AAST-Cairo

EC210

Solid State Electronics

Fall 2016

Lecture 5

Solution of Schrodinger Eq.
Heisenberg Uncertainty Principle

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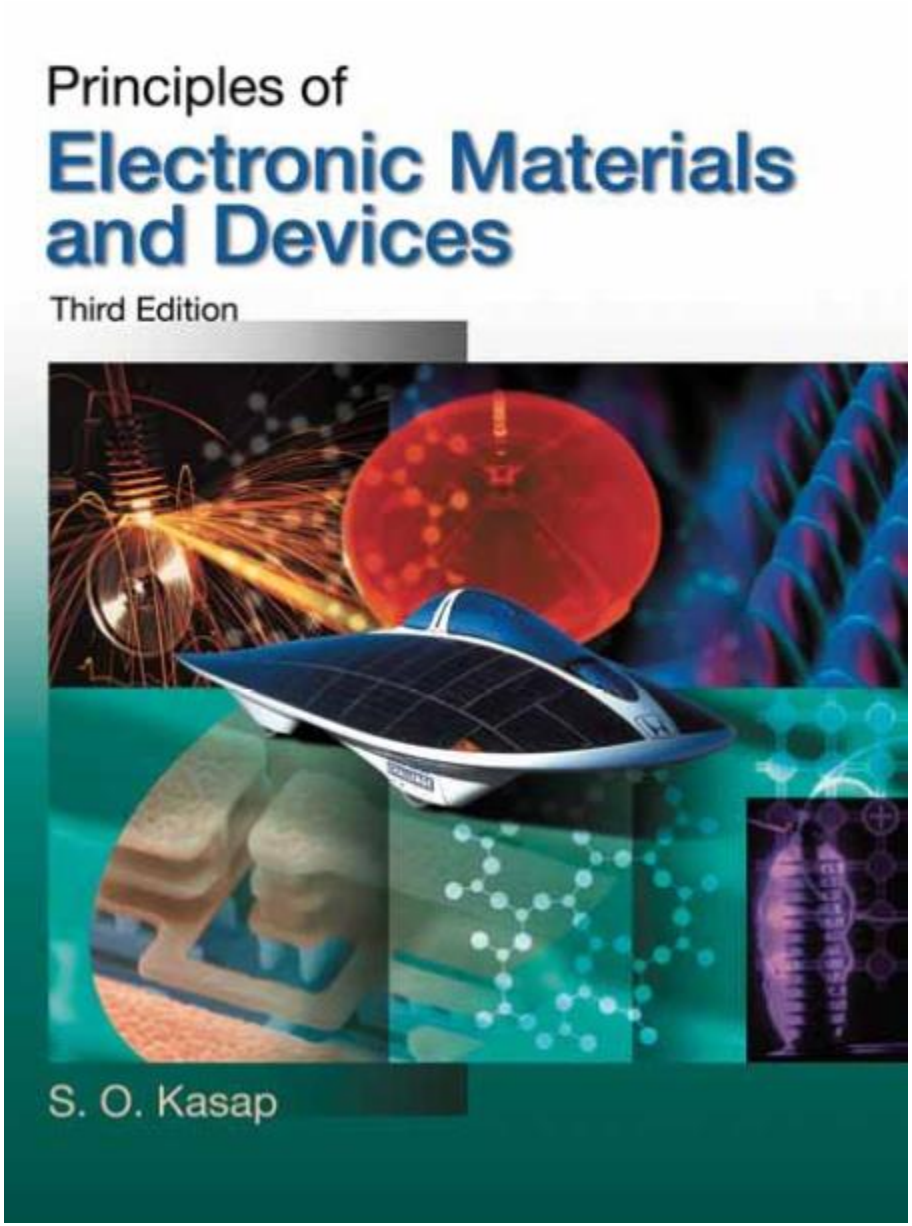

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Kasap:

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Schrodinger's Eqn. (SE) as 2nd Order Differential Equation

