Basic Circuit Theorems, Laws and Techniques

Dr. : Mohamed Hassan Abd El-azeem

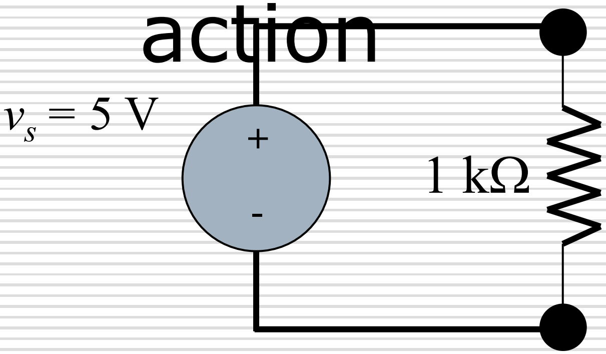
Lecture 2

Contents:

1. **Part 1 : Ohm's Law**
   Kirchhoff's Laws
   Voltage Divider
   Current Divider
2. **Part 2 : Source Transformation**

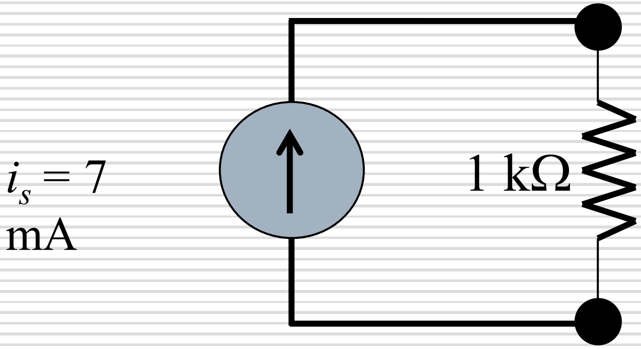
2.1 Ohm's Law

- Current and voltage sources in action



- Compute current across R

$$i = \frac{v}{R} = \frac{5V}{10^3\Omega} = 5mA$$

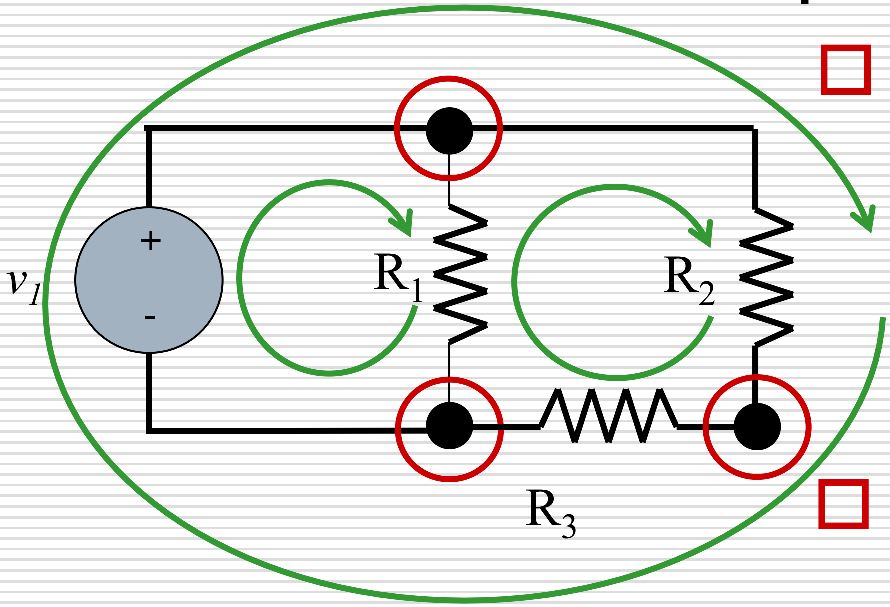


