Solid State Electronics EC210
Arab Academy for Science and Technology
AAST – Cairo
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Lecture 9 Part b
Density of States

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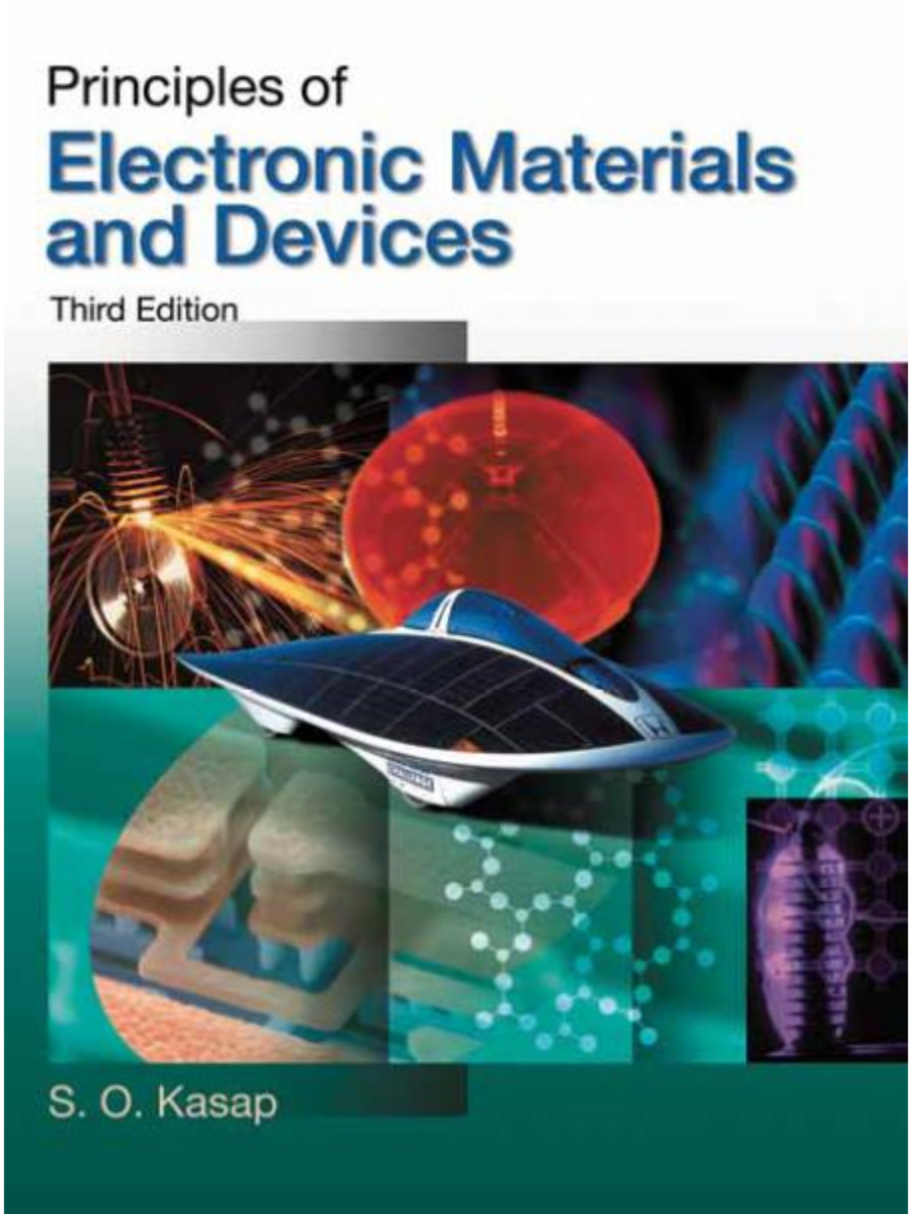

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Introduction

Given a 1D, 2D or 3D material:

How many electrons are allowed to be present within an energy band (such as valence of conduction bands)

- Must find the number of allowed "states" within an Energy range in the E-K diagram, per unit volume (or area or length) of the material
- Each state can hold up to two electrons with different spins
- The band theory does NOT allow infinite numbers of electrons within an energy range
- From the E-K diagram velocity can be found, now only the number of electrons are needed to calculate current

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Pages

- Kasap:
  - p. 305-308

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Energy Levels

- **1D ($L_x = a$) :**

$$E_n = \frac{\hbar^2 {k_n}^2}{2m} = \frac{h^2}{8m\,a^2} n^2$$

- **2D (assuming $L_x = L_y = a$):**

$$E_n = \frac{\hbar^2 {k_n}^2}{2m} = \frac{h^2}{8m\,a^2} ({n_x}^2 + {n_y}^2) = \frac{h^2}{8m\,a^2} n'^2$$

