Solid State Electronics EC210
Arab Academy for Science and Technology
AAST – Cairo
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Lecture 11:

Fermi Level Calculations

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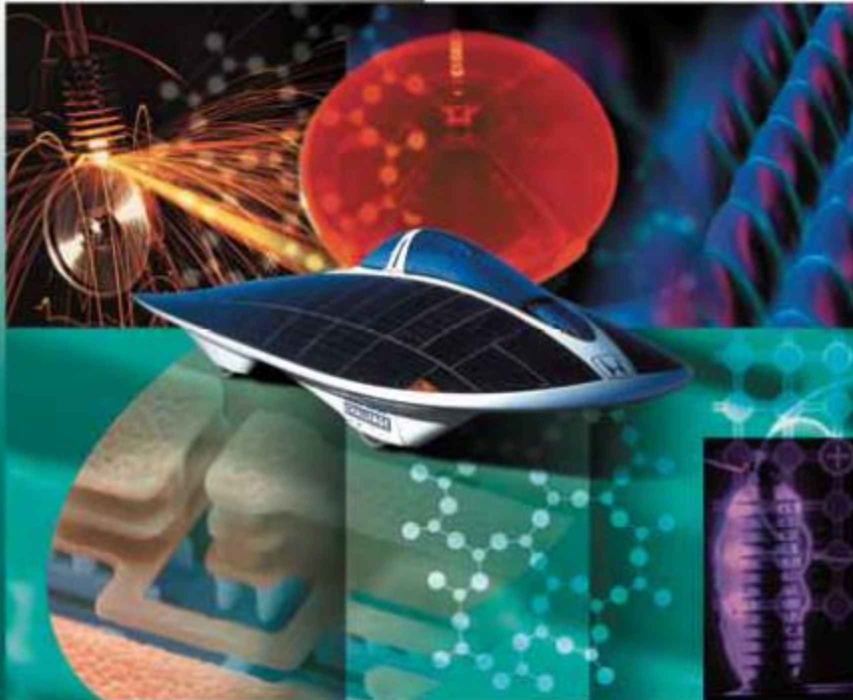

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Pages

- Kasap: p380-401

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Free Carrier Concentration

- Using the density of states and the Fermi function we may calculate the CONCENTRATION of carriers present in the conduction and valence bands at any given temperature

  * We do this by INTEGRATING the density of OCCUPIED states over the range of energies relevant to either band

- $$N.\ of\ \ Electrons\ \ in\ \ Conduction\ \ Band\ \ per\ unit\ volume:$$

$$n = \int_{E_c}^{Ectop} N_{sc}(E) f(E) dE$$

$$N.\ of\ \ Holes\ \ in\ \ Valence\ \ Band\ \ per\ unit\ volume:$$

$$p = \int_{Evbottom}^{E_v} N_{sv}(E)\left[1 - f(E)\right] dE$$

⇒ Note here how we write the probability for occupation of HOLE states as 1-*f*(*E*)

⇒ By substituting the expressions for the electron and hole densities of states into these integrals we may calculate *n* and *p* for any given *T* and E_F

Carriers in Conduction and Valence Bands: Intrinsic Case ($n=p=n_i$)

$E_c+\chi$

CB

E_c

E_F

E_v

VB

0

(a)

E

(b)

E

$[1-f(E)]$

For electrons

E_F

For holes

$f(E)$

(c)

E

Area = n

E_c

$n_E(E)$

E_v

$p_E(E)$

Area = p

$n_E(E)$ or $p_E(E)$

(d)

(a) Energy band diagram. (b) Density of states (number of states per unit energy per unit volume). (c) Fermi-Dirac probability function (probability of occupancy of a state). (d) The product of $g(E)$ and $f(E)$ is the energy density of electrons in the CB (number of electrons per unit energy per unit volume). The area under $n_E(E)$ vs. E is the electron concentration in the conduction band.

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Carriers in Conduction and Valence Bands: n-Type Case (n≈ N_D^+)

