AAST-Cairo
EC210
Solid State Electronics
Spring 2015

Lecture 4

Electron Equivalent Wavelength, Introduction to Schrodinger Wavefunction, Quantum Well

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Kasap:

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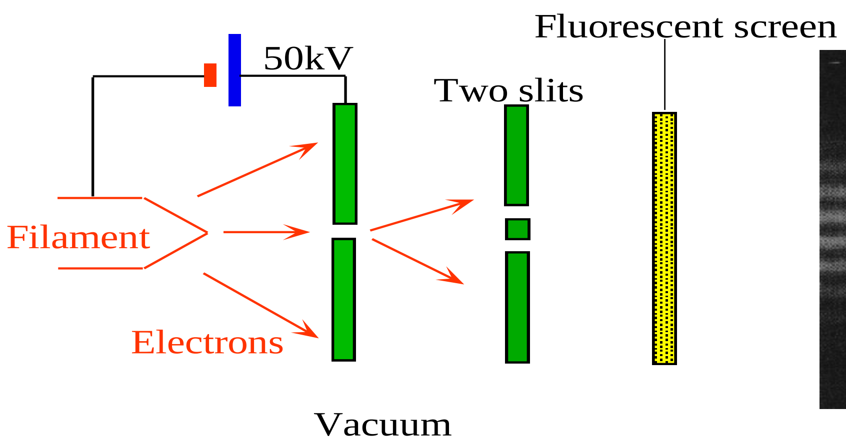
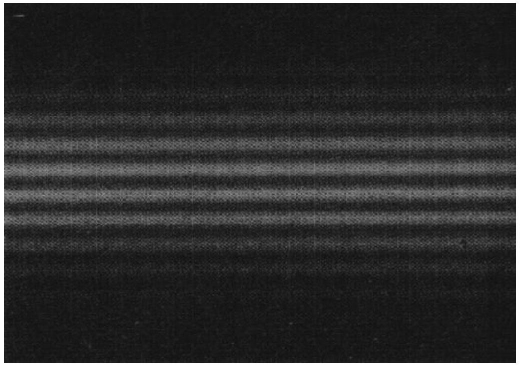
Young's Experiment





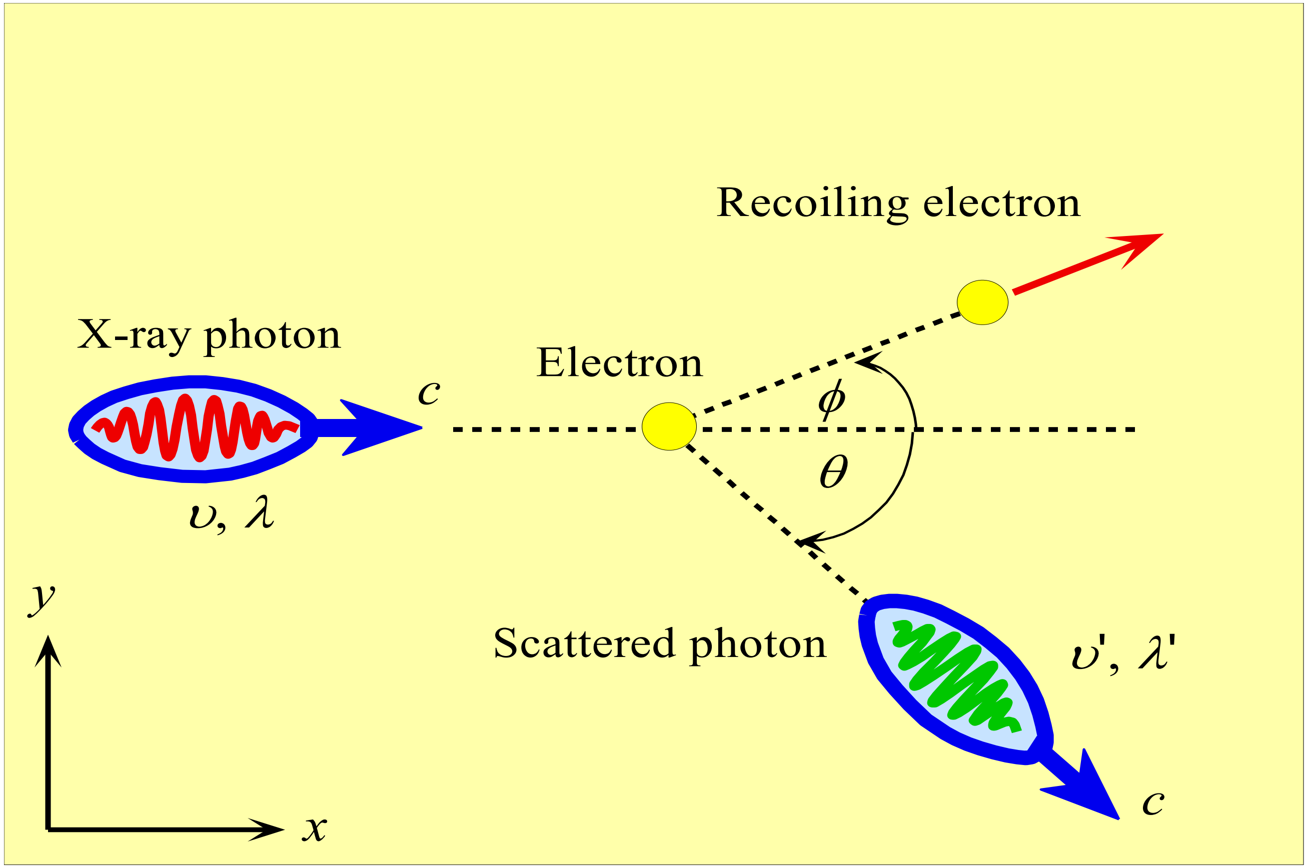
Electron diffraction fringes on the screen