- Compute voltage across R

$$V = iR = 7mA \cdot 1k\Omega = 7V$$

2.2 Kirchhoff's Laws

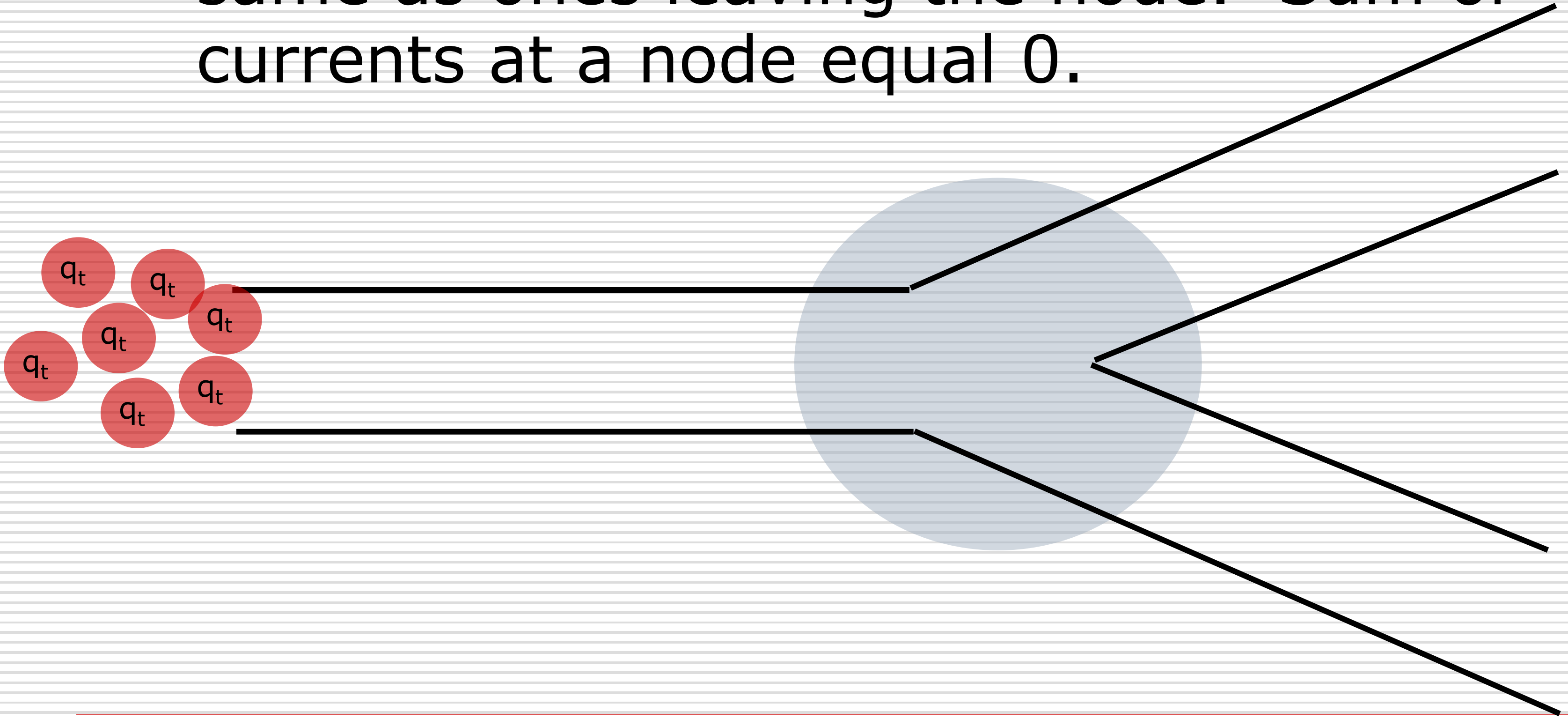
- Nodes and loops



- Nodes are points were 2 or more elements meet
- Loops are closed paths around the circuit starting and ending at a node

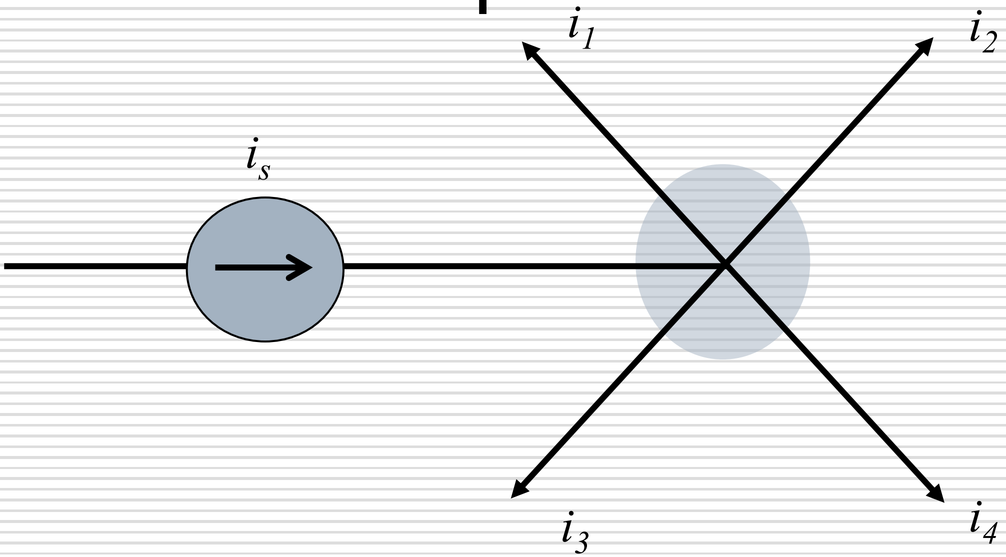
2.2.1 Kirchhoff's Current Law

- Charges entering a node are exactly the same as ones leaving the node. Sum of currents at a node equal 0.



