Solid State Electronics EC210
Arab Academy for Science and Technology
AAST – Cairo
Spring 2015

Lecture 9
Band Theory:
E-K Diagram, Energy Gaps and Effective Mass

Lecture Notes Prepared by:
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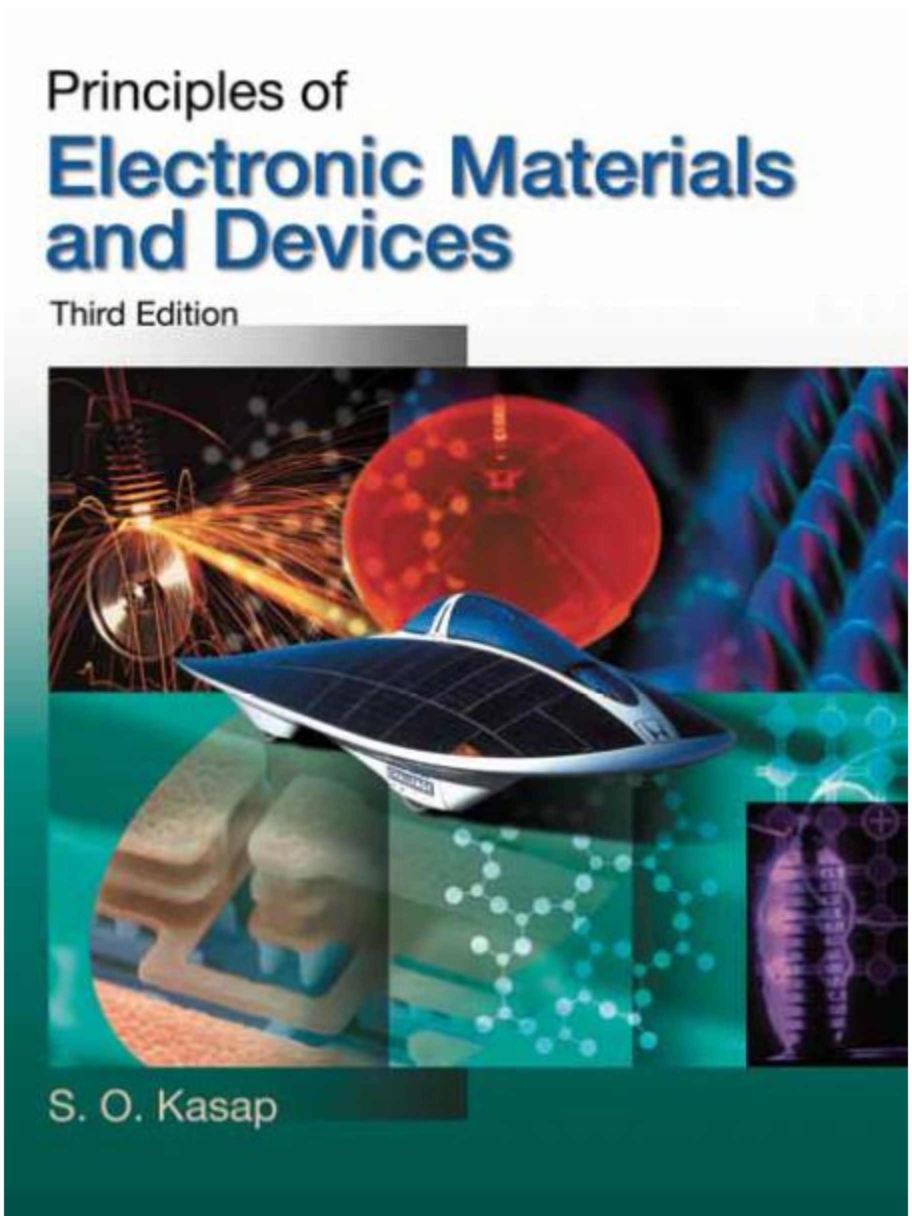
Principles of
Electronic Materials and Devices
Third Edition
S. O. Kasap

Pages

- Kasap:
  - P.355 (Kronig Penny)
  - P.303-304, p. 454-455 (Effective Mass)