Rewriting Schrodinger eqn.:

$$\frac{d^2\psi(x)}{dx^2} = -\frac{2m}{\hbar^2}[E - U(x)]\psi(x)$$

$$\psi''(x) = -k^2\,\psi(x)$$

This is a 2nd order DE , with solution:

$$\psi(x)=Ae^{jkx} + Be^{-jkx}$$

Need to apply boundary conditions to find A, B

Note: $E - U(x) = KE = p^2/2m = \hbar^2 k^2/2m$

$$\rightarrow k^2 = \frac{2m}{\hbar^2}[E - U(x)]$$

General Solution of Schrodinger 2nd Order Differential Equation

The solution of this differential equation is in the form:

$$\Psi(x) = A\, e^{jkx} \quad \text{or} \quad B\, e^{-jkx}$$

For time-dependent wavefunction:

$$\Psi(x,t) = \Psi(x) e^{-j\omega t}$$

$$\Psi(x,t) = A\, e^{j(kx-\omega t)} \quad \textit{or} \quad B\, e^{-j(kx+\omega t)}$$

Applications of Schrödinger equation
Free Particle

A free particle whose mass is m and total energy is E:

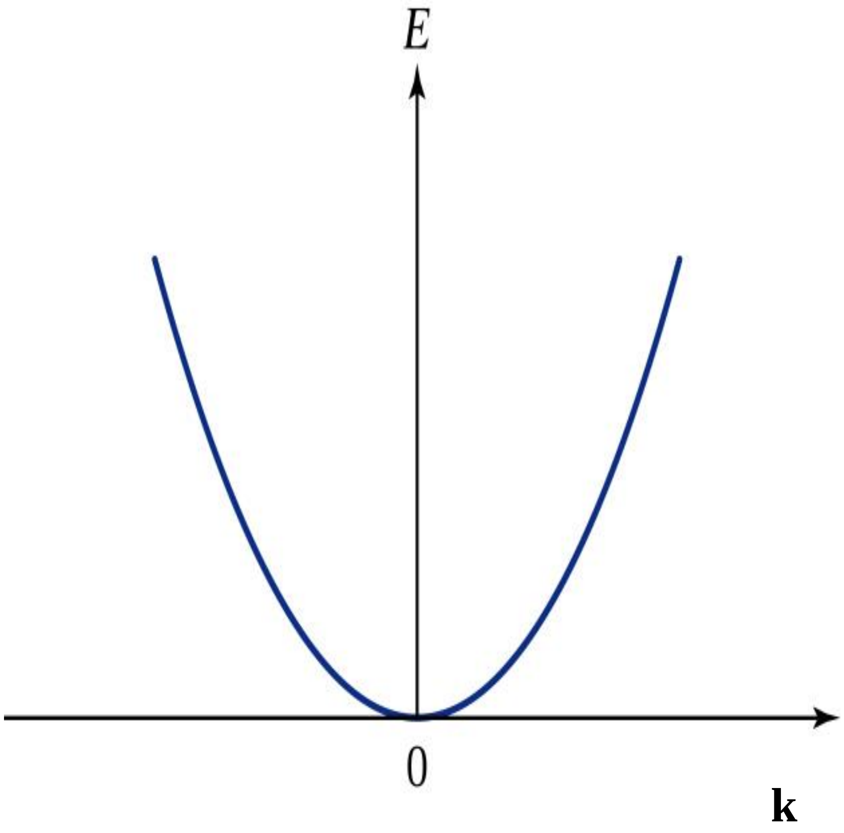
Let U(x)=0 in Schrodinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+U(x)\psi(x)=E\psi(x)$$

$$\frac{d^2\psi(x)}{dx^2}=-\frac{2mE}{\hbar^2}\psi(x) \qquad k^2=\frac{2mE}{\hbar^2}$$

$$\frac{d^2\psi(x)}{dx^2}=-k^2\psi(x)$$

Free Particle (No Potential Energy)



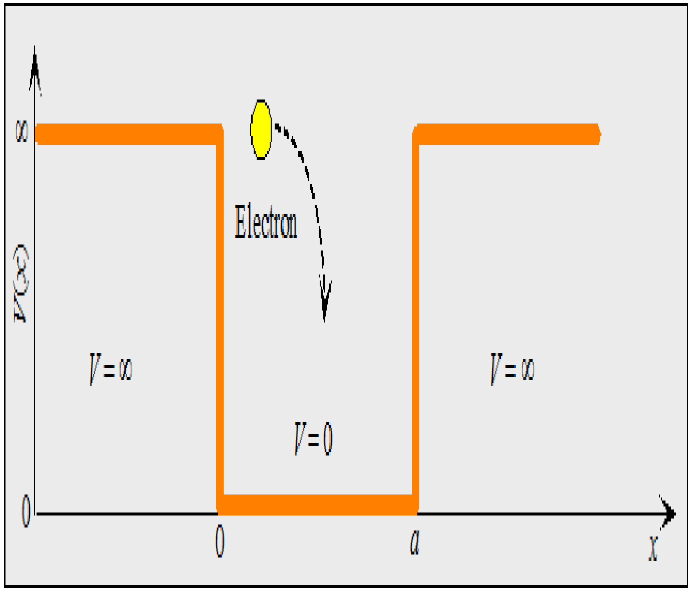
$$E = \frac{\hbar^2 k^2}{2m}$$

Free Particle

Applications

(1) Particle in an Infinite Potential Well: Particle in a Box (1-D):

