



**Solid State Electronics EC210**  
**Arab Academy for Science and Technology**  
**AAST – Cairo**  
**Spring 2016**

## **Lecture 9- part a**

# **Effective Mass**

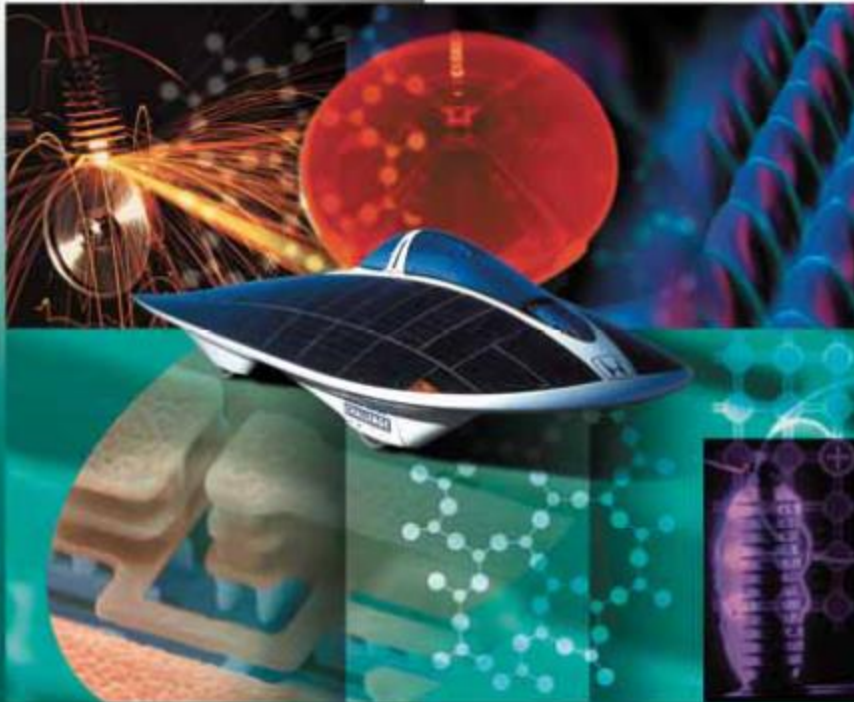
*Original Lecture Notes Prepared by:*

**Dr. Amr Bayoumi, Dr. Nadia Rafat**



# Principles of Electronic Materials and Devices

Third Edition



S. O. Kasap

These PowerPoint color diagrams can only be used by instructors if the 3<sup>rd</sup> Edition has been adopted for his/her course. Permission is given to individuals who have purchased a copy of the third edition with CD-ROM Electronic Materials and Devices to use these slides in seminar, symposium and conference presentations provided that the book title, author and © McGraw-Hill are displayed under each diagram.



From *Principles of Electronic Materials and Devices, Third Edition*, S.O. Kasap (© McGraw-Hill, 2005)

Original Lecture Notes Prepared by:  
Dr. Amr Bayoumi, Dr. Nadia Rafat

Lecture 9 Part 1 : Effective Mass

Solid State Electronics EC210, Spring 2016  
Arab Academy for Science and Technology  
AAST – Cairo,

