Solid State Electronics EC210
Arab Academy for Science and Technology
AAST – Cairo
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Lecture 9- part a
Effective Mass

Original Lecture Notes Prepared by:
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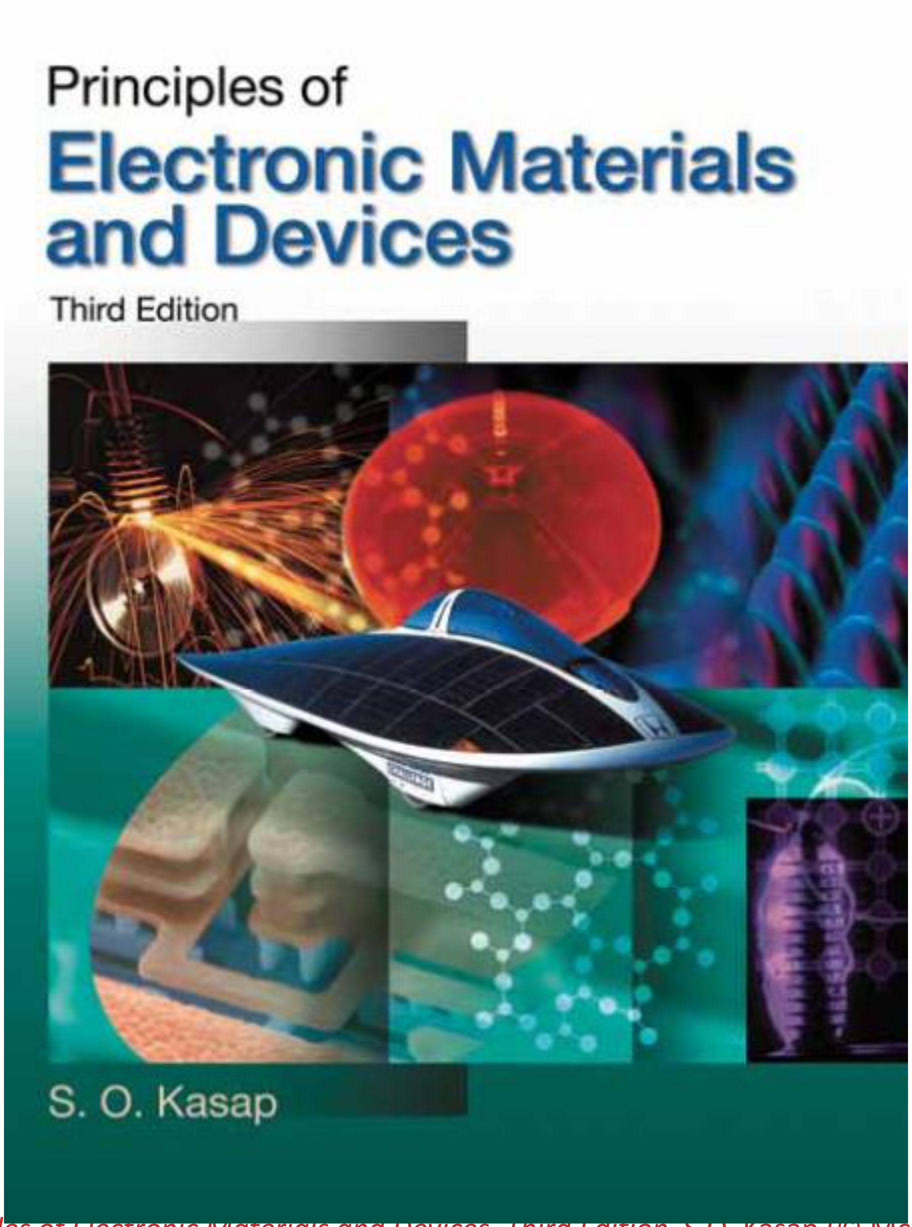