A particle of mass m is completely trapped in a region and is moving freely between x=0 and x=L

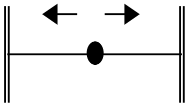


$V(x)=0, \quad \text{if } 0 \leq x \leq a$

- $V(x)=\infty, \text{ if } x<0 \text{ or } x>a$
- ***Boundary conditions on ψ:***

Ψ must be continuous across any boundary

$\Psi(0)=0=\Psi(a)$



Particle in a Box Solution

$$\psi(x) = Ae^{jkx} + Be^{-jkx}$$

$$at\ x = 0:\ \psi(0^-) = \psi(0^+) = 0 \rightarrow 0 = A + B \rightarrow B = -A$$

$$\psi(x) = A(e^{jkx} - e^{-jkx}) = 2A\,j\,\sin(kx)$$

$$at\ x = a:\ \psi(a^-) = \psi(a^+) = 0 \rightarrow \sin(ka) = 0$$

$$\rightarrow ka = \pm n\pi \rightarrow kn = \pm\frac{n\pi}{a}$$

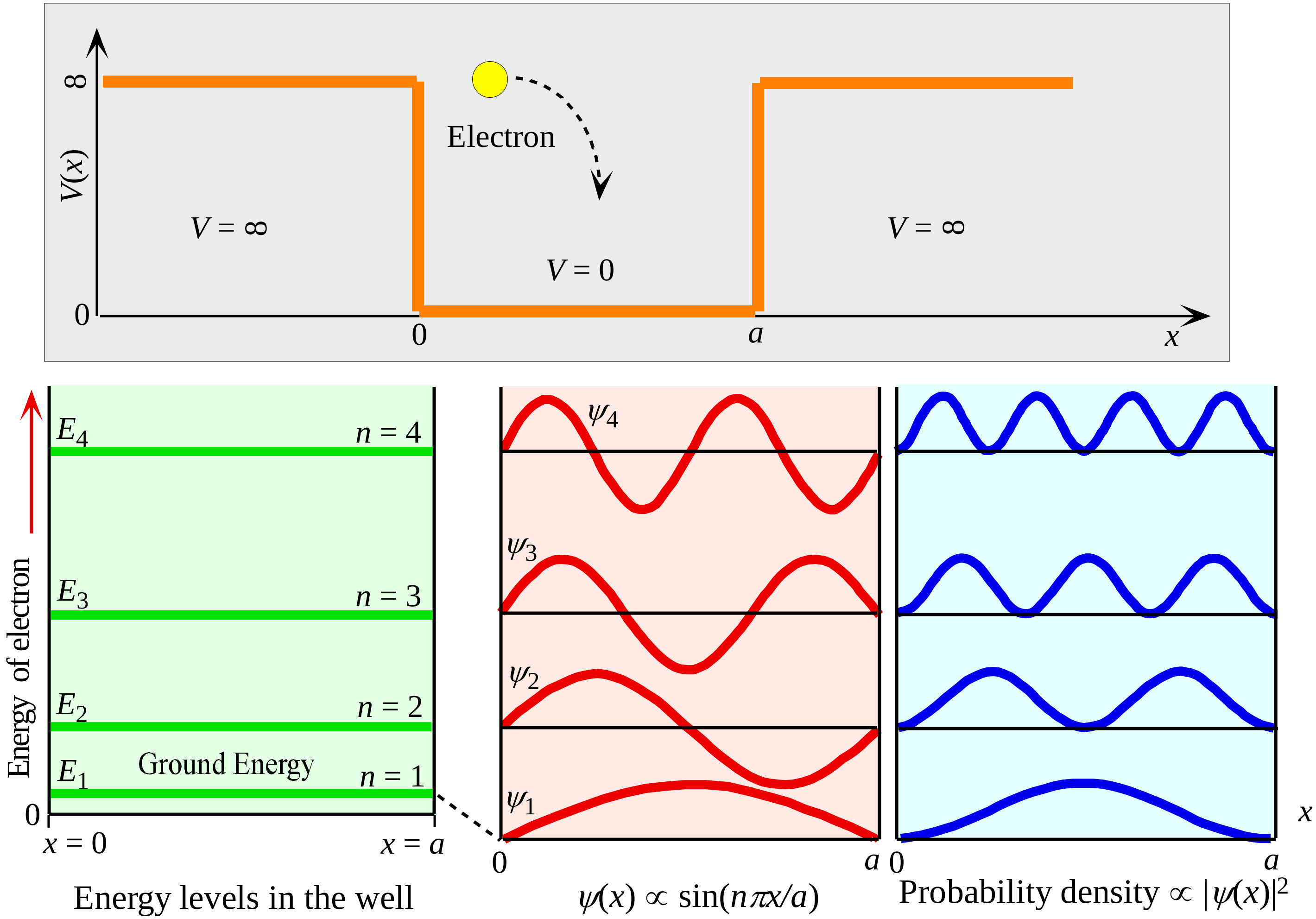
To find the value of A:

Probability to find the particle in $0 \le x \le a$ **is** *1:*

$$\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = \int_{0}^{a} |\psi(x)|^2 \, dx = 1$$

$$\psi_n(x) = j\sqrt{\frac{2}{a}}\sin(k_n x)$$

$$k_n = \frac{n\pi}{a}$$

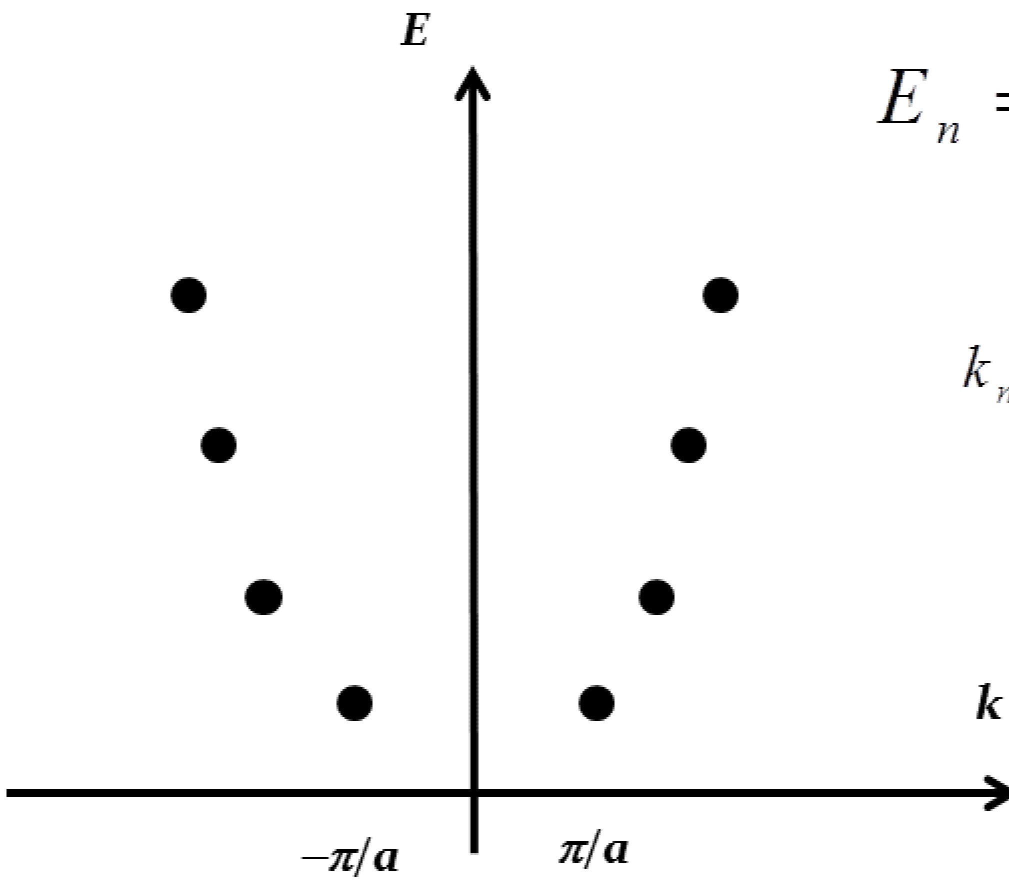


Electron in a one-dimensional infinite PE well. The energy of the electron is quantized. Possible wavefunctions and the probability distributions for the electron are shown.