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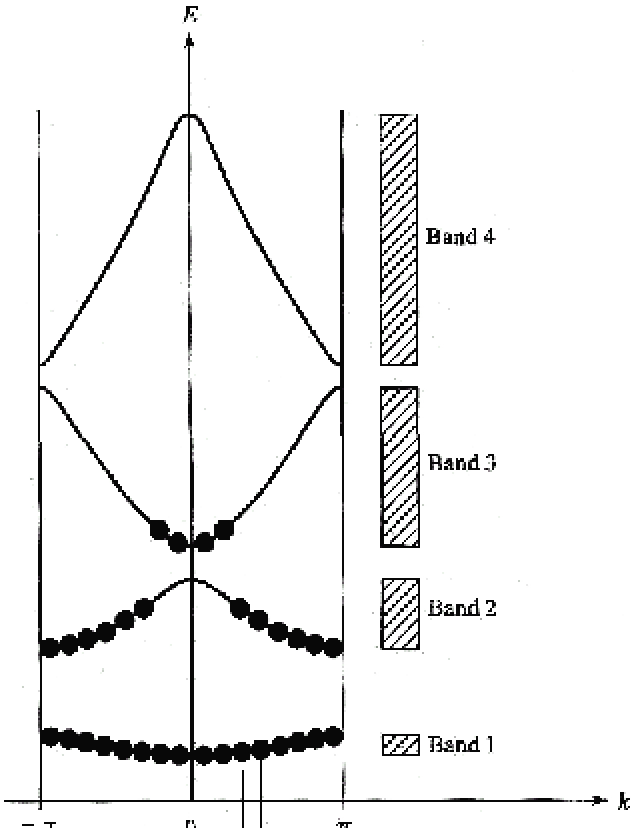
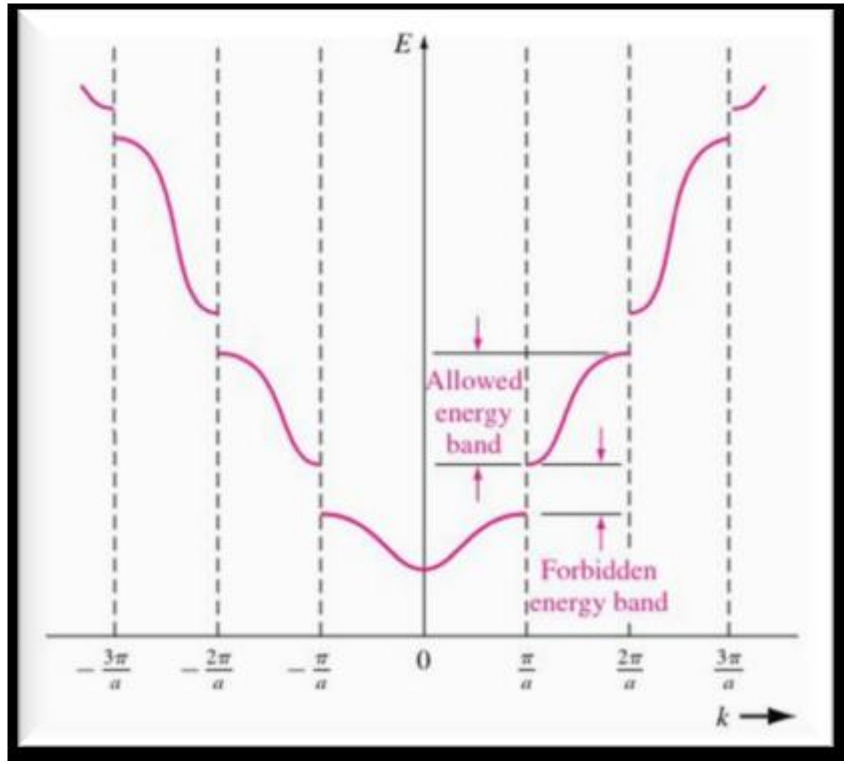
E-K Diagram usingKronig-Penney Model





