



AAST-Cairo  
EC210  
Solid State Electronics  
Fall 2016

Lecture 4  
Introduction to Wavefunction, Schrodinger  
Eqn.

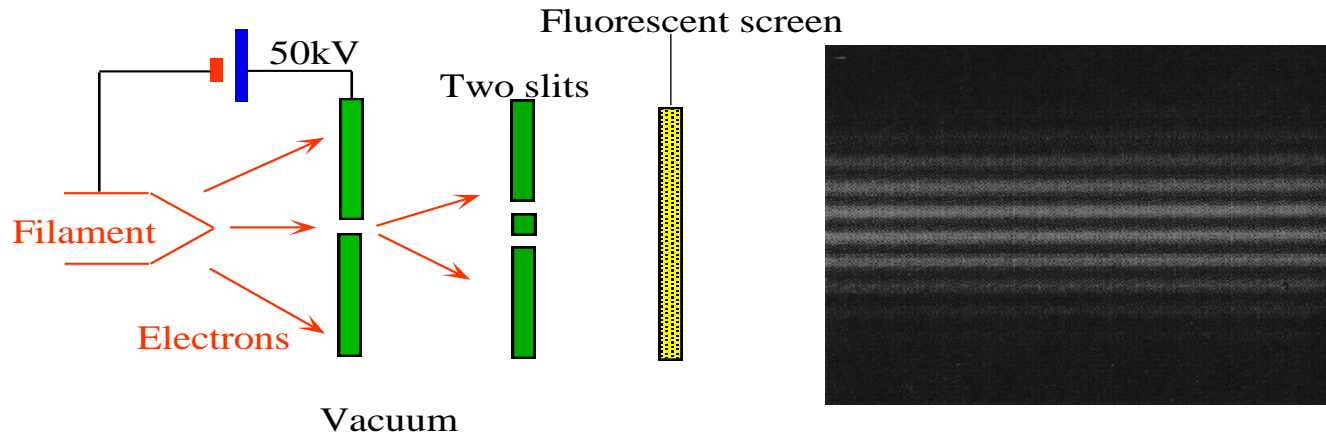
Original Lecture Notes Prepared by:  
Dr. Amr Bayoumi, Dr. Nadia Rafat



Kasap:

- Page 199-201
- Page 205-212

# Young's Experiment



Electron diffraction fringes on the screen

Fig 3.12: Young's double slit experiment with electrons involves an electron gun and two slits in a cathode ray tube (CRT) (hence in vacuum). Electrons from the filament are accelerated by a 50 kV anode voltage to produce a beam which is made to pass through the slits. The electrons then produce a visible pattern when they strike a fluorescent screen (e.g. a TV screen) and the resulting visual pattern is photographed (pattern from C. Jönsson, D. Brandt, S. Hirschi, *Am. J. Physics*, **42**, Fig. 8, p. 9, 1974).

From *Principles of Electronic Materials and Devices, Second Edition*, S.O. Kasap (© McGraw-Hill, 2002)  
<http://Materials.Usask.ca>



# Schrödinger equation: Wavefunction

- The Schrödinger equation plays the role of Newton's laws in classical mechanics
- Explains the wave-like nature of electrons using photon-like concepts

Assuming:

$$\psi(x, t) = \psi(x) \phi(t) = \psi(x) e^{-i\omega t} = \psi(x) e^{-i\frac{E}{\hbar}t}$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$



# 1-D Time-independent Schrödinger Equation

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

- **$U(\mathbf{x})$ : Potential Energy**
- **E: Total energy**
- **Equivalent of energy conservation equation in classical mechanics.**
- **Predicts the shape of the wave function.**
- **System is defined by potential energy, boundary conditions**