- **3D (assuming $L_x = L_y = L_z = a$) :**

$$E_n = \frac{\hbar^2 {k_n}^2}{2m} = \frac{h^2}{8m\,a^2} ({n_x}^2 + {n_y}^2 + {n_z}^2) = \frac{h^2}{8m\,a^2} n'^2$$

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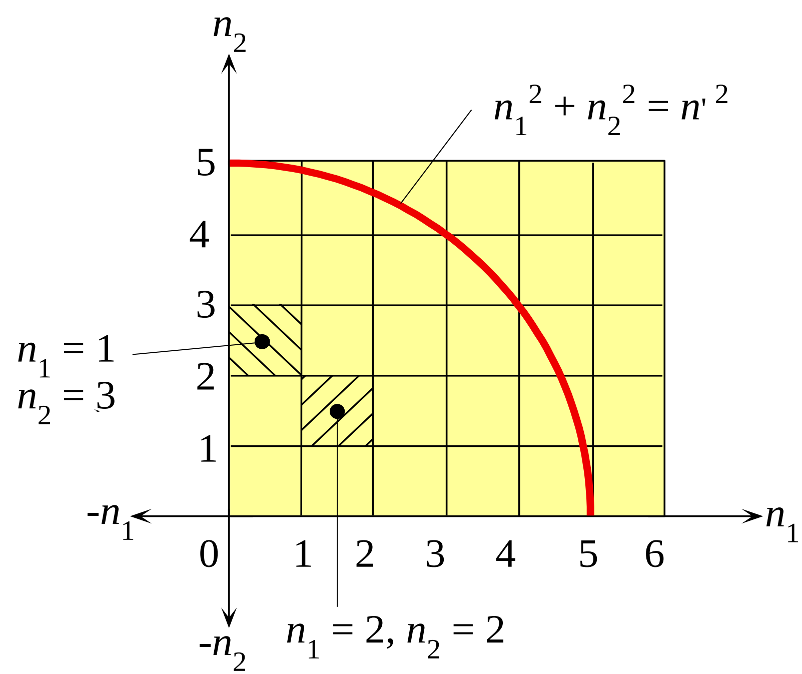
Density of States (2D)

- -k and +k are equivalent in k_x kx and k_y directions
- Number of k states within n' = Area of quarter circle with radius n':

$$N'_{2D} = \frac{\pi\, n'^2}{4} = \frac{\pi}{4}\frac{8ma^2}{h^2}E'$$

- Each state can carry a maximum of two electrons
- **Maximum** Number of electrons /unit Area of material = $2\ electrons\ * N'_{2D}$/Area:

$$S_{2D}(E) = \frac{2\,N'_{2D}}{a^2} = 2[\frac{\pi}{4}\frac{8m}{h^2}E']$$



Each state, electron wavefunction in the crystal, can be represented by a box at n_1,n_2.

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Fig 4.21

Density of States (3D)

- Number of k-states per unit area per unit volume within E':

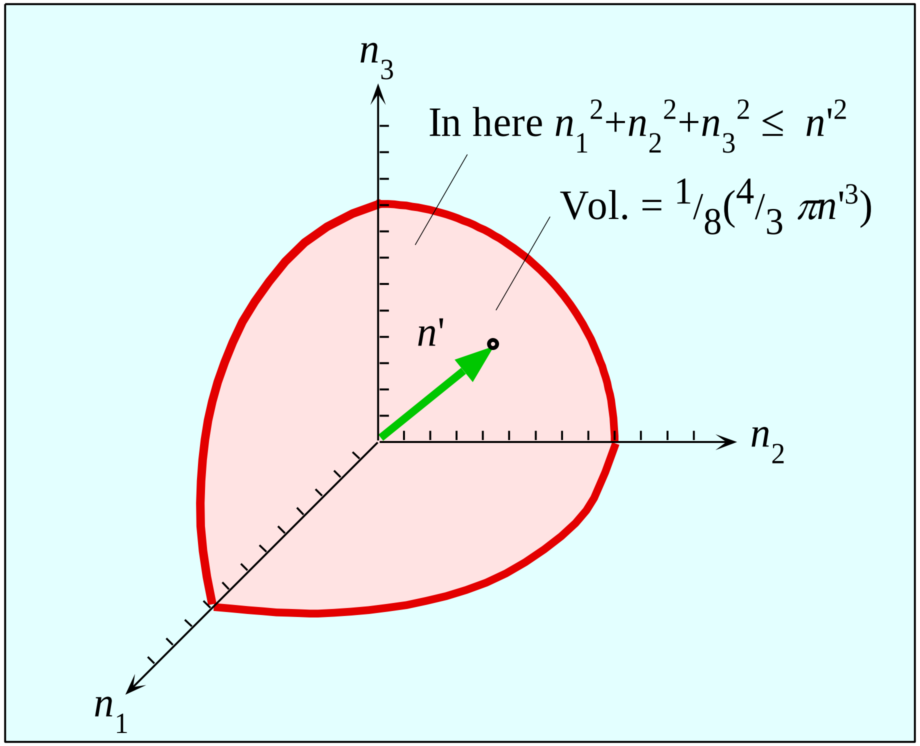
$$N'_{3D}(E') = \frac{\frac{4}{3}\pi\, n'^3}{8}$$