Source: Dr. M. Fedawy's Lecture notes

Fig 4.52

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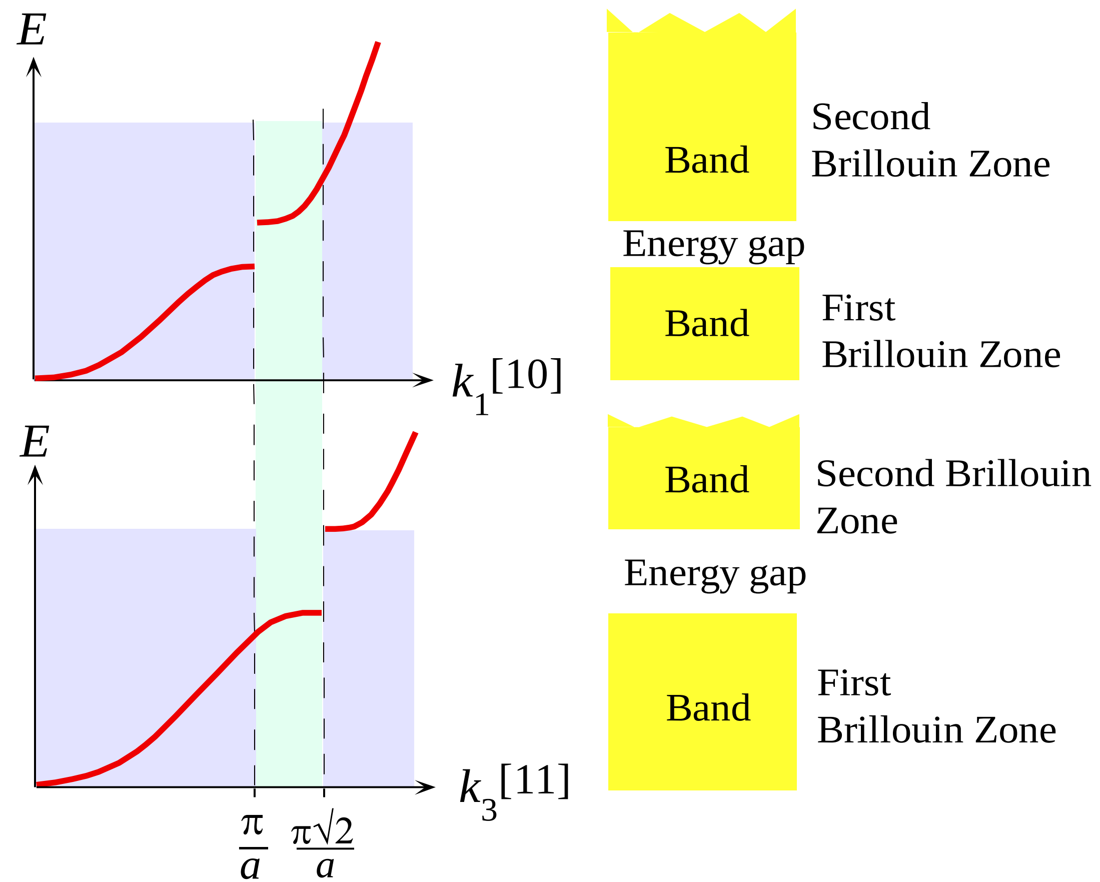
Energy Gap (Bandgaps, Eg)



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The *E*-*k* behavior for the electron along different directions in the two dimensional crystal. The energy gap along [10] is at π/a whereas it is at $\pi\sqrt{2}/a$ along [11].