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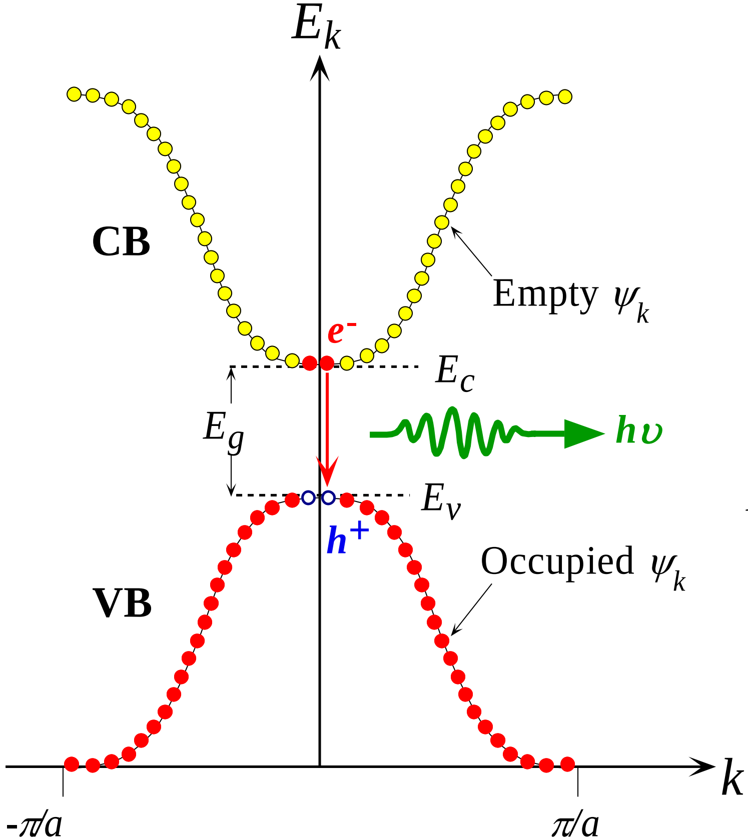
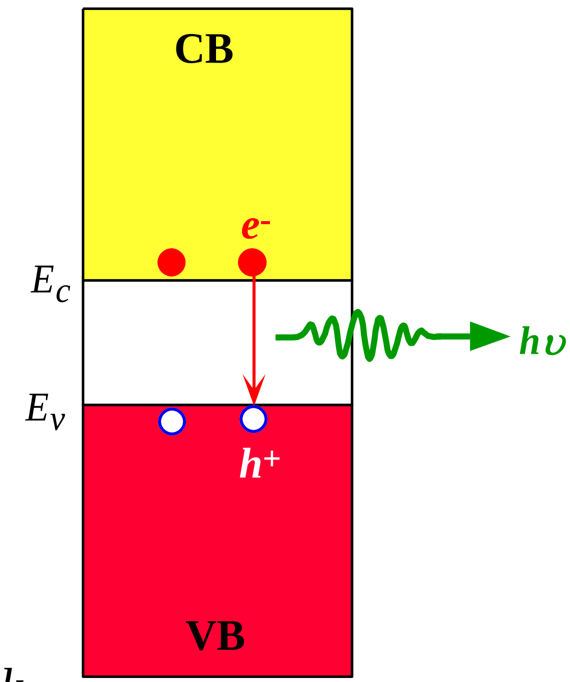
Pages

- Kasap:
  - P.303-304, p. 454-455 (Effective Mass)

The *E*-*k* Diagram



The Energy Band Diagram



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The E-k diagram of a direct bandgap semiconductor such as GaAs. The E-k curve consists of many discrete points each point corresponding to a possible state, wavefunction y k(x), that is allowed to exist in the crystal. The points are so close that we normally draw the E-k relationship as a continuous curve. In the energy range Ev to Ec there are no points (yk(x) solutions).

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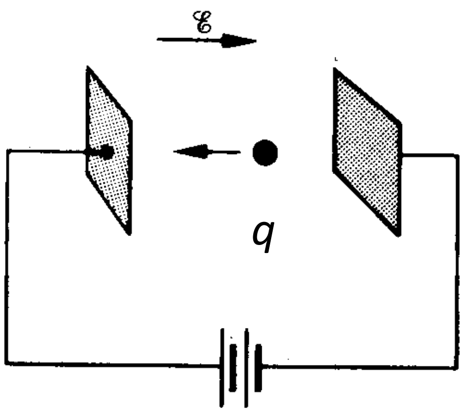
Fig 5.49