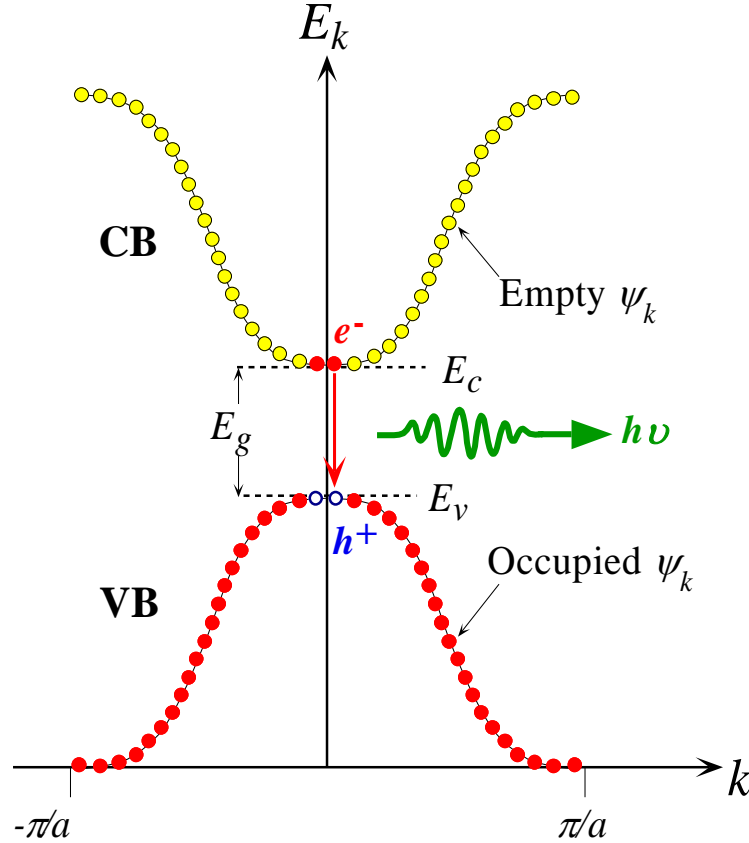


# Pages

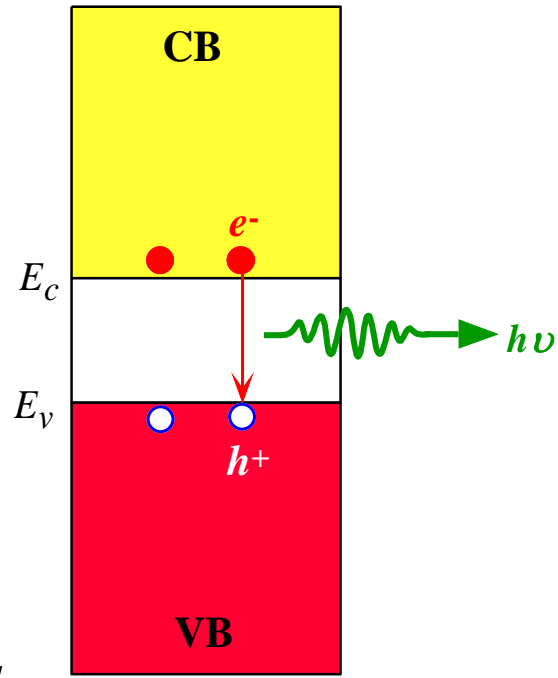
- Kasap:
  - P.303-304, p. 454-455 (Effective Mass)

*From Principles of Electronic Materials and Devices, Third Edition, S.O. Kasap (© McGraw-Hill, 2005)*

### The $E-k$ Diagram



### The Energy Band Diagram



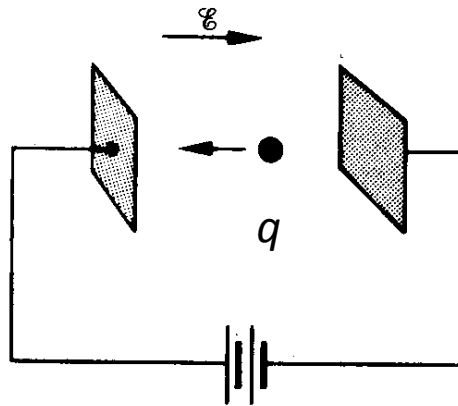
From *Principles of Electronic Materials and Devices, Third Edition*, S.O. Kasap (© McGraw-Hill, 2005)

The  $E-k$  diagram of a direct bandgap semiconductor such as GaAs. The  $E-k$  curve consists of many discrete points each point corresponding to a possible state, wavefunction  $\psi_k(x)$ , that is allowed to exist in the crystal. The points are so close that we normally draw the  $E-k$  relationship as a continuous curve. In the energy range  $E_v$  to  $E_c$  there are no points ( $\psi_k(x)$  solutions).