2.2.1 Kirchhoff's Current Law

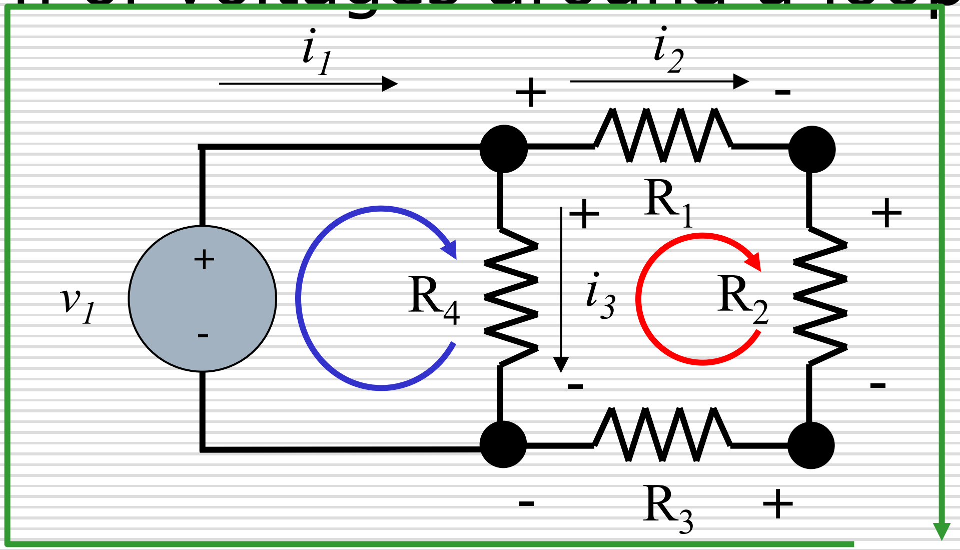
- Circuit Example



$$i_s = i_1 + i_2 + i_3 + i_4 \qquad i_s - i_1 - i_2 - i_3 - i_4 = 0$$

2.2.2 Kirchhoff's Voltage Law

- Sum of voltages around a loop equal 0



Loop 1: $-v_1 + R_1 i_2 + R_2 i_2 + R_3 i_2 = 0$

Loop 2: $-v_1 + R_4 i_3 = 0$

Loop 3: $-R_4 i_3 + R_1 i_2 + R_2 i_2 + R_3 i_2 = 0$

2.3 Kirchhoff's Laws Exercise

- Compute i_1, i_2, & i_3, & p expended for all resistors. Given $R_1 = R_2 = R_3 = 10k\Omega$, $R_4 = 15k\Omega$, & $v_1 = 15V$

Loop Equations

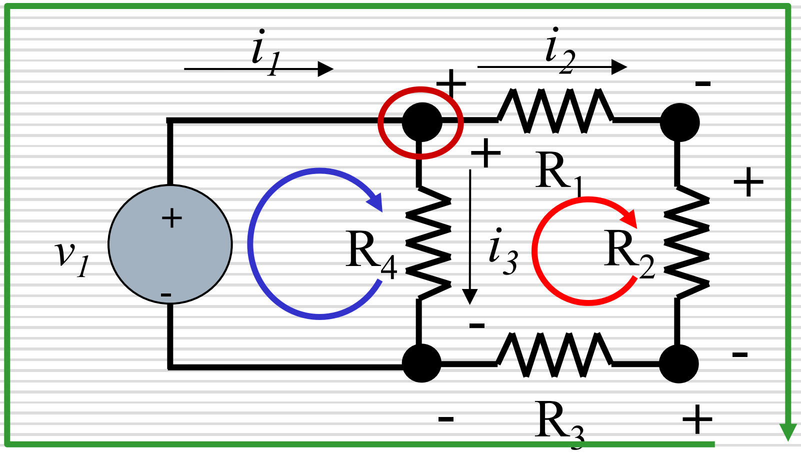
$$-v_1 + i_2(R_1 + R_2 + R_3) = 0$$

$$-v_1 + R_4 i_3 = 0$$

$$-R_4 i_3 + R_1 i_2 + R_2 i_2 + R_3 i_2 = 0$$

Node Equation(s)

$$i_1 - i_2 - i_3 = 0$$



$$-v_1 + R_4 i_3 = 0 \Rightarrow -15 + 15e3\, i_3 = 0 \Rightarrow i_3 = 1mA$$

$$-v_1 + i_2(R_1 + R_2 + R_3) = 0 \Rightarrow -15 + i_2(30e3) = 0 \Rightarrow i_2 = 0.5mA$$

2.3 Kirchhoff's Laws Exercise

- Given $R_1 = R_2 = R_3 = 10k\Omega$, $R_4 = 15k\Omega$, & $v_1 = 15V$

Loop Equations

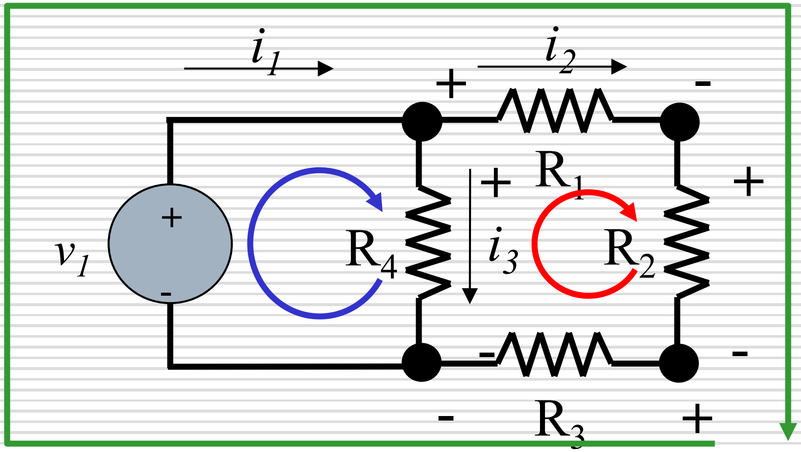
$-v_1 + i_2(R_1 + R_2 + R_3) = 0$

$-v_1 + R_4 i_3 = 0$

$- R_4 i_3 + R_1 i_2 + R_2 i_2 + R_3 i_2 = 0$

Node Equation(s)

$i_1 - i_2 - i_3 = 0$



$i_2 = 0.5mA \quad i_3 = 1mA \quad i_1 - i_2 - i_3 = 0 \Rightarrow \quad i_1 = 1.5mA$