Effective Mass

In vacuum

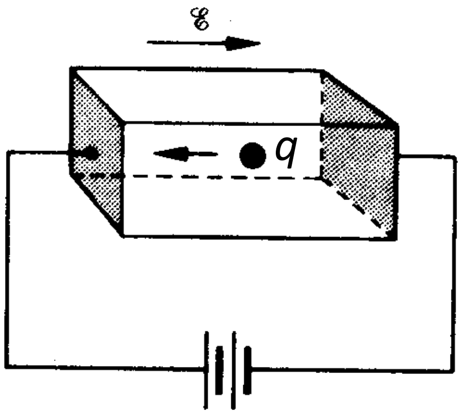


$F = q\,\varepsilon = m_o a$

where

m_o is the electron mass

In semiconductor



$F_{ext} = (-q)\mathbf{E}$

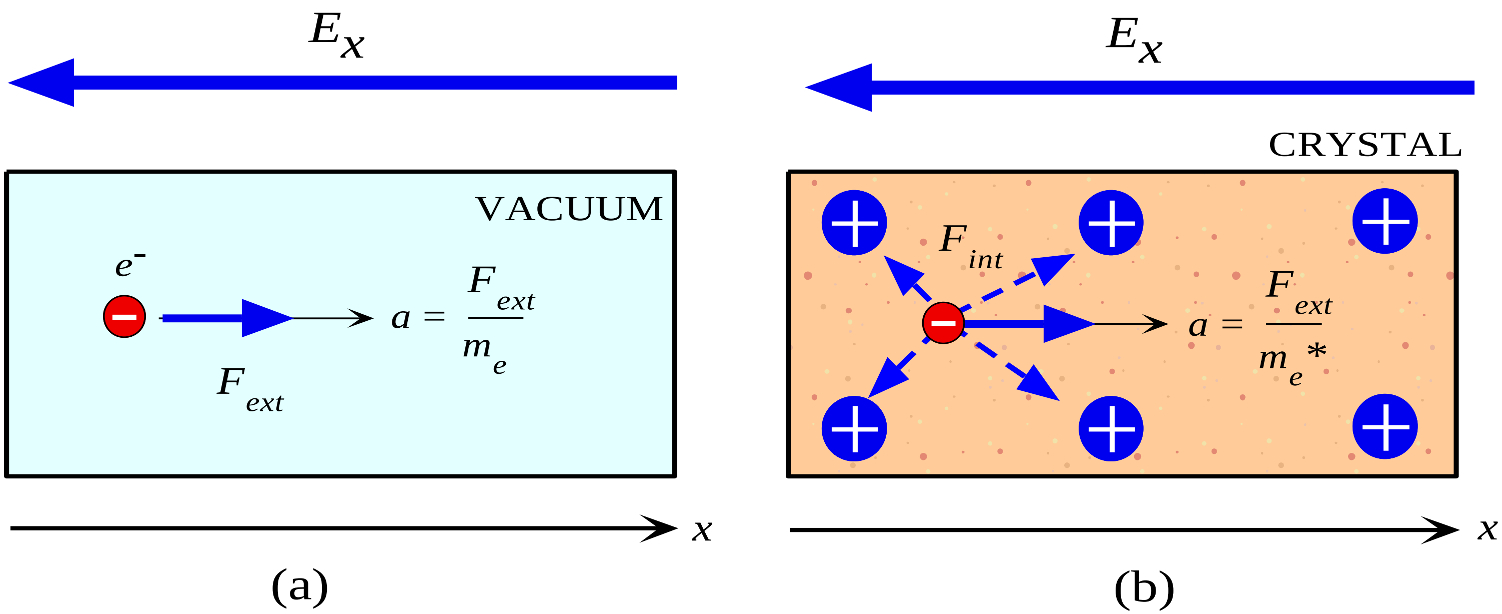
$F_{ext} + F_{int} = m_o a$

$F_{ext} = m_n{}^{*} a$

where

$m_n{}^{*}$ is the electron effective mass

Effective Mass



(a) An external force F_{ext} applied to an electron in vacuum results in an acceleration $a_{vac} = F_{ext}\ / m_e$. (b) An external force F_{ext} applied to an electron in a crystal results in an acceleration $a_{cryst} = F_{ext}\ / m_e^*$. (E_x is the electric field.)

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Fig 4.19

Example of Effective Mass

Table 4.2 Effective mass m_e^* of electrons in some metals

Metal	Ag	Au	Bi	Cu	K	Li	Na	Ni	Pt	Zn
$\frac{m_e^*}{m_e}$	0.99	1.10	0.047	1.01	1.12	1.28	1.2	28	13	0.85

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Effective Mass

Group Velocity defined as the velocity of the *wavefunction* of the electrons (analogous to speed of sinusoidal wave):

$$v_g = \frac{dx}{dt} = \frac{d\omega}{dk}, \qquad dx = v_g\, dt$$

$$\omega = \frac{E}{\hbar} \qquad \rightarrow\ vg = \frac{1}{\hbar}\frac{dE}{dk} \qquad \rightarrow dE = vg\,{}_{\hbar}\, dk$$

$$dE = F_{ext}\, dx = F_{ext}\, vg\, dt \rightarrow F_{ext} = \frac{1}{v_g}\frac{dE}{dt} \rightarrow F_{ext} = \hbar\frac{dk}{dt}$$

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Effective Mass (2)

Acceleration:

$$a = \frac{dv_g}{dt} = \frac{d}{dt}\left[\frac{1}{\hbar}\frac{dE}{dk}\right] = \frac{1}{\hbar}\frac{d}{dk}\left[\frac{dE}{dt}\right] = \frac{1}{\hbar}\frac{d}{dk}\left[\frac{dE}{dk}\frac{dk}{dt}\right]$$

$$a = \frac{1}{\hbar}\frac{d^2E}{dk^2}\frac{dk}{dt} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}\hbar\frac{dk}{dt} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}F_{ext}$$

Using $F_{ext} = m^* a$

$$m^* = \left[\frac{1}{\hbar^2}\frac{d^2E}{dk^2}\right]^{-1} = \hbar^2\left[\frac{d^2E}{dk^2}\right]^{-1}$$

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