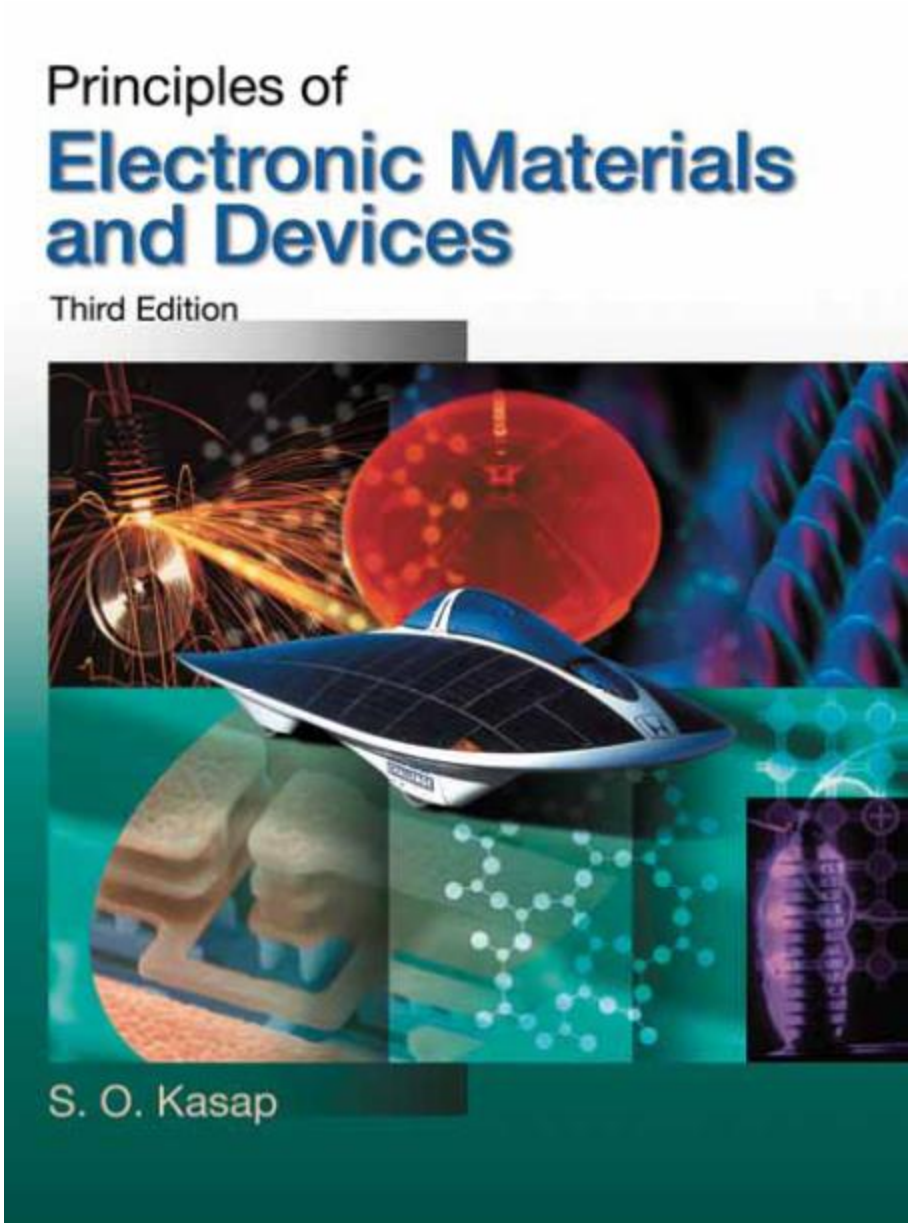
Solid State Electronics EC210
Arab Academy for Science and Technology
AAST – Cairo
Spring 2016

Lecture 6:

Free Carrier Concentrations
Fermi Level Calculation

Original Lecture Notes Prepared by:
Dr. Amr Bayoumi, Dr. Nadia Rafat



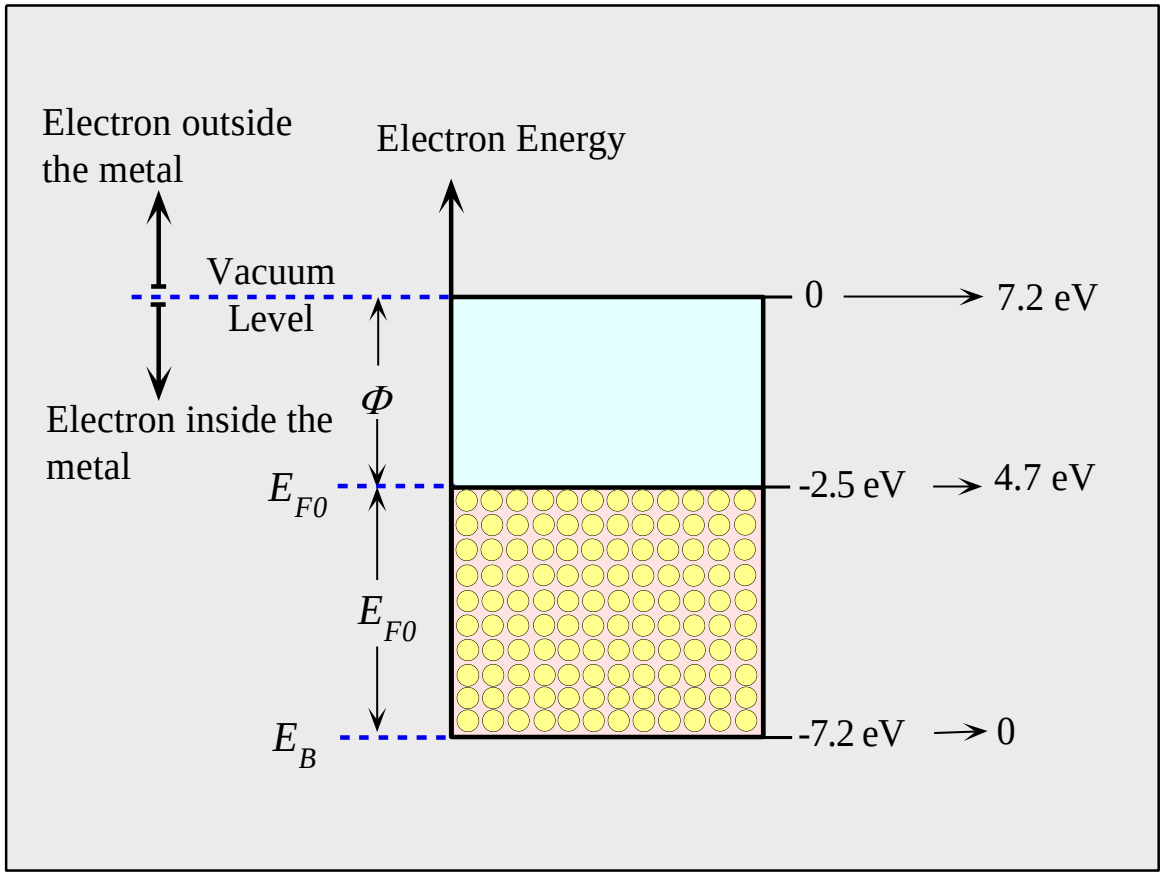
These PowerPoint color diagrams can only be used by instructors if the 3rd Edition has been adopted for his/her course. Permission is given to individuals who have purchased a copy of the third edition with CD-ROM Electronic Materials and Devices to use these slides in seminar, symposium and conference presentations provided that the book title, author and © McGraw-Hill are displayed under each diagram.

Pages

Kasap:

- **p380-401**

Metal



Typical electron energy band diagram for a metal All the valence electrons are in an energy band which they only partially fill. The top of the band is the vacuum level where the electron is free from the solid ($PE = 0$).

Example Workfunctions for Metals

- **Workfunction = Φ = Energy from Fermi Level to Vacuum**

Table 4.1 Fermi energy and work function of selected metals

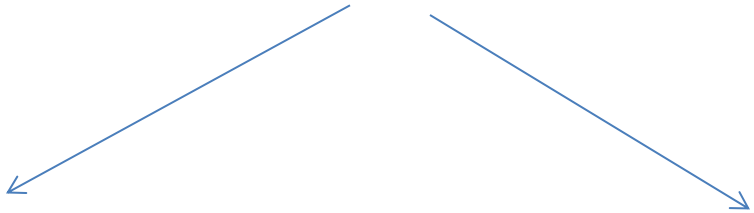
	Metal							
	Ag	**Al**	**Au**	**Cs**	**Cu**	**Li**	**Mg**	**Na**
Φ (eV)	4.5	4.28	5.0	2.14	4.65	2.3	3.7	2.75
E_{FO} (eV)	5.5	11.7	5.5	1.58	7.0	4.7	7.1	3.2

- **Also for Semiconductors:**
  - Workfunction = Φ =Energy from Fermi Level to Vacuum

