psi_B - \frac{V_{ds}}{2}\right) V_{ds} - \frac{2\sqrt{2\varepsilon_{si} q N_A}}{3C_{ox}} \left[(2\Psi_B + Vds)^{\frac{3}{2}} + (2\Psi_B)^{\frac{3}{2}}\right]]$$

Charge Sheet Approximation: Strong Inversion, Linear Current Regime

At small Vds, after power series expansion , and keeping first terms:

$$I_{ds\ linear} = \mu_{neff}\frac{W}{L}C_{ox}\left(V_{gs} - V_{FB} - 2\Psi_B - \frac{\sqrt{2\varepsilon_{si}qN_A(2\Psi_{Bulk})}}{C_{ox}}\right)V_{ds}$$

$$= \mu_{neff}\frac{W}{L}C_{ox}(V_{gs} - V_t)V_{ds}$$

Charge Sheet Approximation, Strong Inversion: **Large V_{ds}: Parabolic Current Regime**

$$I_{ds} = \mu_{neff} \frac{W}{L} [C_{ox}(V_{gs} - V_t)V_{ds} - \frac{m}{2} V_{ds}{}^2]$$

$$m = 1 + \frac{\sqrt{\varepsilon_{si} q N_A/(4\Psi_B)}}{C_{ox}} = 1 + \frac{C_{dm}}{C_{ox}} = 1 + \frac{3tox}{W_{dm}}$$

m= Body Coefficient

W_{dm}= Depletion Width at Ψs=2ΨB

C_{dm}= Depletion Capacitance at Ψs=2ΨB:

$$V_t = V_{FB} + (2m - 1)(2\Psi_B)$$

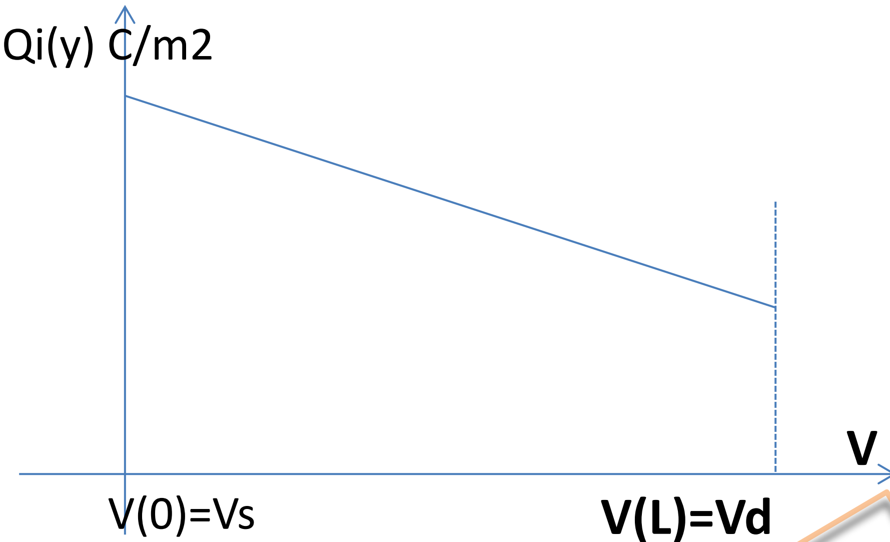
Charge Sheet Dependence on y in Strong Inversion, using Charge Sheet Approx.