# Boundary Conditions

(1) **Wavefunction Magnitude is Continuous:**

$$\psi(x^-) = \psi(x^+)$$

(2) **Wavefunction Slope is Continuous:**

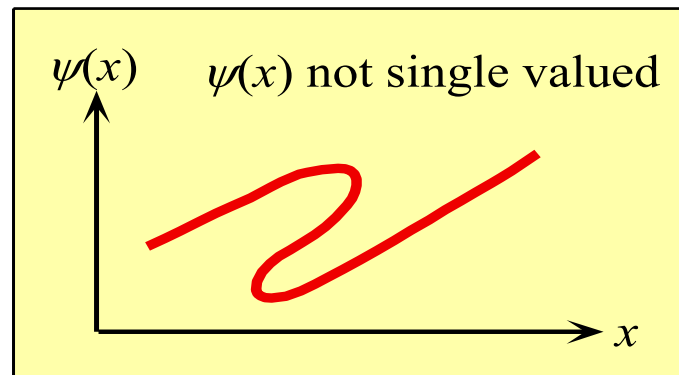
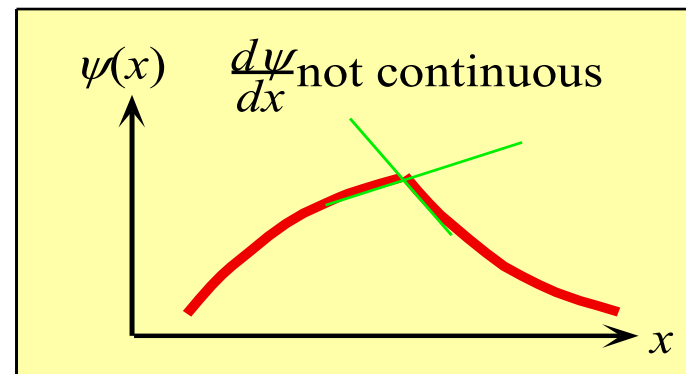
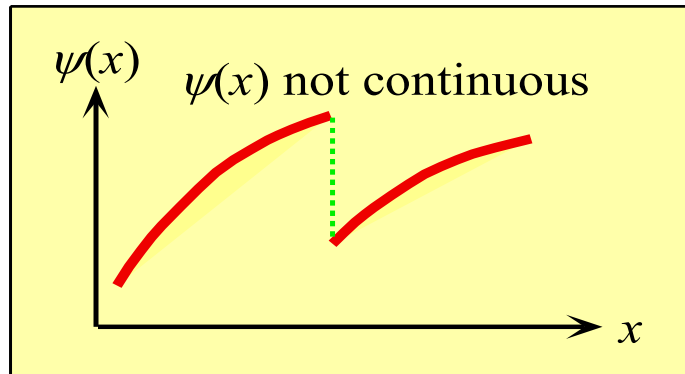
$$\frac{d\Psi(x^-)}{dx} = \frac{d\Psi(x^+)}{dx}$$

(3) **Normalization Condition:**

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$



# Unacceptable Forms of Wavefunctions



Unacceptable forms of  $\psi(x)$

Fig 3.14



# Applications

## (1) Particle in an Infinite Potential Well: Particle in a Box (1-D):

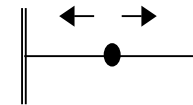
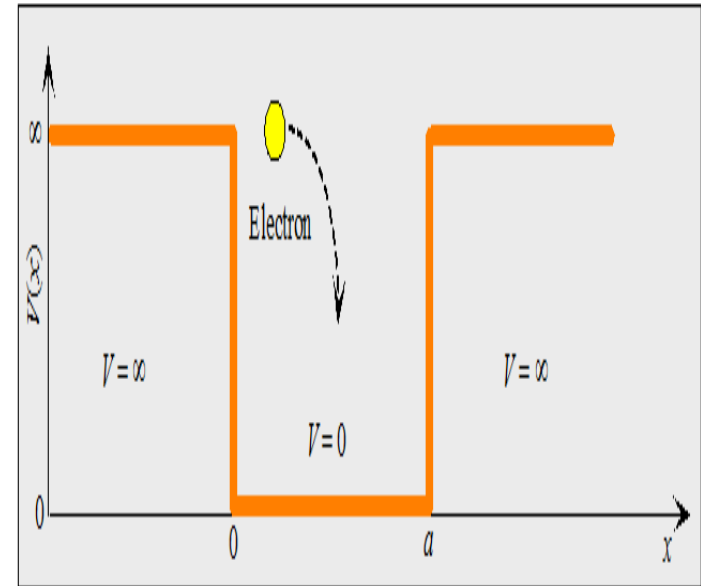
A particle of mass  $m$  is completely trapped in a region and is moving freely between  $x=0$  and  $x=L$