$p_{R1} = p_{R2} = p_{R3} = (i_2)^2 R = (0.5mA \times 0.5mA) 10e3 = 2.5mW$

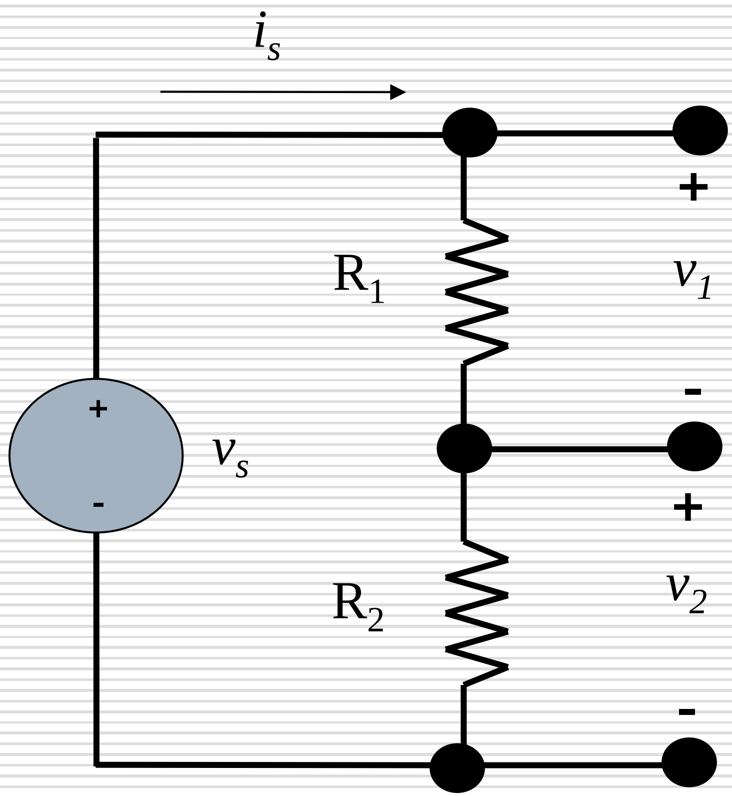
$p_{R1} = (i_3)^2 R = (1mA)^2 15e3 = 15mW$

2.4 Voltage Divider

$$i_s = \frac{v_s}{R_1 + R_2}$$

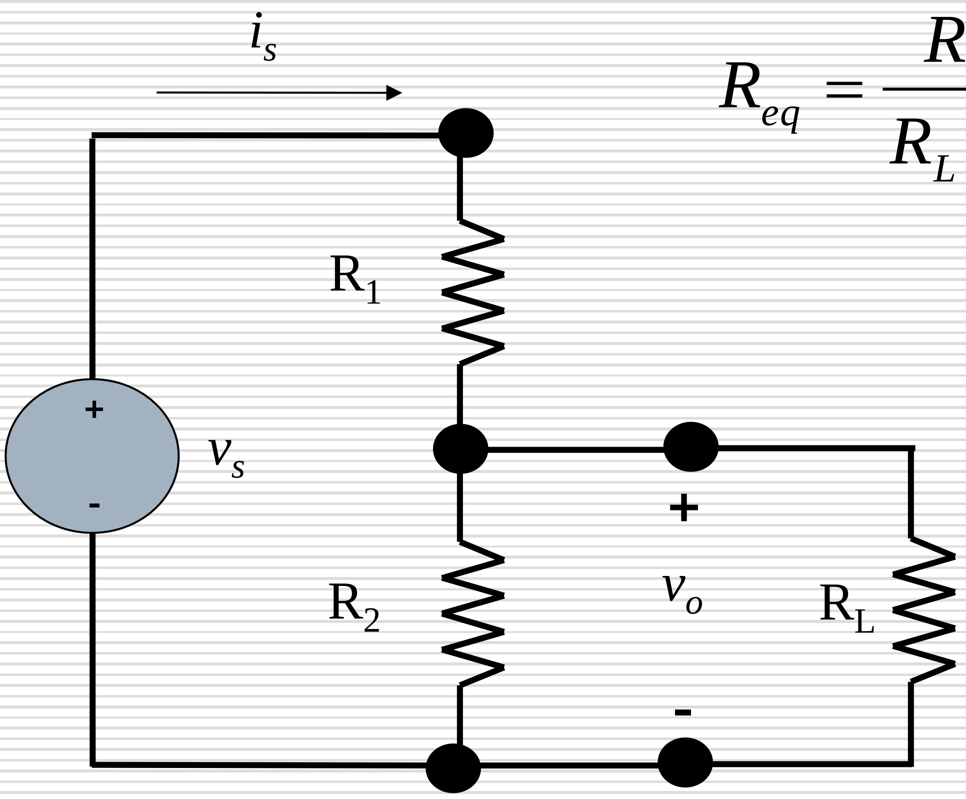
$$v_1 = \frac{R_1}{R_1 + R_2} v_s$$

$$v_2 = \frac{R_2}{R_1 + R_2} v_s$$



Voltage Divider

- Find v_o in terms of v_s



$$R_{eq} = \frac{R_L R_2}{R_L + R_2}$$

$$V_o = \frac{R_{eq}}{R_1 + R_{eq}} V_s$$

$$V_o = \frac{\dfrac{R_L R_2}{R_L + R_2}}{R_1 + \dfrac{R_L R_2}{R_L + R_2}} V_s$$

