COLLEGE OF ENGINEERING & TECHNOLOGY

Department: Electronics and Communications Engineering

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Course Title: Advanced Engineering Mathematics Fall 2014

Course No.: EC760 Problem Set #5

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2D Finite Difference Method

1) A 2D square potential box with $W = H = 2\mu m$, have a grid with 2x2 internal nodes, with the following boundary conditions for potentials:

   $V(0, y) = V(W, y) = 10V$

   $V(x, 0) = V(x, H) = 0V$

   If the potential inside the box follows a 2D Laplace equation, plot the 2D grid, and start from all zeros initial solution for internal nodes, then:

   a) Find the potentials at the internal grid nodes, using Standard *Liebmann's Method* (i.e. $\boldsymbol{\lambda = 1}$), after 3 iterations

   b) Repeat part a, if the *Overrelaxation Factor* λ is equal to 1.5

   c) Solve directly using LU factorization

   d) Compare the final results for the three previous methods for all internal nodes in a table.

1D Finite Difference Method

2) Assume a 1D rod with length L=10μm follows the following Poisson's Equation:

$$\frac{d^2V}{dx^2} = -2\ V/\mu m$$

with the following boundary conditions:

   $V(0) = -1V, \quad V(L) = 2V$

   a) If the rod is divided into two internal nodes, find the potentials at the internal grid nodes, using Standard *Liebmann's Method* (i.e. $\boldsymbol{\lambda = 1}$), after 3 iterations

   b) Repeat part a, if the *Over-relaxation Factor* λ is equal to 1.5

   c) Solve directly using LU factorization

   d) Compare the final results for the three previous methods for all internal nodes in a table.

1D Finite Element Method

3) If the rod in Question 2 is divided into 3 finite elements of equal length the rod, assume that:

$$Electric\ Field = \frac{dV(x)}{dx} = continuous\ at\ Element\ Boundaries$$

a) Find the Linear Interpolation functions for each element describing the potential distribution within the element as a function of its local boundary values

b) Use *Galerkin's Method* to express the residues in each of these elements

c) Write down the *Local Matrix* equations for these elements

d) Assemble the *Global Matrix* equations

e) Find the potential at the center of each of these elements

f) Compare the potentials at the elements boundaries with those obtained from Question 2 using 1D Finite Difference Method

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2D Finite Element Method

4) For Problem 1):

a) Redraw the grid after labeling for 2D square Finite Elements Method

b) Write down the Linear Interpolation Functions for any of these elements describing the potential distribution within the element as a function of its local boundary values

c) If one of these elements has the following local boundary conditions, as function of its local x-y coordinates:

$$V(0{,}0) = 0V, \qquad V(0, w) = 1V, \qquad V(0, l) = 0.5V, \qquad V(w, l) = 0.8V$$

Where l and w are the size of the element in the x and y local coordinates, respectively.

Find the voltage in the center of this element