$$U(x)=0, \quad \text{if } 0 \leq x \leq a$$

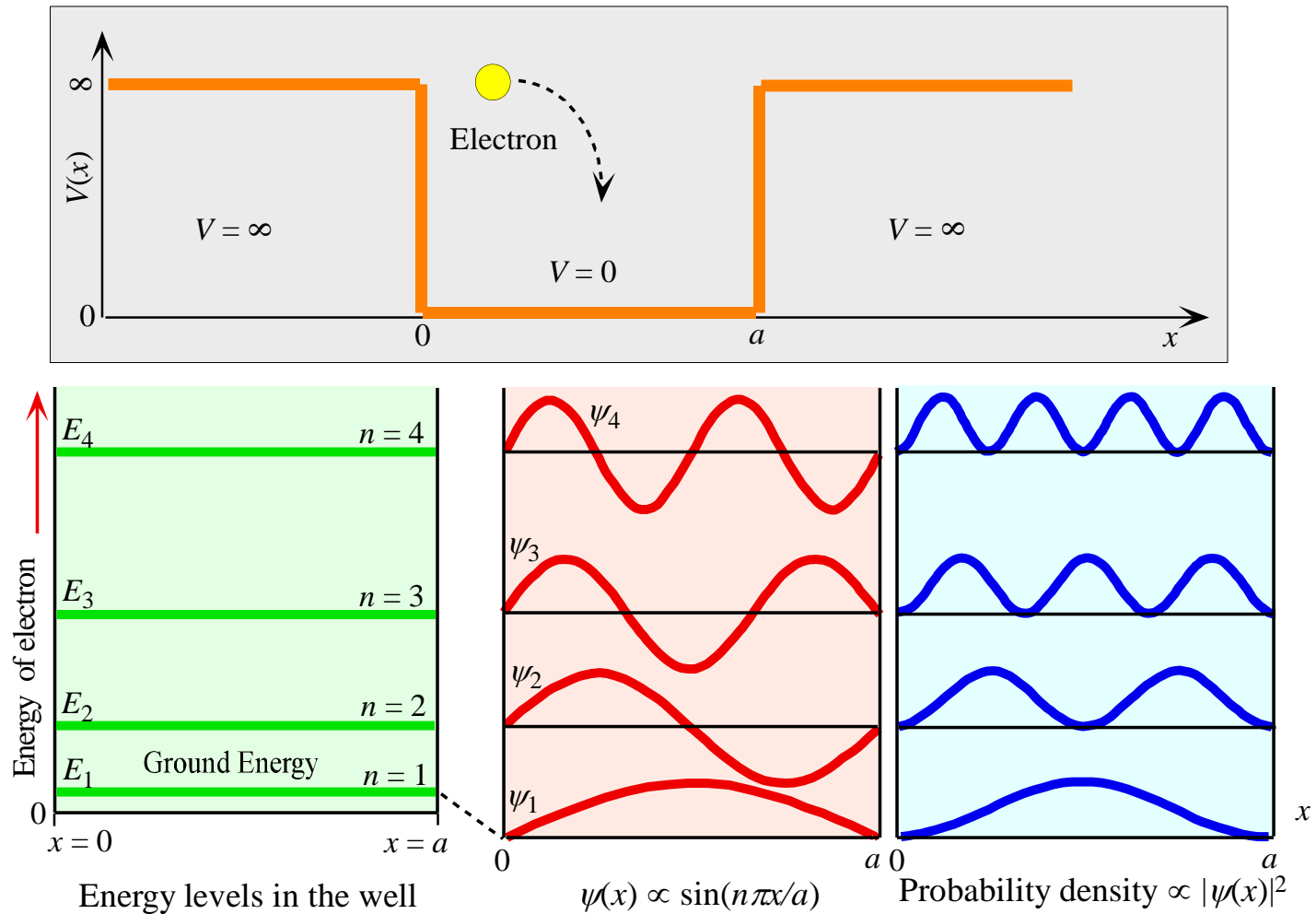
- $U(x)=\infty$ , if  $x < 0$  or  $x > a$
- **Boundary conditions on  $\psi$ :**

*$\Psi$  must be continuous across any boundary*

$$\Psi(0)=0=\Psi(a)$$







Electron in a one-dimensional infinite PE well. The energy of the electron is quantized. Possible wavefunctions and the probability distributions for the electron are shown.

Fig 3.15