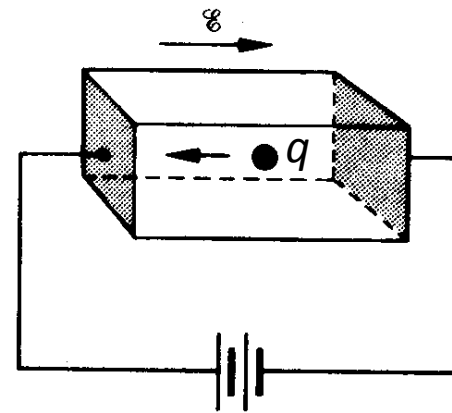
From *Principles of Electronic Materials and Devices, Third Edition*, S.O. Kasap (© McGraw-Hill, 2005)

# Effective Mass

In vacuum



In semiconductor



$$F = q \varepsilon = m_0 a$$

where

$m_0$  is the electron mass

$$F_{ext} = (-q)\mathbf{E}$$

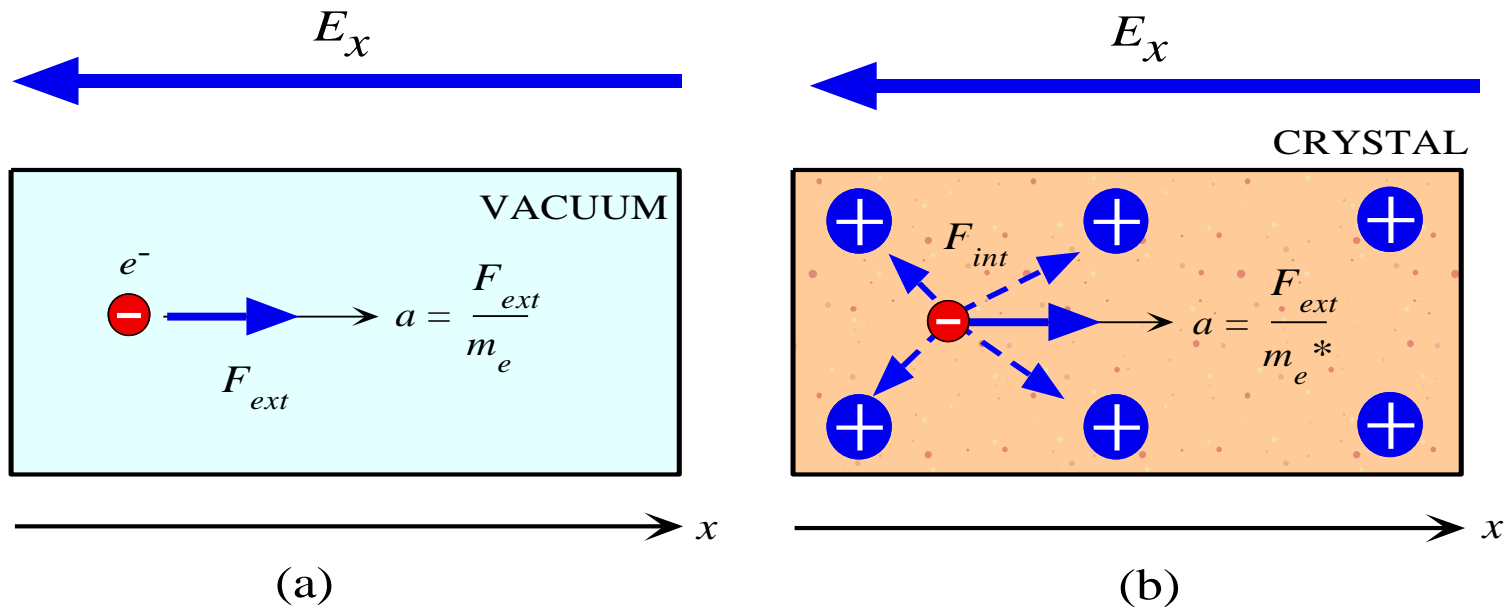
$$F_{ext} + F_{int} = m_0 a$$

$$F_{ext} = m_n^* a$$

where

$m_n^*$  is the electron effective mass

# Effective Mass



(a) An external force  $F_{ext}$  applied to an electron in vacuum results in an acceleration  $a_{vac} = F_{ext} / m_e$ . (b) An external force  $F_{ext}$  applied to an electron in a crystal results in an acceleration  $a_{cryst} = F_{ext} / m_e^*$ . ( $E_x$  is the electric field.)

From *Principles of Electronic Materials and Devices, Third Edition*, S.O. Kasap (© McGraw-Hill, 2005)

From *Principles of Electronic Materials and Devices, Third Edition*, S.O. Kasap (© McGraw-Hill, 2005)  
 Original Lecture Notes Prepared by:  
 Dr. Amr Bayoumi, Dr. Nadia Rafat

Lecture 9 Part 1 : Effective Mass

Solid State Electronics EC210, Spring 2016  
 Arab Academy for Science and Technology  
 AAST – Cairo,



# Example of Effective Mass

**Table 4.2** Effective mass  $m_e^*$  of electrons in some metals

Metal	Ag	Au	Bi	Cu	K	Li	Na	Ni	Pt	Zn
$\frac{m_e^*}{m_e}$	0.99	1.10	0.047	1.01	1.12	1.28	1.2	28	13	0.85

From *Principles of Electronic Materials and Devices, Third Edition*, S.O. Kasap (© McGraw-Hill, 2005)

Original Lecture Notes Prepared by:  