$$= \frac{R_L R_2}{R_1 \left(R_L + R_2\right) + R_L R_2} V_s$$

2.5 Current Divider

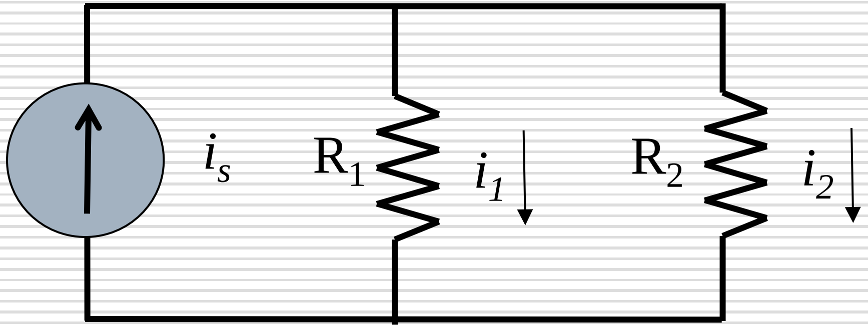
- Find i_1 and i_2 in terms of i_s

$$i_s = i_1 + i_2 \qquad i_1 R_1 = i_2 R_2$$

$$i_2 = i_1 \frac{R_1}{R_2} \Rightarrow i_s = i_1 + i_1 \frac{R_1}{R_2} = i_1 \left(\frac{R_2 + R_1}{R_2} \right)$$

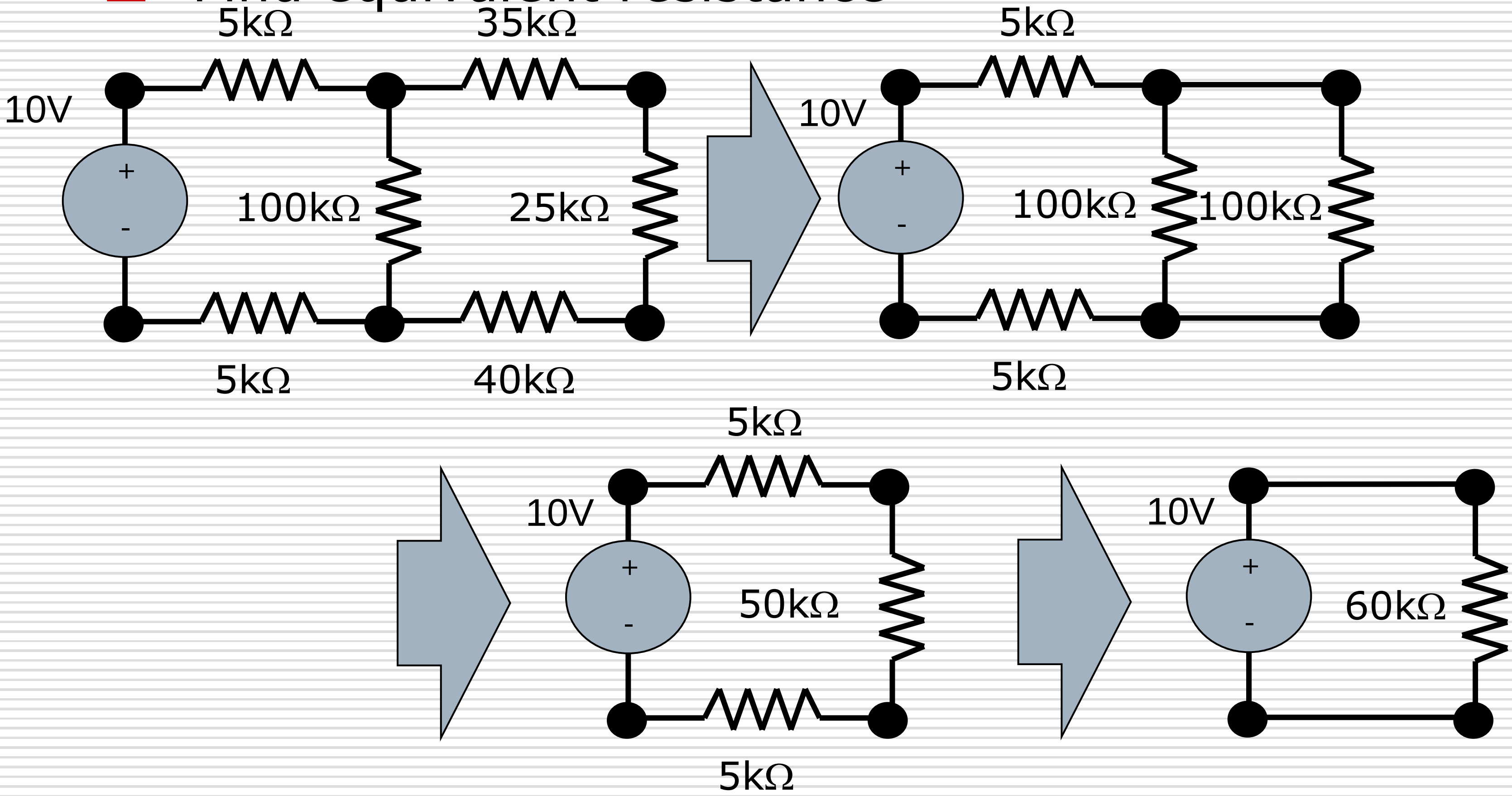
$$i_1 = i_s \left(\frac{R_2}{R_1 + R_2} \right)$$