Free Carriers

- **(Free Conduction Electrons and Free Valence Holes)**

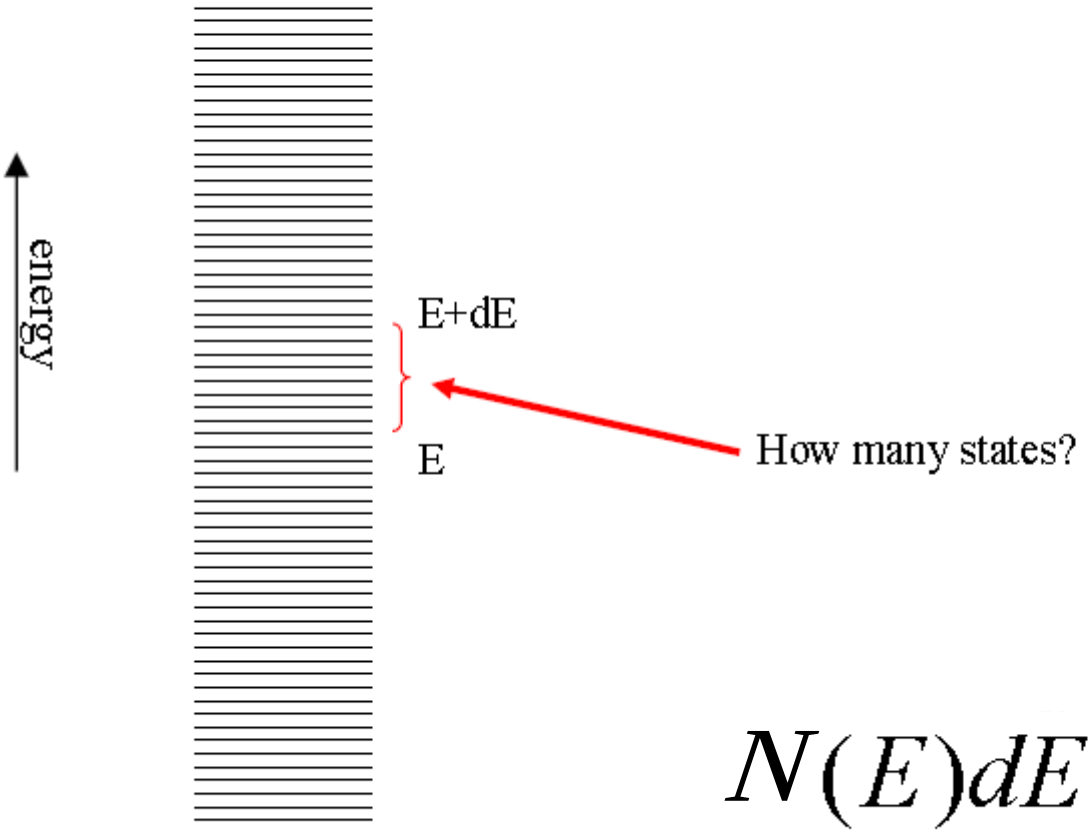
Concentration of carriers



Density of States

Distribution of carriers (Fermi Dirac Distribution)

Density of states:



$$N(E)dE = ?$$

Number of states with energy between E and E + dE *per volume*.

Free Carrier Concentration

• Using the density of states and the Fermi function we may calculate the CONCENTRATION of carriers present in the conduction and valence bands at any given temperature

* We do this by INTEGRATING the density of OCCUPIED states over the range of energies relevant to either band

•

$$N.\ of\ \ Electrons\ \ in\ \ Conduction\ \ Band\ \ \ per\ unit\ volume:$$

$$n = \int_{E_c}^{E_{top}} N_{sc}(E) f(E) dE$$

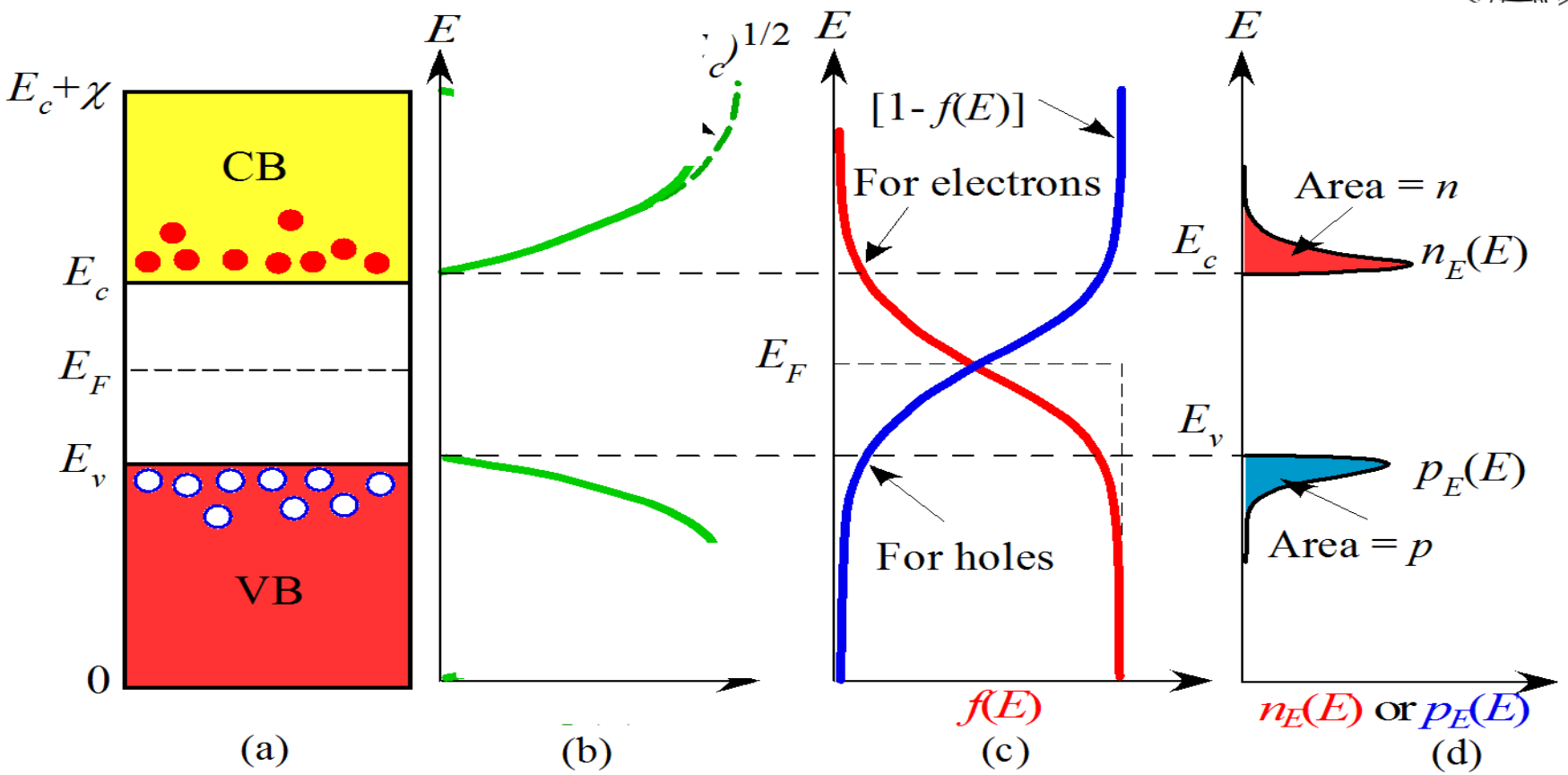
$$N.\ of\ \ Holes\ \ in\ \ Valence\ \ Band\ \ \ per\ unit\ volume:$$