Fig 3.12: Young's double slit experiment with electrons involves an electron gun and two slits in a cathode ray tune (CRT) (hence in vacuum). Electrons from the filament are accelerated by a 50 kV anode voltage to produce a beam which is made to pass through the slits. The electrons then produce a visible pattern when they strike a fluoresecent screen (e.g. a TV screen) and the resulting visual pattern is photographed (pattern from C. Jönsson, D. Brandt, S. Hirschi, Am. J. Physics, **42**, Fig. 8, p. 9, 1974.

From *Principles of Electronic Materials and Devices, Second Edition*, S.O. Kasap (© McGraw-Hill, 2002) http://Materials.Usask.Ca

http://www.youtube.com/watch?v=DfPeprQ7oGc

Compton's Scattering



Scattering of an x-ray photon by a "free" electron in a conductor.

Fig 3.9

From *Principles of Electronic Materials and Devices, Third Edition*, S.O. Kasap (© McGraw-Hill, 2005)

Compton Scattering

- Kinetic Energy of Scattered *electron*
  - $KE = \frac{1}{2} m v_e^2 = h\upsilon - h\upsilon'$
  - *υ is the frequency of light f, v_e is the electron velocity*
- Momentum of *photon*
  - $p = h / \lambda$
  - $\lambda = 2\pi / k$
- Energy of photon
  - $E = h\upsilon = (h/2\pi)\,\omega$

Schrödinger equation: Wavefunction

-The Schrödinger equation plays the role of Newton's laws in classical mechanics

-Explains the wave-like nature of electrons using photon-like concepts

Assuming:

$$\psi(x,t) = \psi(x)\,\phi(t) = \psi(x)\,e^{-i\omega t} = \psi(x)\,e^{-i\frac{E}{\hbar}t}$$

$$e^{-i\theta} = \cos\theta + i\,sin\,\theta$$

1-D, time-independent Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + \boldsymbol{V}(x)\psi(x) = E\psi(x)$$