$$i_2 = i_s \left(\frac{R_1}{R_1 + R_2} \right)$$



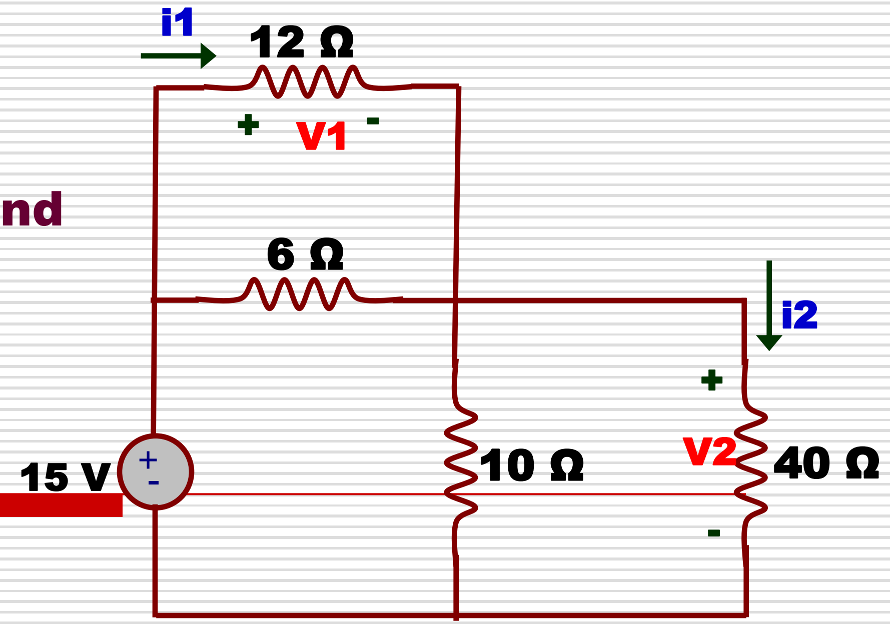
Note: Resistors in Series & Parallel

- Find equivalent resistance



Example 2-1

Find v1 and v2 in the circuit shown in Fig., Also calculate i1 and i2 and the power dissipated in the 12-Ω and 40-Ω resistors.

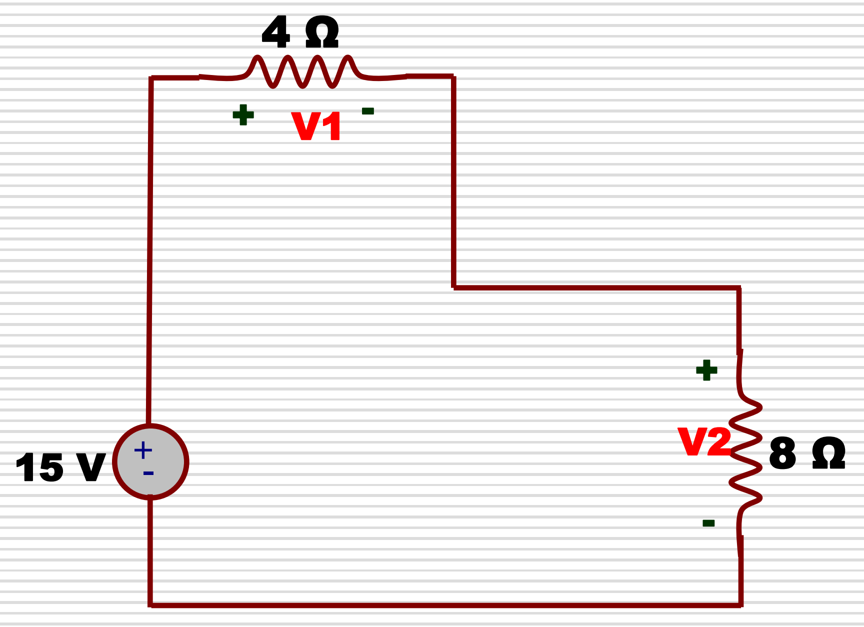


Solution:-

$$12 \parallel 6 = \frac{12*6}{12+6} = 4\ \Omega$$

$$40 \parallel 10 = \frac{40*10}{40+10} = 8\ \Omega$$

$$V_1 = 15\ \frac{4}{4+8} = 5\ V \qquad V_2 = 15\ \frac{8}{4+8} = 10\ V$$



$$i_1 = \frac{V_1}{12\ \Omega} = \frac{5\ V}{12\ \Omega} = 416.7\ mA$$

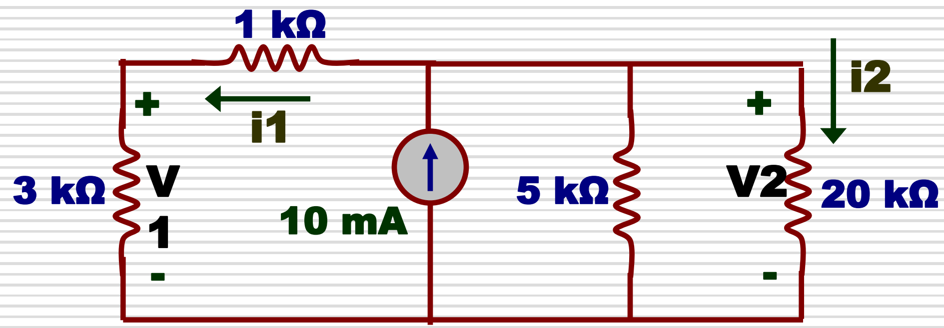
$$i_2 = \frac{V_2}{40\ \Omega} = \frac{10\ V}{40\ \Omega} = 250\ mA$$

$$P_{12\Omega} = (i_1)^2\ 12 = \frac{(V_1)^2}{12} = 2.083\ W$$

$$P_{40\Omega} = (i_2)^2\ 40 = \frac{(V_2)^2}{40} = 2.5\ W$$

Example 2.2

For the circuit shown in Fig., find:
(a) *v1* and *v2*, (b) the power dissipated in the 3-k and 20-k resistors, and
(c) the power supplied by the current source.