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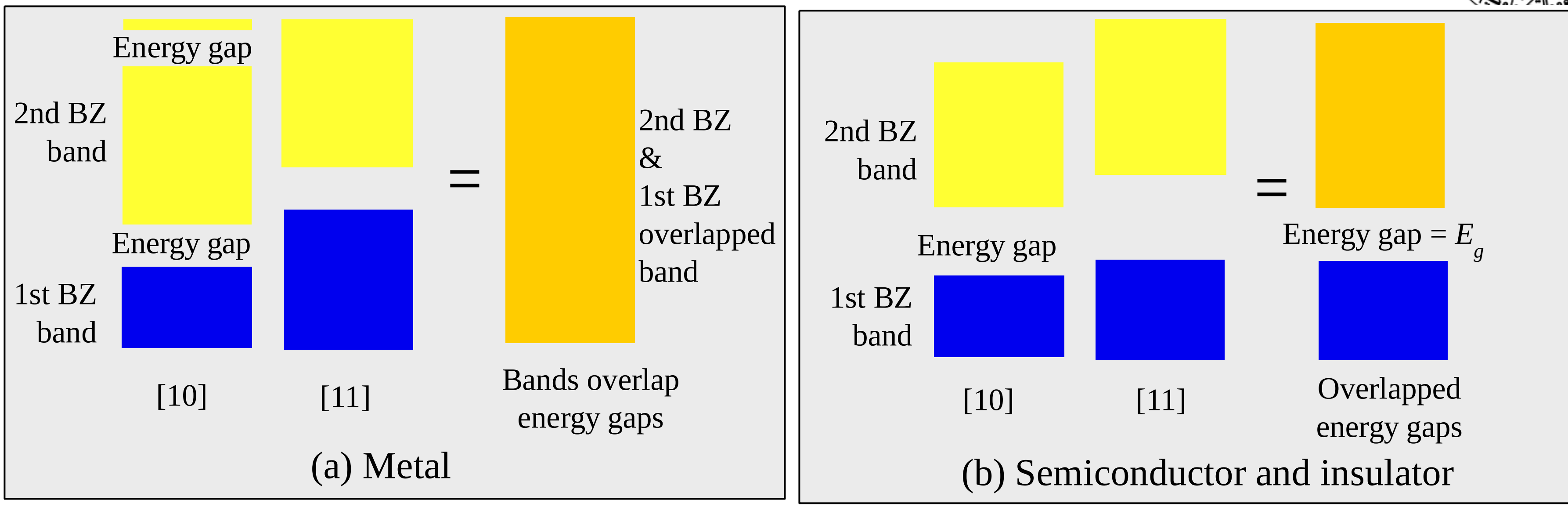
Fig 4.54

Energy Gaps (Eg)



(a) Metal: For the electron in a metal there is no apparent energy gap because the 2nd BZ (Brillouin Zone) along [10] overlaps the 1st BZ along [11]. Bands overlap the energy gaps. Thus the electron can always find any energy by changing its direction.
(b) Semiconductor or insulator: For the electron in a semiconductor there is an energy gap arising from the overlap of the energy gaps along [10] and [11] directions. The electron can never have an energy within this energy gap, E_g.

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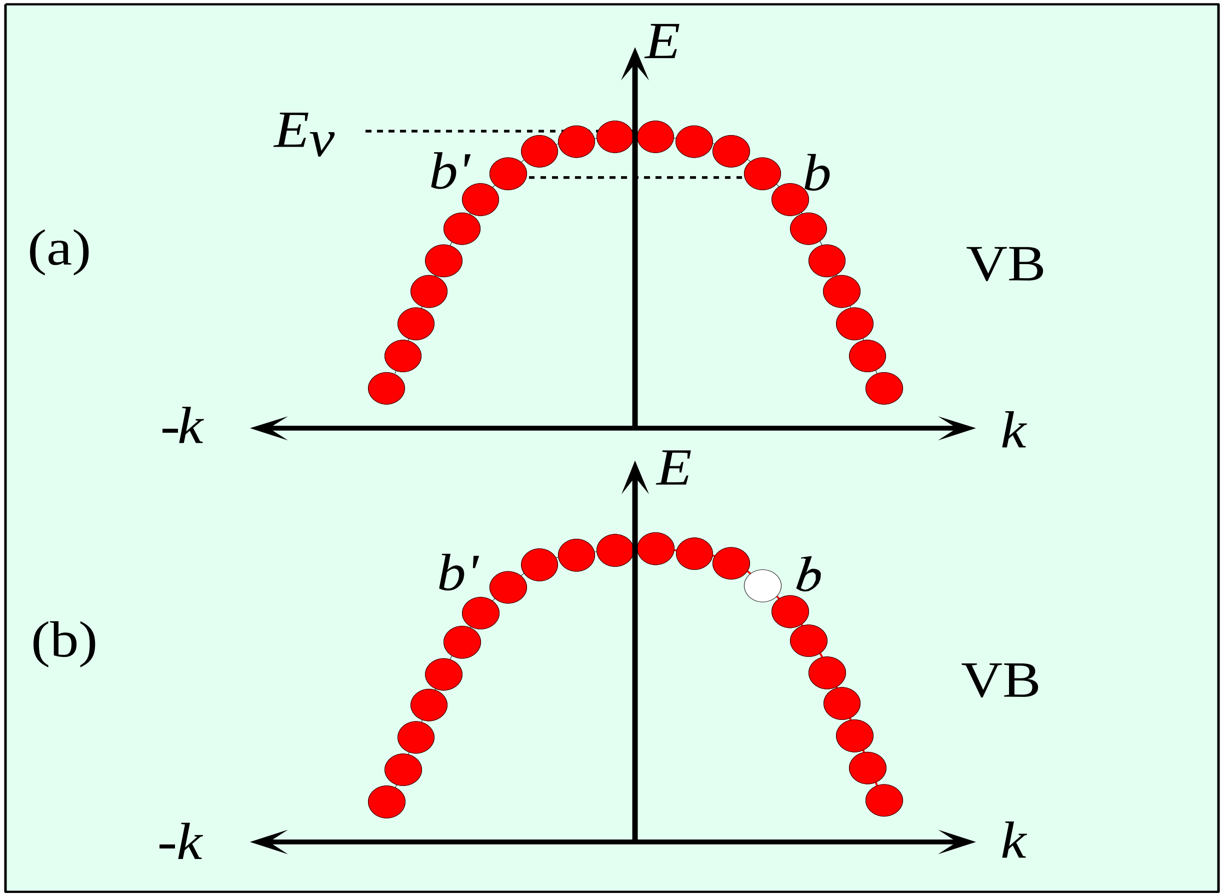
Fig 4.55