$$p = \int_{E_{vbottom}}^{E_v} N_{sv}(E)\left[1 - f(E)\right] dE$$

⇒ Note here how we write the probability for occupation of HOLE states as 1-*f*(*E*)

⇒ By substituting the expressions for the electron and hole densities of states into these integrals we may calculate *n* and *p* for any given *T* and E_F

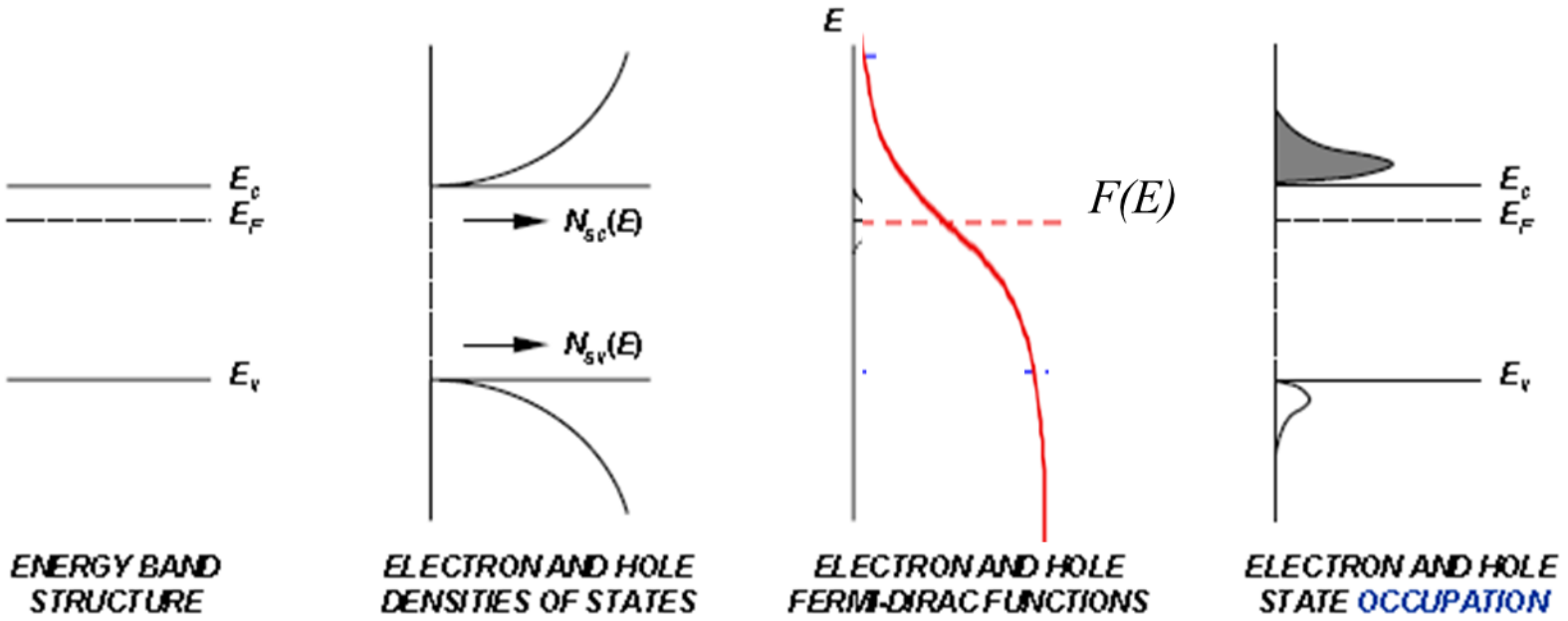
Carriers in Conduction and Valence Bands: Intrinsic Case ($n=p=n_i$)



(a) Energy band diagram. (b) Density of states (number of states per unit energy per unit volume). (c) Fermi-Dirac probability function (probability of occupancy of a state). (d) The product of $g(E)$ and $f(E)$ is the energy density of electrons in the CB (number of electrons per unit energy per unit volume). The area under $n_E(E)$ vs. E is the electron concentration in the conduction band.