Before pinch-off, where $V(y) \le 2\Psi_B$, the *sqrt* term in $Q_i(V)$ can be expanded as:

$$Q_i(V) = Qs - Q_d = C_{ox}\left(V_{gs} - V_t - m\,V(y)\right)$$

At $y = 0 \;\rightarrow\; V(0) = Vs$

At $y = L \;\rightarrow\; V(L) = Vd$



At Pinch-off:

$Q_i(V) = 0, \qquad V_{dsat} = constant$

$C_{ox}\left(V_{gs} - V_t - m\,V_{dsat}\right)=0$

$$\boldsymbol{V_{dsat} = \frac{V_{gs} - V_t}{m}}$$

$V(y)$ Dependence on y in Strong Inversion

$$I_{ds} \int_0^y dy = \mu_{n\,eff}\, W \int_0^V [-Qi(y)]dV$$

$$I_{ds}\, y = \mu_{neff}\, W \int_0^V \left[C_{ox}\left(V_{gs} - V_t - m\, V(y)\right)\right]dV$$

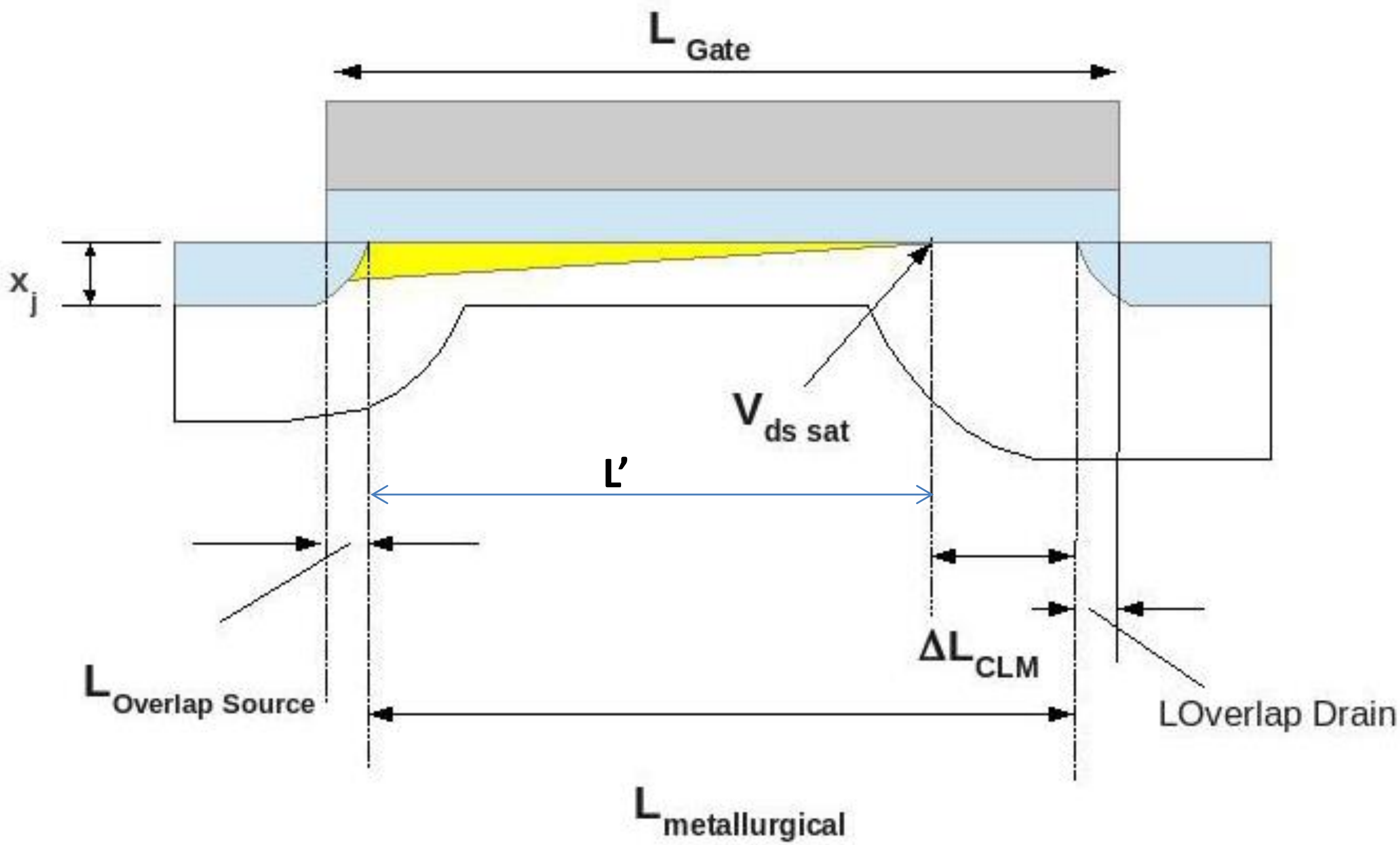
$$\left\{\mu_{n\,eff}\, \frac{W}{L} C_{ox}\left[\left(V_{gs} - V_t\right)V_{ds} - \frac{m}{2} V_{ds}{}^2\right]\right\} y$$