E-



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(a) In a full valence band there is no net contribution to the current. There are equal numbers of electrons (e.g. at *b* and *b'*) with opposite momenta. (b) If there is an empty state (*hole*) at *b* at the top of the band then the electron at *b'* contributes to the current.

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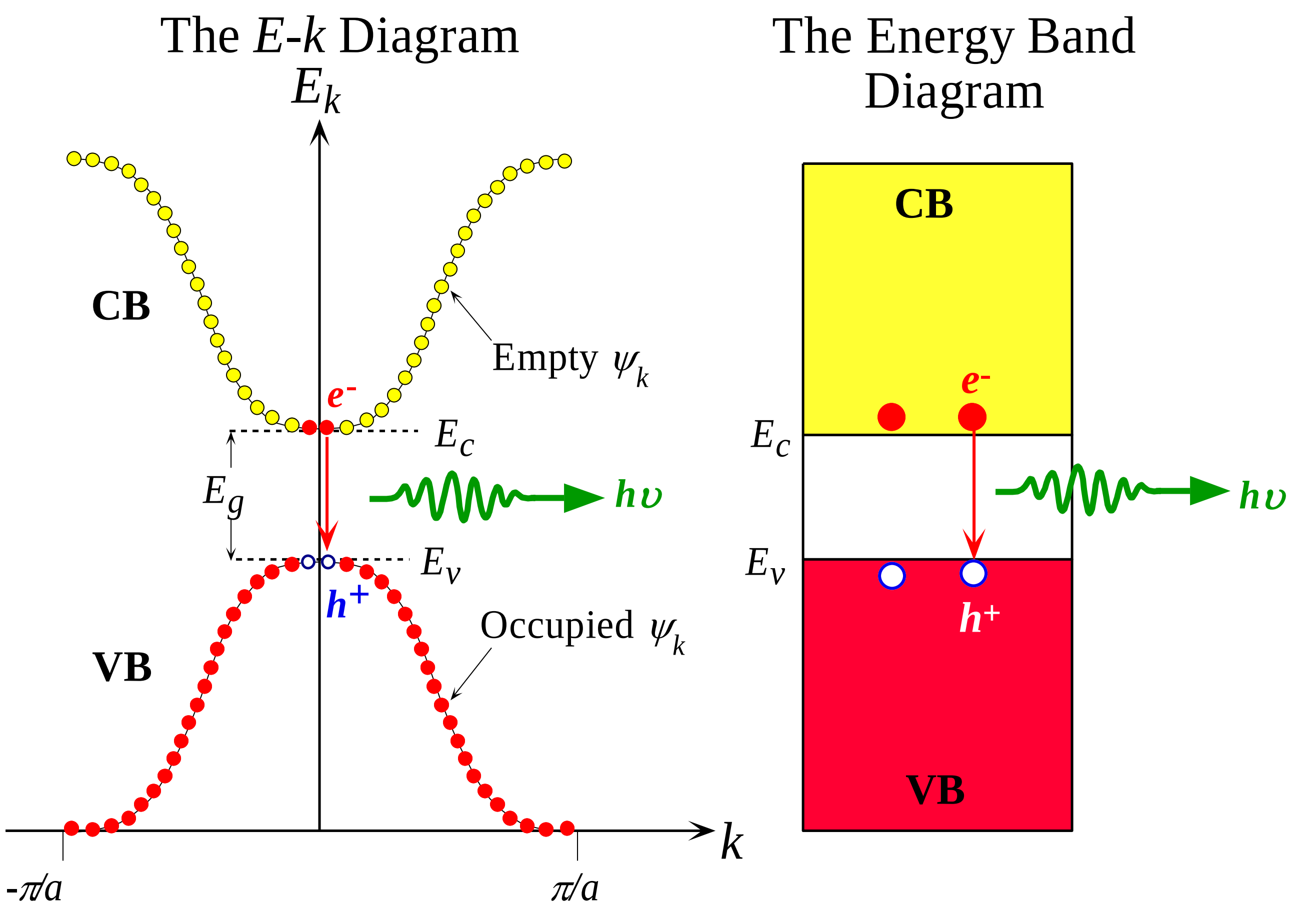
Fig 5.52