Carriers in Conduction and Valence Bands: n-Type Case (n≈ N_D^+)

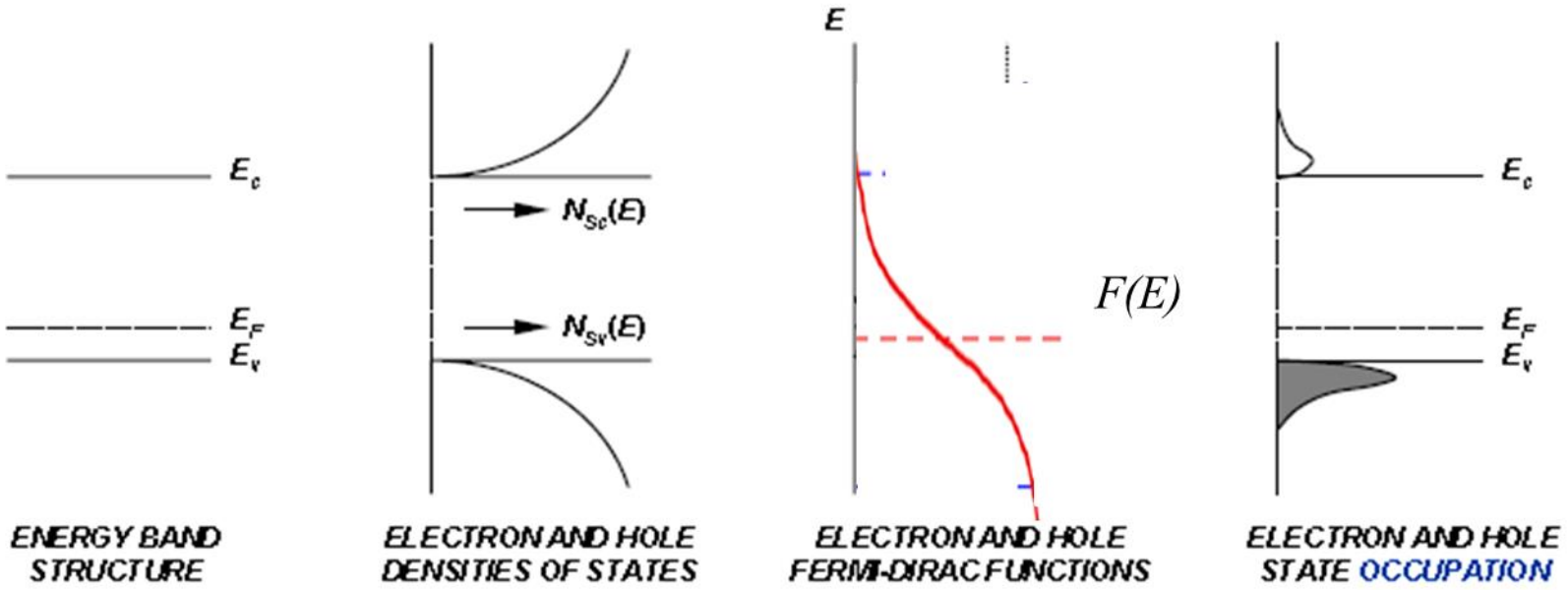
n Type Semiconductors



n > p

Carriers in Conduction and Valence Bands: p-Type Case (p≈ N_A^-)

p Type Semiconductors



p > n

Free Carrier Concentration(2)

$$n = N_C \exp\left[-\frac{(E_C - E_F)}{kT}\right] \qquad N_C = 2\left(\frac{m_e^* kT}{2\pi\hbar^2}\right)^{3/2}$$

Eff. DOS CB

$$p = N_V \exp\left[-\frac{(E_F - E_V)}{kT}\right] \qquad N_V = 2\left(\frac{m_h^* kT}{2\pi\hbar^2}\right)^{3/2}$$

Eff. DOS VB

We can now calculate the np product by multiplying the above equations.