Solution:-

$$i_1 = i_S \frac{R_{eq}}{4\ k\Omega} \qquad i_2 = i_S \frac{R_{eq}}{20\ k\Omega} \qquad \frac{1}{R_{eq}} = \frac{1}{20\ k\Omega} + \frac{1}{5\ k\Omega} + \frac{1}{4\ k\Omega}$$

$$R_{eq} = 2\ k\Omega \longrightarrow i_1 = 10\ mA \frac{2\ k\Omega}{4\ k\Omega} = 5\ mA \qquad i_2 = 10\ mA \frac{2\ k\Omega}{20\ k\Omega} = 1\ mA$$

$$V_1 = i_1\ 3\ k\Omega = 15\ V \qquad V_2 = i_2\ 20\ k\Omega = 20\ V$$

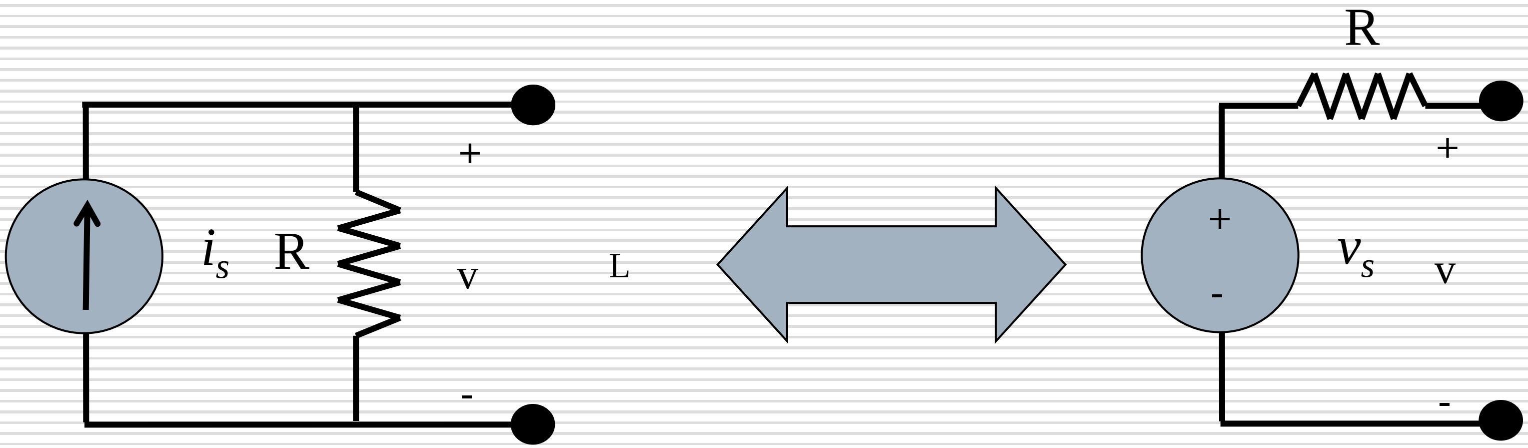
$$P_{3\text{-}k} = (i_1)^2\ 3\ k\Omega = \frac{(V_1)^2}{3\ k\Omega} = 75\ mW \qquad P_{20\text{-}k} = (i_2)^2\ 20\ k\Omega = \frac{(V_2)^2}{20\ k\Omega} = 20\ mW$$

$$P_{10\text{-}mA} = -\ i_S * V_2 = -\ 10\ mA * 20\ V = -\ 200\ mW$$

2.6 Source Transformation

- We can convert a voltage source with series resistor to a current source with parallel resistor.