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The E-k diagram of a direct bandgap semiconductor such as GaAs. The E-k curve consists of many discrete points each point corresponding to a possible state, wavefunction y k(x), that is allowed to exist in the crystal. The points are so close that we normally draw the E-k relationship as a continuous curve. In the energy range Ev to Ec there are no points (yk(x) solutions).

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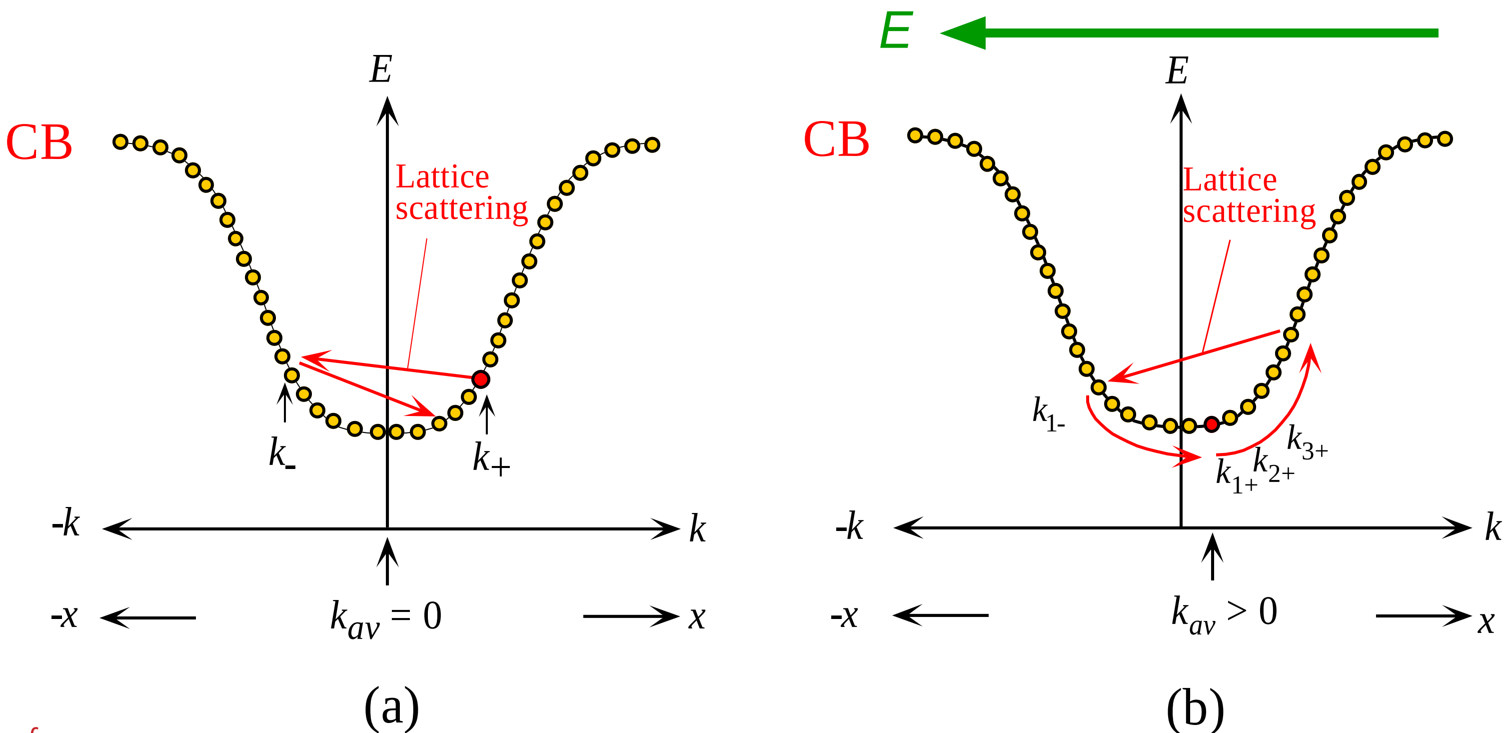
Fig 5.49