$$np = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

Carrier Concentration (n_i) and Mass Action Law

$$np = N_C N_V \exp\left(-E_g / kT\right)$$

Mass action law:

$$np = ni^2$$

$$n_i^2(T_1) = N_C(T_1)\, N_V(T_1) \exp(-E_g / k_B T_1)$$

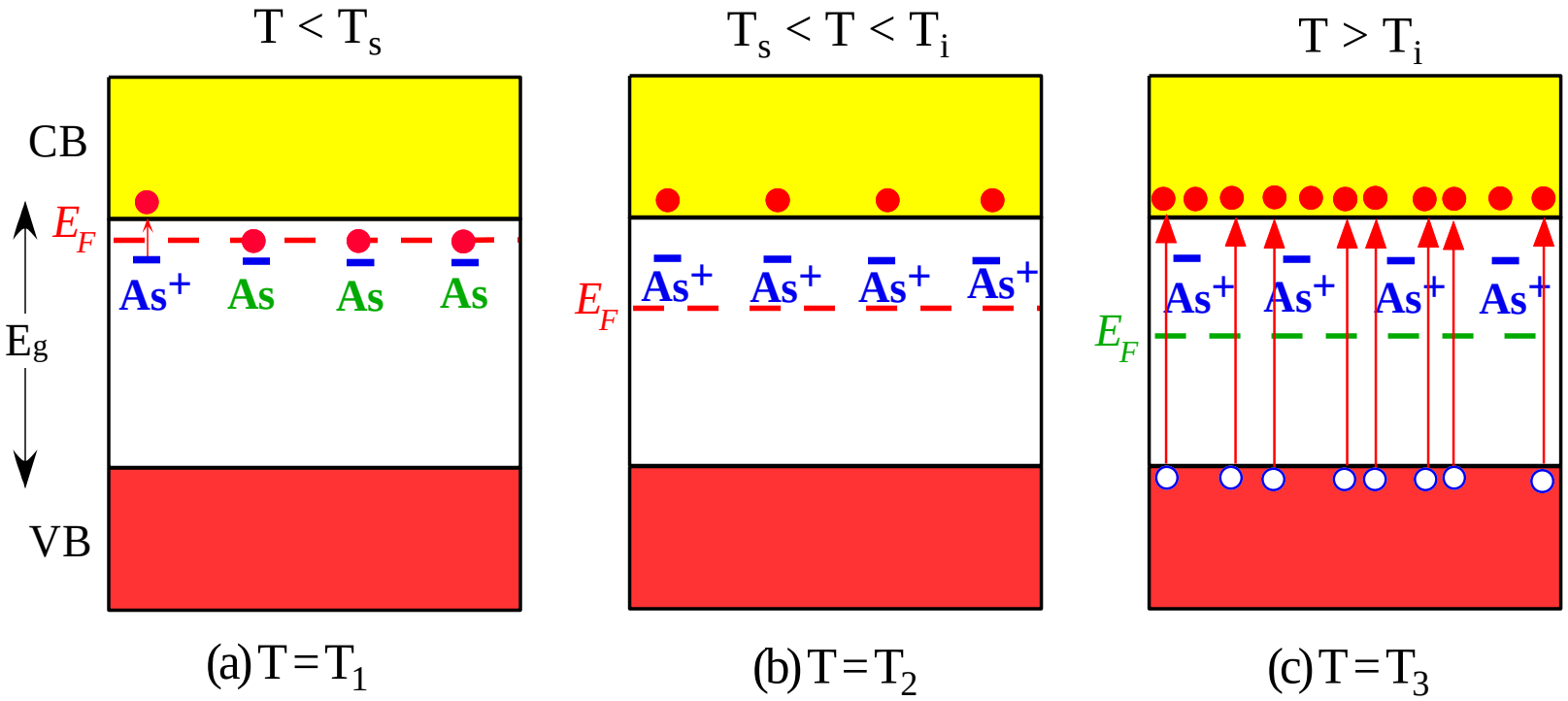
Temperature Dependence of N_C & N_V

- **Both N_c and N_v have $T^{3/2}$ temperature dependence (T is in K)**

$$N_C(T_1) = N_C(300K)\left(\frac{T_1}{300K}\right)^{3/2}$$

$$N_V(T_1) = N_V(300K)\left(\frac{T_1}{300K}\right)^{3/2}$$

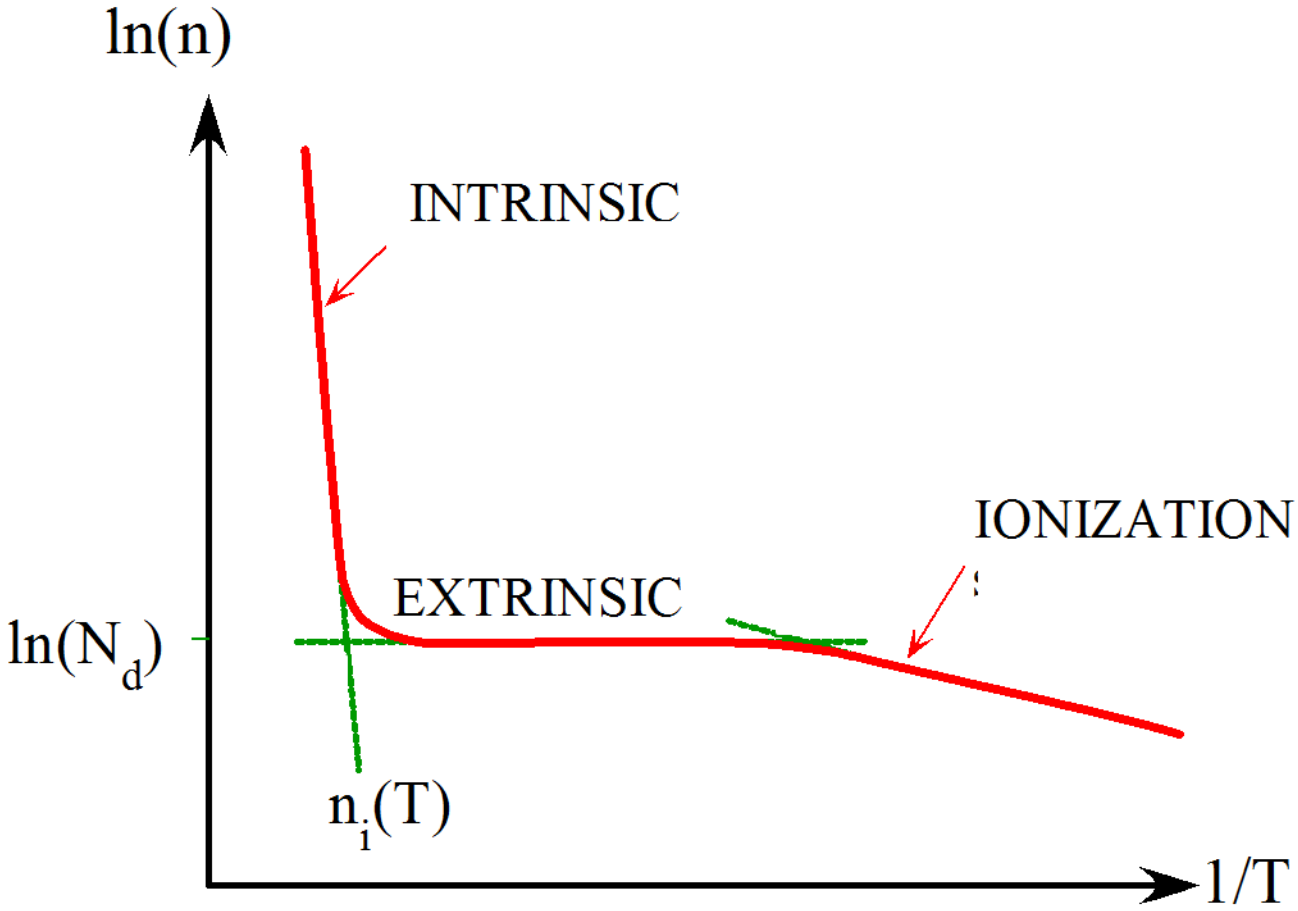
Temperature Dependence



(a) Below T_S, the electron concentration is controlled by the ionization of the donors. (b) Between T_S and T_i, the electron concentration is equal to the concentration of donors since they would all have ionized. (c) At high temperatures, thermally generated electrons from the VB exceed the number of electrons from ionized donors and the semiconductor behaves as if intrinsic.

Fig

Temperature Dependence of Free Carriers