$$= \frac{\pi}{6}\left[\frac{8ma^2}{h^2}E'\right]^{3/2}$$

- **Max**. Number of electrons within E'/ unit Volume of material:

$$S_{3D}(E) = 2\,\frac{N'_{3D}(E')}{a^3}$$

$$= \frac{\pi}{3}\left[\frac{8m}{h^2}E'\right]^{3/2}$$



In three dimensions, the volume defined by a sphere of radius n' and the positive axes n_1, n_2 and n_3, is all the possible combinations of poisitive n_1, n_2 and n_3, values which satisfy $n_1^2+n_2^2+n_3^2 \leq n'^2$.

Fig 4.22

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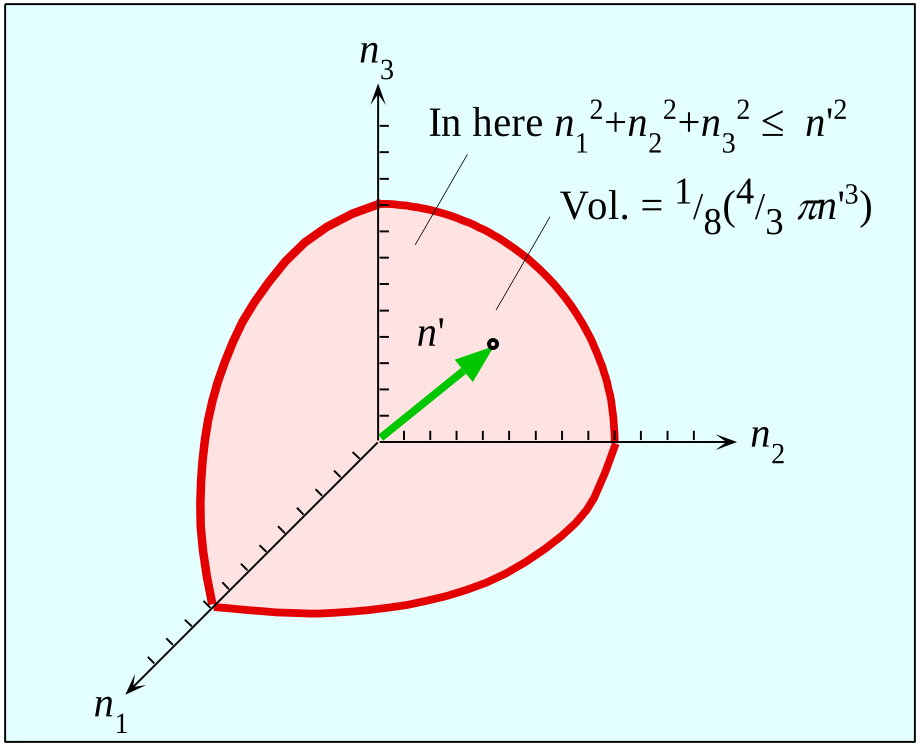
Density of States (3D)

- Density of States = $g(E)$= Maximum Number of electrons per unit Energy per unit Volume:

$$g_{3D}(E) = \frac{d}{dE}\left\{\frac{N_{electrons}(E)}{\text{Vol}}\right\}$$

$$= \frac{d}{dE}\{S_{3D}(E)\}$$



$$= \frac{d}{dE}\left\{\frac{\pi}{3}\left[\frac{8m}{h^2}E\right]^{\frac{3}{2}}\right\}$$

$$= (\pi/2)\left[\frac{8m}{h^2}\right]^{\frac{3}{2}}\{\boldsymbol{E^{1/2}}\}$$

In three dimensions, the volume defined by a sphere of radius n' and the positive axes n_1, n_2 and n_3, is all the possible combinations of poisitive n_1, n_2 and n_3, values which satisfy $n_1^2+n_2^2+n_3^2 \le n'^2$.

Fig 4.22

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Total Number of Electrons allowed between E1 and E2 on the E-K Diagram

$$N_{e,Total} = \int_{E1}^{E2} g(E)dE$$

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