Fig 3.15

Particle in an Infinite Potential Well: Quantized Energy Levels

Particle in a potential well



$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

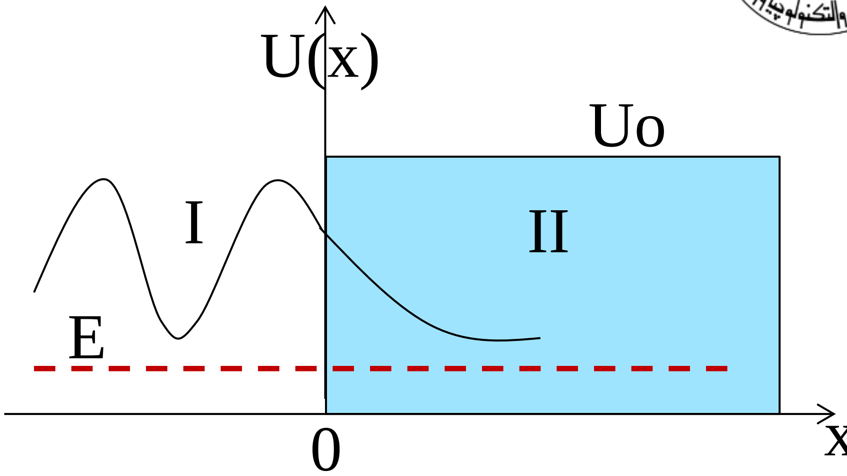
$$k_n = \frac{n\pi}{a}$$

Step Barrier (E<Uo)

Region I:

Free particle: $k^2 = \frac{2mE}{\hbar^2}$

$$\Psi_{\text{I}}(x) = Ae^{jkx} + Be^{-jkx}$$



Region II:

$$\alpha^2 = 2m(U_0 - E)/\hbar^2$$

$$\Psi_{\text{II}}(x) = Ce^{-\alpha x} + 0$$

Step Barrier (E>Uo)

Region I:

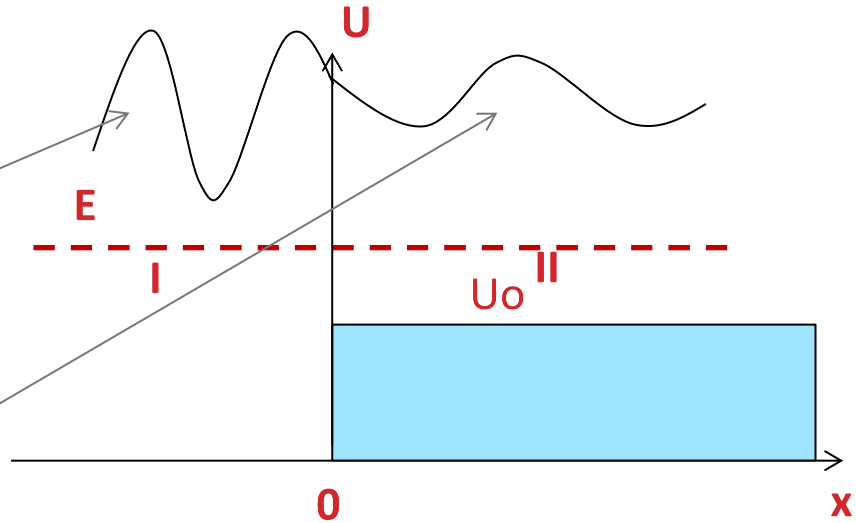
Free particle: $k^2 = \frac{2mE}{\hbar^2}$

$$\Psi_{\mathrm{I}}(x) = Ae^{jkx} + Be^{-jkx}$$

Region II:

$$k_{II}{}^2 = -\alpha^2 = 2m(E - Uo)/\hbar^2$$

$$\Psi_{\mathrm{II}}(x) = Ce^{jk_{II}x} + De^{-jk_{II}x}$$



3.4 Heisenberg's Uncertainty Principle

$$\Delta x \cdot \Delta p_x \geq \hbar$$

We can not exactly and simultaneously know both the position and momentum of particle along a given coordinate

$$\Delta t \cdot \Delta E \geq \hbar$$