n Type Semiconductors

E_c
E_F
E_v

ENERGY BAND STRUCTURE

$N_{sc}(E)$
$N_{sv}(E)$

ELECTRON AND HOLE DENSITIES OF STATES

E
$F(E)$

ELECTRON AND HOLE FERMI-DIRAC FUNCTIONS

E_c
E_F
E_v

ELECTRON AND HOLE STATE OCCUPATION

n > p

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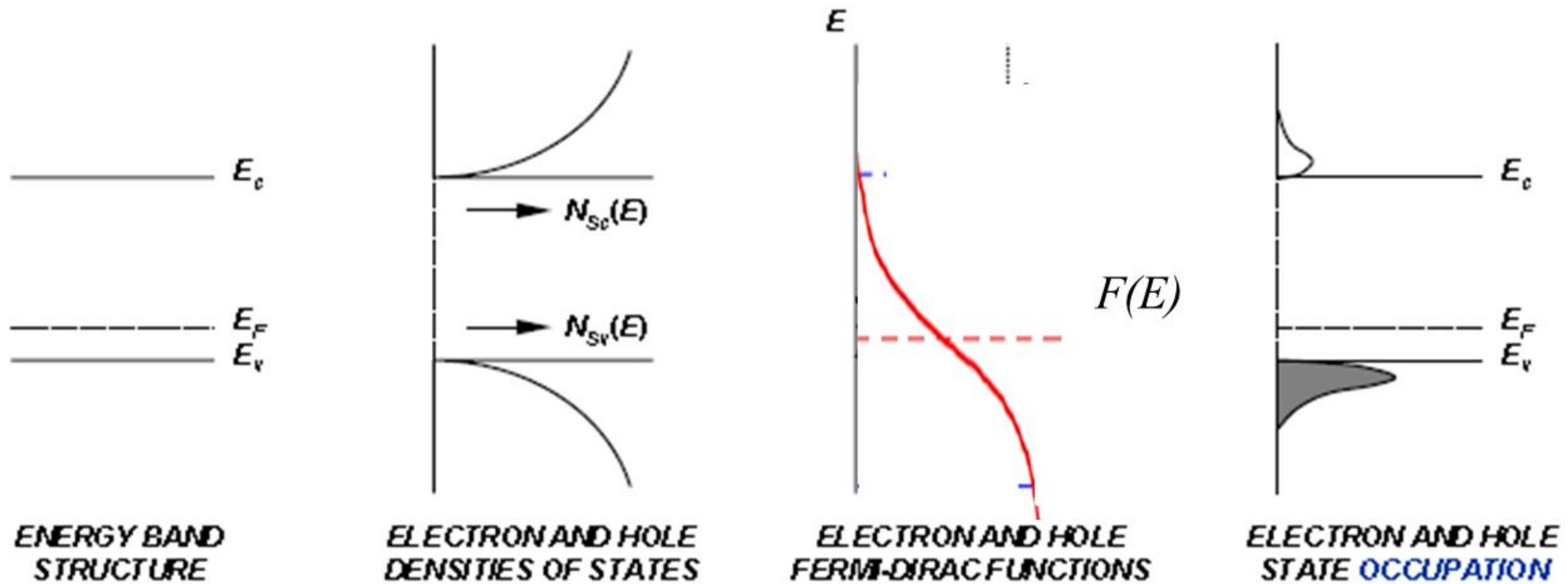
Carriers in Conduction and Valence Bands: p-Type Case (p≈ N_A^-)

p Type Semiconductors



p > n

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Solution of Integral for n,p

$$n = \int_{Ec}^{\infty} g_{3D}(E)\ f_{FD}\ (E)\ dE$$

$$p = \int_{EV}^{E_V} g_{3D}(E)\ [1 - f_{FD}(E)]dE$$

$$g_{3D}(E) = (\pi/2)\ [\frac{8m^*}{h^2}]^{\frac{3}{2}}\ (E - E_c)^{1/2}$$

Where E has been replaced by $E - E_c$, or $E - E_V$ for holes

In Conduction or Valence Bands:

$$f_{FD} \sim f_{MB} = \exp\left[-\frac{E - E_F}{k_B T}\right]$$

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Solution of Integral for n, p

$$I_1 = \int_0^{\infty} x^{1/2}\, e^{-x} dx = \sqrt{\pi}/2$$

- Use $x = \frac{E - E_C}{K_B T}$:

$$\int_{E-Ec=0}^{\infty} \sqrt{\frac{E-E_C}{K_B T}}\, e^{-\left(\frac{E-E_C}{K_B T}\right)} d\left(\frac{E-E_C}{K_B T}\right) = \frac{\sqrt{\pi}}{2}$$

Exercise: Derive the Effective Density of States for electrons in CB and holes in VB

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Free Carrier Concentration

$$n = N_C \exp\left[-\frac{(E_C - E_F)}{kT}\right] \qquad N_C = 2\left(\frac{m_e^* kT}{2\pi\hbar^2}\right)^{3/2}$$ Eff. DOS CB

$$p = N_V \exp\left[-\frac{(E_F - E_V)}{kT}\right] \qquad N_V = 2\left(\frac{m_h^* kT}{2\pi\hbar^2}\right)^{3/2}$$ Eff. DOS VB

We can now calculate the np product by multiplying the above equations.

$$np = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

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Intrinsic Concentration (n_i) and Mass Action Law

$$np = N_C N_V \exp\left({}^{-E_g}\!/\!{}_{kT} \right)$$

Mass action law:

$$np = n_i^2$$

- **Both N_c and N_v have $T^{3/2}$ temperature dependence (T is in K)**
  - **$N_C(T_1)/N_C(300K) = (T_1/300K)^{3/2}$**
  - **$N_V(T_1)/N_V(300K) = (T_1/300K)^{3/2}$**