The temperature dependence of the electron concentration in an n-type semiconductor.

Fig 5.15

Example on Temperature Dependence of n_i in n-Type Si

Si, N_D=1x10^{16}cm^{-3}, N_c (300K)=3.2x10^{19}cm^{-3}, N_V(300K)=1.8x10^{19}cm^{-3}, E_g=1.12eV, k_B=8.617x10^{-5} eV K^{-1}

Find N_C, N_V, n_i, p, n, for T=200K, 300K, 400K:

$$\frac{N_C(T_1)}{N_C(300K)} = \left(\frac{T_1}{300K}\right)^{3/2} \qquad \frac{N_V(T_1)}{N_V(300K)} = \left(\frac{T_1}{300K}\right)^{3/2}$$

$$n_i^2(T_1) = N_C(T_1)\,N_V(T_1)\exp(-E_g/kT_1)$$

$n \approx N_D^+ = N_D$ (200-400K is considered extrinsic range: All N_D donor atoms are ionized, $N_D >> n_i$)

$$n\,p = ni^2 \;\rightarrow\; p = ni^2/n$$

At T=200K in Si: $kT = ?\; n_i = ?$

At T=400K in Si: $kT = ?\; ni = ?$

Example on Temperature Dependence of n_i in n-Type Si

At T=200K in Si:

$kT = (8.617\text{x}10^{-5})(200\text{K}) = 0.0172\text{eV}$

$N_C(200\text{K}) = N_C(300\text{K})\left(\frac{200}{300}\right)^{3/2} = 3.2\text{x}10^{19}(0.666)^{1.5} = 2.13\text{x}10^{19}\text{cm}^{-3}$