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(a) In the absence of a field, over a long time, average of all *k* values is zero, there is no net momentum in any one particular direction. (b) In the presence of a field *E* in the -*x* direction, the electron accelerates in the +*x* direction increasing its *k* value along *x* until it is scattered to a random *k* value. Over a long time, average of all *k* values is along the +*x* direction. Thus the electron drifts along +*x*.

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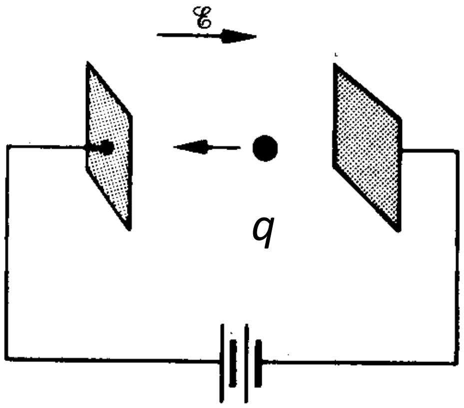
Effective Mass

In vacuum

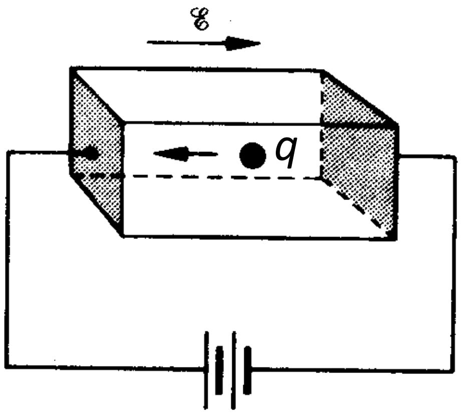


$F = q\,\varepsilon = m_o a$

where

m_o is the electron mass

In semiconductor



$F_{ext} = (-q)\mathbf{E}$

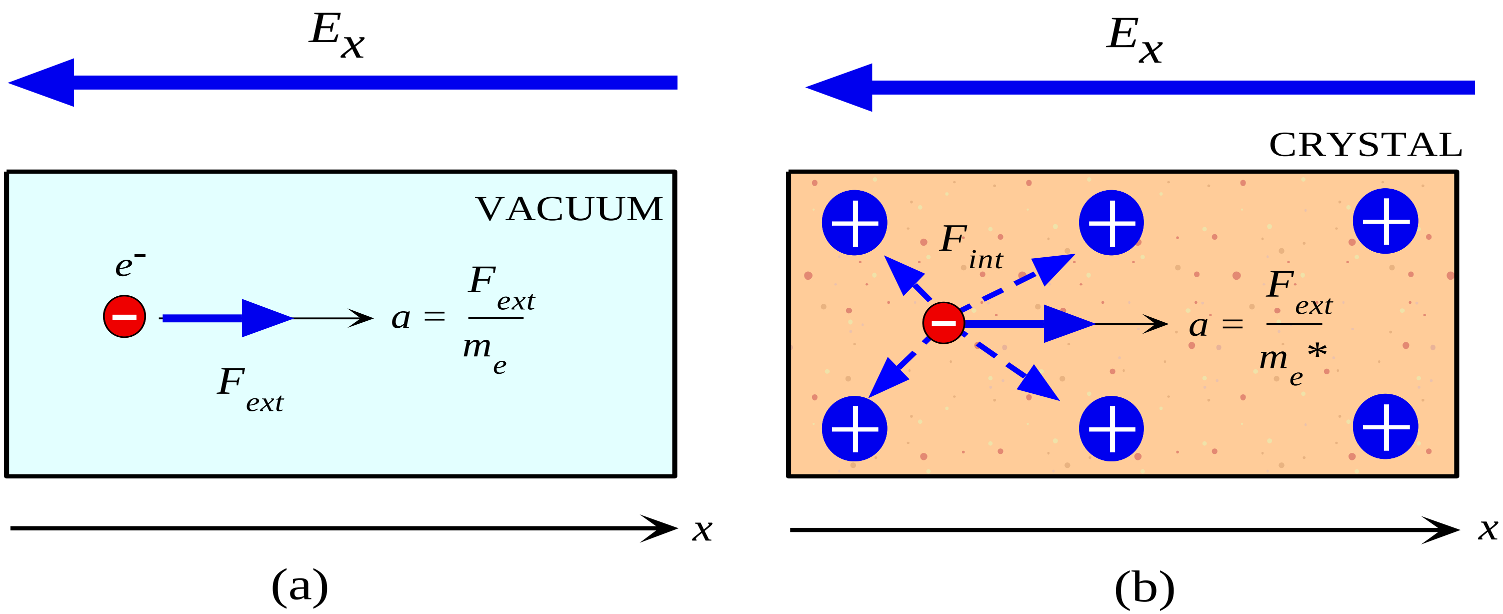
$F_{ext} + F_{int} = m_o a$

$F_{ext} = m_n^* a$

where

m_n^* is the electron effective mass

Effective Mass



(a) An external force F_{ext} applied to an electron in vacuum results in an acceleration $a_{vac} = F_{ext} \ / \ m_e$. (b) An external force F_{ext} applied to an electron in a crystal results in an acceleration $a_{cryst} = F_{ext} \ / \ m_e^*$. (E_x is the electric field.)

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Fig 4.19

Example of Effective Mass