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Temperature Dependence

$T < T_s$ $T_s < T < T_i$ $T > T_i$

CB

E_F

E_g

VB

As^+ As As As

As^+ As^+ As^+ As^+

As^+ As^+ As^+ As^+

(a) $T = T_1$ (b) $T = T_2$ (c) $T = T_3$

(a) Below T_S, the electron concentration is controlled by the ionization of the donors. (b) Between T_S and T_i, the electron concentration is equal to the concentration of donors since they would all have ionized. (c) At high temperatures, thermally generated electrons from the VB exceed the number of electrons from ionized donors and the semiconductor behaves as if intrinsic.

Fig

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Temperature Dependence of Free Carriers

ln(n)

INTRINSIC

IONIZATION

EXTRINSIC

$\ln(N_d)$

$n_i(T)$

1/T

The temperature dependence of the electron concentration in an n-type semiconductor.

Fig 5.15

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600°C 400°C 200°C 27°C 0°C

Intrinsic Concentration (cm^{-3})

10^{18}
10^{15}
10^{12}
10^{9}
10^{6}
10^{3}

2.4×10^{13} cm^{-3}

Ge

1.45×10^{10} cm^{-3}

Si

2.1×10^{6} cm^{-3}

GaAs

1 1.5 2 2.5 3 3.5 4

1000/T (1/K)

The temperature dependence of the intrinsic concentration.

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Fig 5.16

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Example on Determination of n_i and E_C-E_F in n-Type Si

- Assume for Si at 300K:

N_D=1x10^{16}cm^{-3}, N_C=3.2x10^{19}cm^{-3}, N_V=1.8x10^{19}cm^{-3}, E_g=1.12eV, kT=25.85meV

$$n \approx N_D = N_c \exp(-(E_C - EF)/kT)$$

$$\ln\left(\frac{N_D}{N_c}\right) = -\frac{(E_C - EF)}{kT}$$

$E_c - EF = kT \ln(N_c/N_D)$ =0.0258 ln(3.2x1019/1016) eV

Exercise: Repeat for T=200K and 400K

$n_i^2 = N_c N_v \exp(-E_g/kT)$= (3.2x10^{19})(1.8x10^{19})exp(-1.12/0.0258) cm^{-6}

At T=300K in Si: n_i = 0.93x10^{10}cm^{-3}

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Example on Temperature Dependence of n_i in n-Type Si

Si, N_D=1x10^{16}cm^{-3}, N_c (300K)=3.2x10^{19}cm^{-3}, N_V(300K)=1.8x10^{19}cm^{-3}, E_g=1.12eV, k_B=8.617x10^{-5} eV K^{-1}

Find N_C, N_V, n_i, p, n, for T=200K, 300K, 400K:

$$\frac{N_C(T_1)}{N_C(300K)} = \left(\frac{T_1}{300K}\right)^{3/2} \qquad \frac{N_V(T_1)}{N_V(300K)} = \left(\frac{T_1}{300K}\right)^{3/2}$$

$$n_i^2 = N_c N_v \exp(-E_g/kT)$$

n≈N_D^+ =N_D (200-400K is considered extrinsic range: All N_D donor atoms are ionized, $N_D >> n_i$)

$$n\,p = ni^2 \rightarrow p = ni^2/n$$

At T=200K in Si: $kT = ?\; n_i = ?$

At T=400K in Si: $kT = ?\; ni = ?$

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Example on Temperature Dependence of n_i in n-Type Si

At T=200K in Si:

$kT = (8.617\text{x}10^{-5})(200\text{K}) = 0.0172\text{eV}$

$N_C(200\text{K}) = N_C(300\text{K})\left(\frac{200}{300}\right)^{3/2} = 3.2\text{x}10^{19}(0.666)^{1.5} = 2.13\text{x}10^{19}\text{cm}^{-3}$

$N_V(200\text{K}) = N_V(300\text{K})\left(\frac{200}{300}\right)^{3/2} = 1.8\text{x}10^{19}(0.666)^{1.5} = 1.2\text{x}10^{19}\text{cm}^{-3}$

$n_i^2(200K) = (2.13\text{x}1019)(1.2\text{x}1019)\exp\left(-\frac{1.12}{0.0172}\right) = 1.34\text{x}10^{10}\text{cm}^{-6}$

$n_i = 1.16\text{x}10^{5}\ \text{cm}^{-3}$

$p = ni^2/n = ni^2/ND = (1.34\text{x}10^{10})/(10^{16}) = 1.34\text{x}10^{-6}\ \text{cm}^{-3}$

At T=400K in Si: $kT = ?\ ni = ?$

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