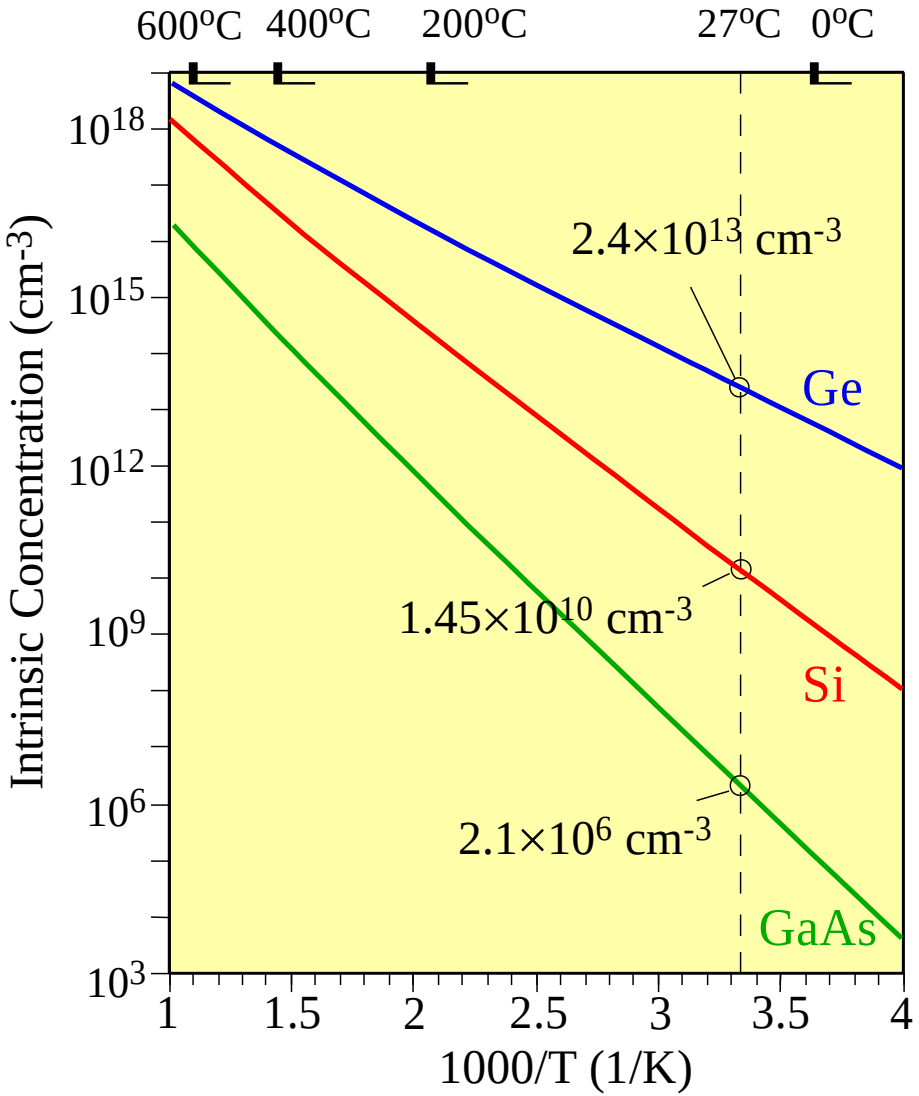
$N_V(200\text{K}) = N_V(300\text{K})\left(\frac{200}{300}\right)^{3/2} = 1.8\text{x}10^{19}(0.666)^{1.5} = 1.2\text{x}10^{19}\text{cm}^{-3}$

$n_i^2(200K) = (2.13\text{x}1019)(1.2\text{x}1019)\exp\left(-\frac{1.12}{0.0172}\right) = 1.34\text{x}10^{10}\text{cm}^{-6}$

$n_i = 1.16\text{x}10^{5}\ \text{cm}^{-3}$

$p = ni^2/n = ni^2/ND = (1.34\text{x}10^{10})/(10^{16}) = 1.34\text{x}10^{-6}\ \text{cm}^{-3}$

At T=400K in Si: $kT = ?\ ni = ?$



The temperature dependence of the intrinsic concentration.

From *Principles of Electronic Materials and Devices, Third Edition*, S.O. Kasap (© McGraw-Hill, 2005)

Fig 5.16

Example on Determination of n_i and E_C-E_F in n-Type Si

- **Assume for Si at 300K:**

N_D=1x10^{16}cm^{-3}, N_c=3.2x10^{19}cm^{-3}, N_V=1.8x10^{19}cm^{-3}, E_g=1.12eV, kT=25.85meV

$$n \approx N_D = N_c \exp(-(E_c - EF)/kT)$$

$$\ln\left(\frac{N_D}{N_c}\right) = -\frac{(E_c - EF)}{kT}$$

$E_c - EF = kT \ln(Nc/ND)$ =0.0258 ln(3.2x1019/1016) eV

Exercise: Repeat for T=200K and 400K

$n_i^2 = N_c N_v \exp(-E_g/kT)$= **(3.2x10^{19})(1.8x10^{19})**exp**(-1.12/0.0258) cm^{-6}**

At T=300K in Si: n_i **= 0.93x10^{10}cm^{-3}**