Table 4.2 Effective mass m_e^* of electrons in some metals

Metal	Ag	Au	Bi	Cu	K	Li	Na	Ni	Pt	Zn
$\frac{m_e^*}{m_e}$	0.99	1.10	0.047	1.01	1.12	1.28	1.2	28	13	0.85

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Effective Mass

Group Velocity defined as the velocity of the wavefunction of the electrons (analogous to speed of sinusoidal wave):

$$v_g = \frac{dx}{dt} = \frac{d\omega}{dk}, \qquad dx = v_g\, dt$$

$$\omega = \frac{E}{\hbar} \qquad \rightarrow \; vg = \frac{1}{\hbar}\frac{dE}{dk} \qquad \rightarrow dE = vg\,_{\hbar}\, dk$$

$$dE = F_{ext}\, dx = F_{ext}\, vg\; dt \rightarrow F_{ext} = \frac{1}{v_g}\frac{dE}{dt} \rightarrow F_{ext} = \hbar\frac{dk}{dt}$$

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Effective Mass (2)

Acceleration:

$$a = \frac{dv_g}{dt} = \frac{d}{dt}\left[\frac{1}{\hbar}\frac{dE}{dk}\right] = \frac{1}{\hbar}\frac{d}{dk}\left[\frac{dE}{dt}\right] = \frac{1}{\hbar}\frac{d}{dk}\left[\frac{dE}{dk}\frac{dk}{dt}\right]$$

$$a = \frac{1}{\hbar}\frac{d^2E}{dk^2}\frac{dk}{dt} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}\hbar\frac{dk}{dt} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}F_{ext}$$

Using $F_{ext} = m^* a$

$$m^* = \left[\frac{1}{\hbar^2}\frac{d^2E}{dk^2}\right]^{-1} = \hbar^2\left[\frac{d^2E}{dk^2}\right]^{-1}$$

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