- ***V*(x): Potential Energy**
- **E: Total energy**

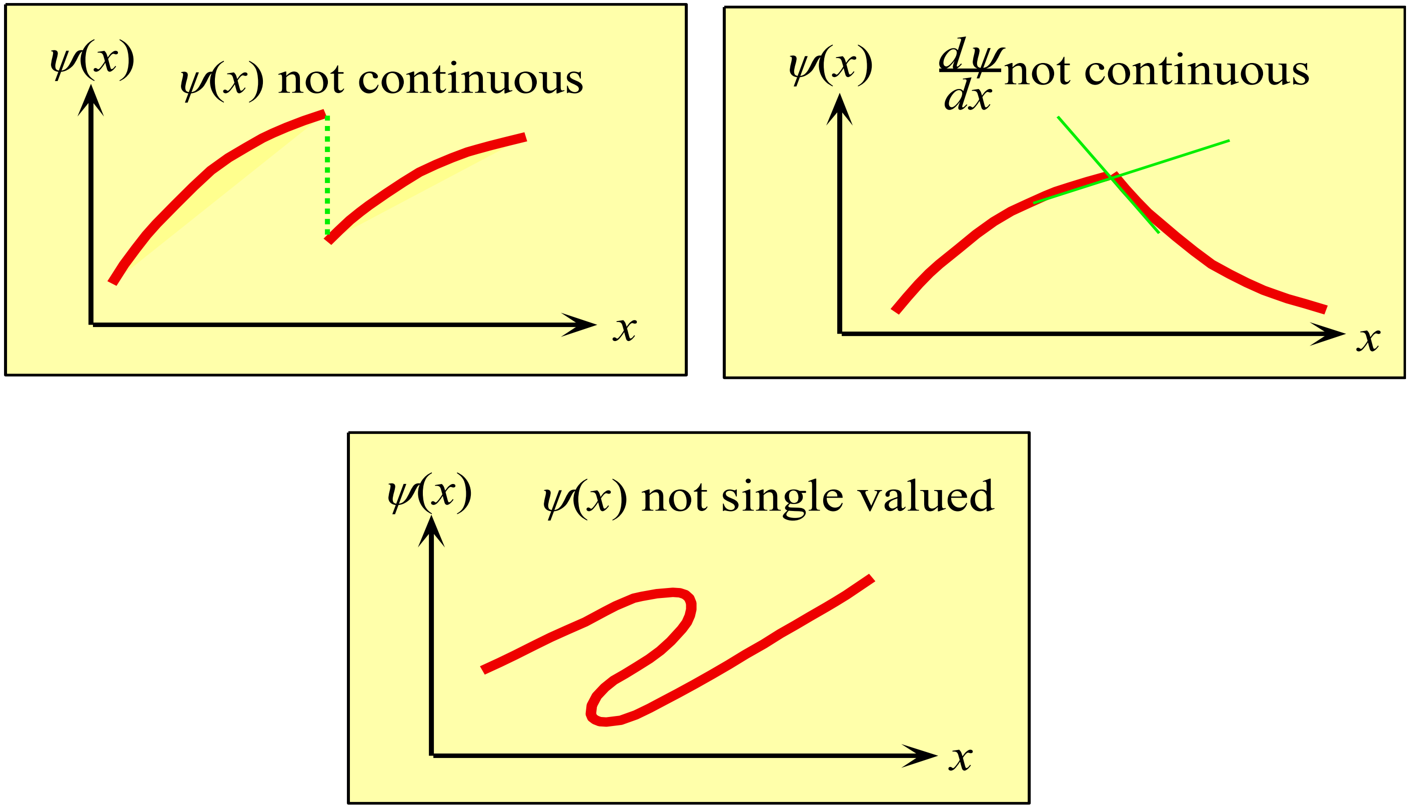
- **Equivalent of energy conservation equation in classical mechanics.**
- **Predicts the shape of the wave function.**
- **System is defined by potential energy, boundary conditions**

Stationary Wavefunction

$$\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = 1$$

Normalization Condition

Unacceptable Forms of Wavefunctions



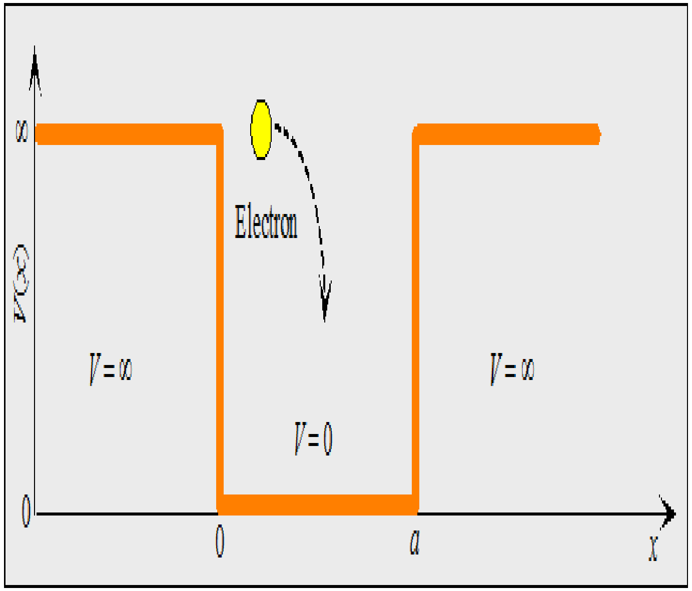
Unacceptable forms of $\psi(x)$

Fig 3.14

Applications

(1) Particle in an Infinite Potential Well: Particle in a Box (1-D):

A particle of mass m is completely trapped in a region and is moving freely between x=0 and x=L

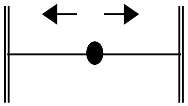


$V(x)=0, \quad \text{if } 0 \le x \le a$

- $V(x)=\infty, \text{ if } x<0 \text{ or } x>a$
- ***Boundary conditions on*** ψ:

  Ψ ***must be continuous across any boundary***

  $\Psi(0)=0=\Psi(L)$



To find the value of A:

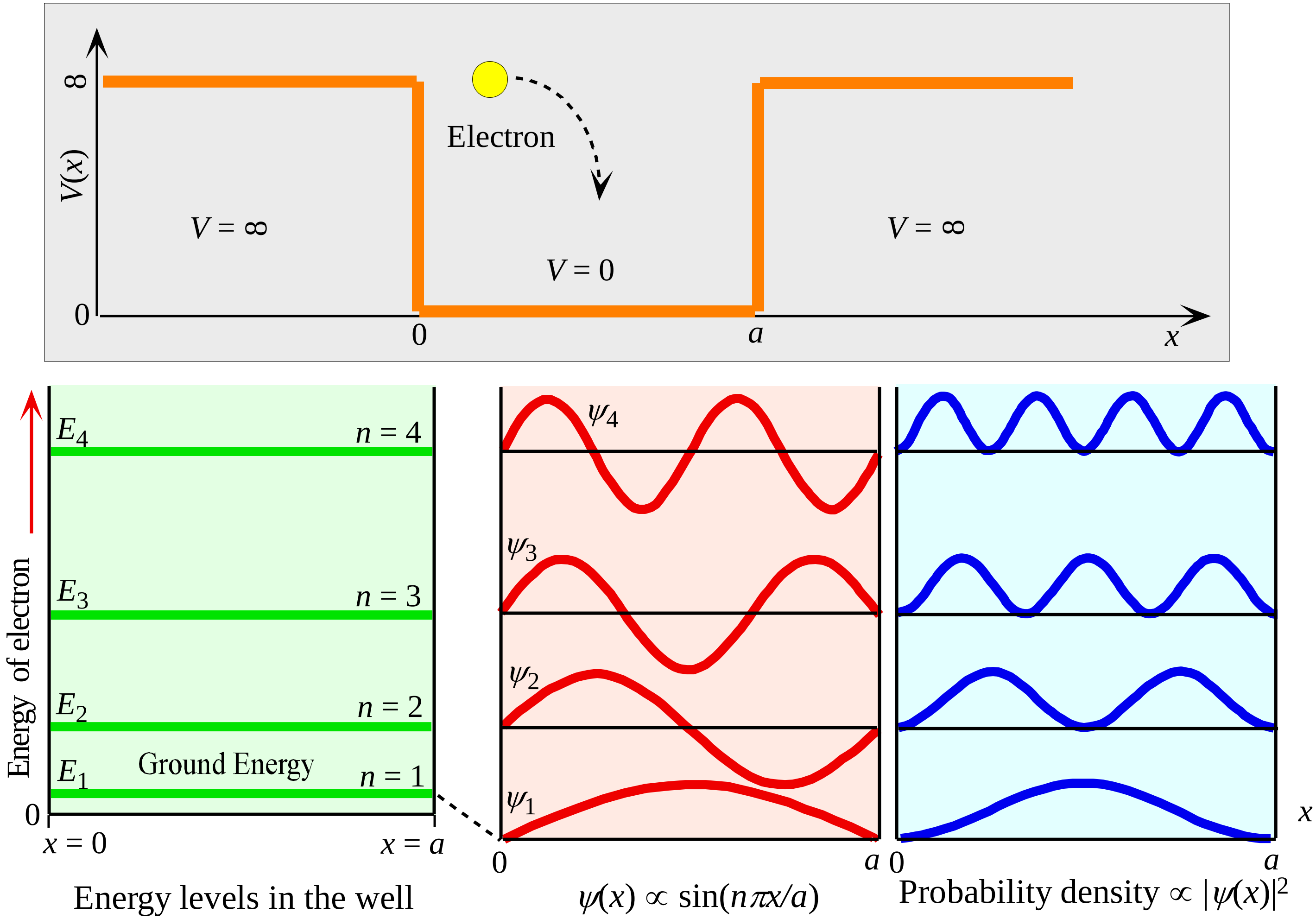
Probability to find the particle in ***0 ≤ x ≤ a*** **is** ***1:***

$$\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = \int_{0}^{a} |\psi(x)|^2 \, dx = 1$$

$$\psi_n(x) = j\sqrt{\frac{2}{a}}\sin(k_n x)$$

wave function

$$k_n = \frac{n\pi}{a}$$



Electron in a one-dimensional infinite PE well. The energy of the electron is quantized. Possible wavefunctions and the probability distributions for the electron are shown.

Fig 3.15

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