Dr. Amr Bayoumi, Dr. Nadia Rafat

Lecture 9 Part 1 : Effective Mass

From *Principles of Electronic Materials and Devices, Third Edition*, S.O. Kasap (© McGraw-Hill, 2005)  
Solid State Electronics, EC210, Spring 2016  
Arab Academy for Science and Technology  
AAST – Cairo, **7**



# Effective Mass

Group Velocity defined as the velocity of the *wavefunction* of the electrons (analogous to speed of sinusoidal wave ):

$$v_g = \frac{dx}{dt} = \frac{d\omega}{dk}, \quad dx = v_g dt$$

$$\omega = \frac{E}{\hbar} \quad \rightarrow \quad v_g = \frac{1}{\hbar} \frac{dE}{dk} \quad \rightarrow \quad dE = v_g \hbar dk$$

$$dE = F_{ext} dx = F_{ext} v_g dt \rightarrow F_{ext} = \frac{1}{v_g} \frac{dE}{dt} \rightarrow F_{ext} = \hbar \frac{dk}{dt}$$

From *Principles of Electronic Materials and Devices, Third Edition*, S.O. Kasap (© McGraw-Hill, 2005)





# Effective Mass (2)

Acceleration:

$$a = \frac{dv_g}{dt} = \frac{d}{dt} \left[ \frac{1}{\hbar} \frac{dE}{dk} \right] = \frac{1}{\hbar} \frac{d}{dk} \left[ \frac{dE}{dt} \right] = \frac{1}{\hbar} \frac{d}{dk} \left[ \frac{dE}{dk} \frac{dk}{dt} \right]$$

$$a = \frac{1}{\hbar} \frac{d^2E}{dk^2} \frac{dk}{dt} = \frac{1}{\hbar^2} \frac{d^2E}{dk^2} \hbar \frac{dk}{dt} = \frac{1}{\hbar^2} \frac{d^2E}{dk^2} F_{ext}$$

Using  $F_{ext} = m^* a$

$$m^* = \left[ \frac{1}{\hbar^2} \frac{d^2E}{dk^2} \right]^{-1} = \hbar^2 \left[ \frac{d^2E}{dk^2} \right]^{-1}$$

From *Principles of Electronic Materials and Devices, Third Edition*, S.O. Kasap (© McGraw-Hill, 2005)