$$= \mu_{n\,eff}\, W C_{ox}\left[\left(V_{gs} - V_t\right)V\,(y) - \frac{m}{2}\, V(y)^2\right]$$

$$\rightarrow V(y) = \frac{V_{gs} - V_t}{m} - \sqrt{\left(\frac{V_{gs} - V_t}{m}\right)^2 - 2\frac{y}{L}\left(\frac{V_{gs} - Vt}{m}\right)V_{ds} + \frac{y}{L} V_{ds}{}^2}$$

$V(y)$ has a $sqrt$ dependence on y

Current Saturation: Pinch-Off



Charge Sheet Approximation, Strong Inversion: $V_{ds} \geq V_{gs} - Vt$: Current Saturation Regime (Pinch-off)

By substituting in the parabolic current equation using:

$$V_{dsat} = (V_{gs} - V_t)/m$$

$$\rightarrow I_{ds} = \mu_{n\,eff} \frac{W}{L} C_{ox} \frac{(V_{gs} - V_t)^2}{2m}$$

For long channel, in Current Saturation (Pinch-off), at $V_{ds} \geq V_{dsat}$:

I_{ds} is independent of V_{ds}

Vds at the end of the inversion channel is fixed at Vdsat

Saturation Vds (V_{dsat}) at Pinchoff

(no approximation by series expansion)

$$Q_i(V) = -C_{ox}[V_{gs} - (V_{FB} + 2\Psi_B + V(y))] + \sqrt{2\varepsilon_{si} q N_A (2\Psi_{Bulk} + V(y))}$$

Using V_{dsat} instead of $V(y)$, and using $Q_i(V_{dsat}) = Qi(L') = 0$:

$$0 = -C_{ox}[Vgs - (V_{FB} + 2\Psi_B + V_{dsat})] + \sqrt{2\varepsilon_{si} q N_A (2\Psi_{Bulk} + V_{dsat})}$$

$$V_{dsat} = V_{gs} - V_{FB} - 2\Psi_B + \frac{\varepsilon_{si} q N_A}{C_{ox}{}^2} - \sqrt{\frac{2\varepsilon_{si} q N_A}{C_{ox} 2}\left(V_{gs} - V_{FB} + \frac{\varepsilon_{si} q N_A}{C_{ox}{}^2}\right)}$$

Exercise:

Compare with: $V_{dsat} = (V_{gs} - V_t)/m$

Qinv Dependence on L in Current Saturation

(Using approximation by series expansion)

$$Q_i(V) = -C_{ox}\left(V_{gs} - V_t - m\,V(y)\right)$$

Using $V_{dsat} = (V_{gs} - V_t)/m$ instead ofV_{ds}:

$$V(y) = \frac{V_{gs} - V_t}{m} - \sqrt{\left(\frac{V_{gs} - V_t}{m}\right)^2 - 2\frac{y}{L}\left(\frac{V_{gs} - V_t}{m}\right)V_{dsat} + \frac{y}{L}V_{dsat}{}^2}$$

$$= \frac{V_{gs} - V_t}{m}\left(1 - \sqrt{1 - \frac{y}{L}}\right)$$

$$Q_i(V) = -C_{ox}\left(V_{gs} - V_t\right)\sqrt{1 - \frac{y}{L}}$$