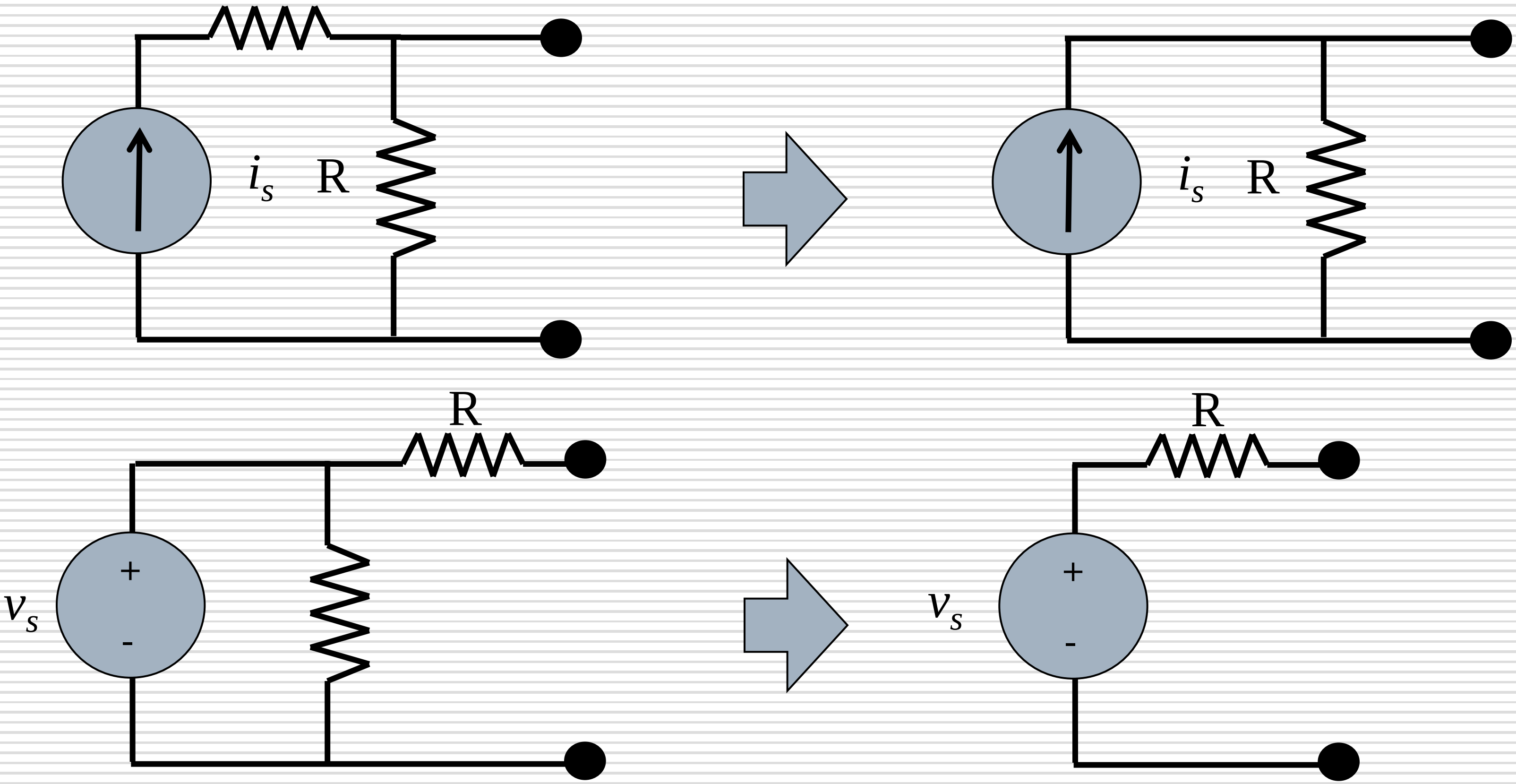
If we place a load then the voltage across the load should be the same



$$v_s = i_s R$$

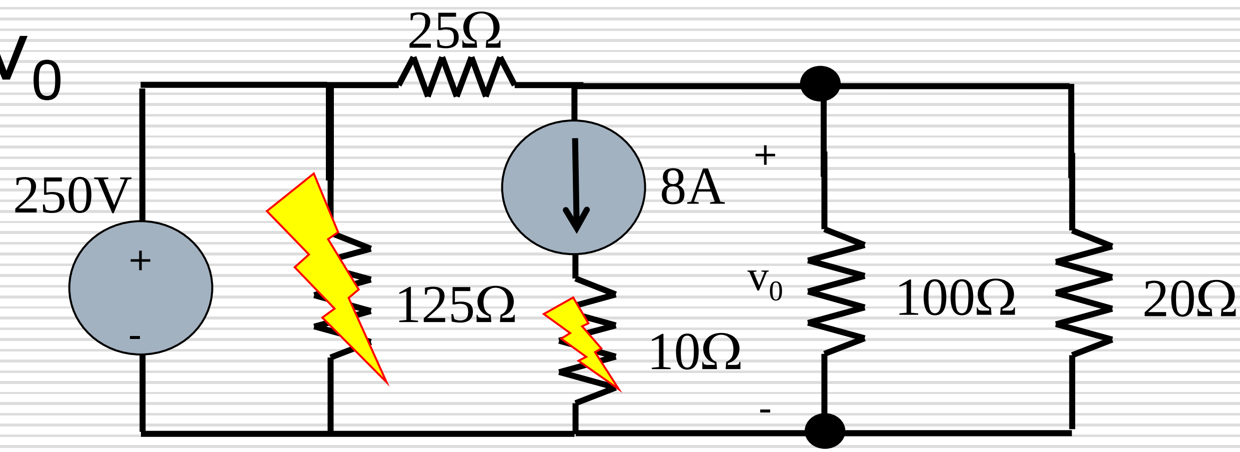
Source Transformation

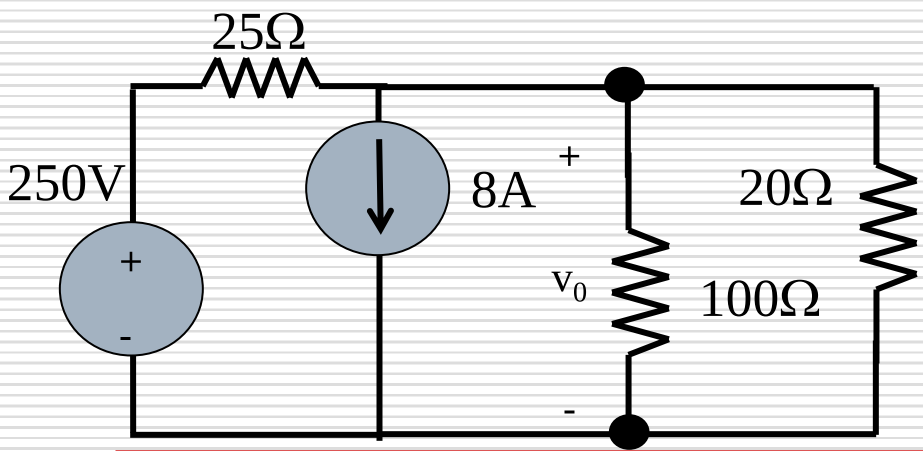
- More simplications

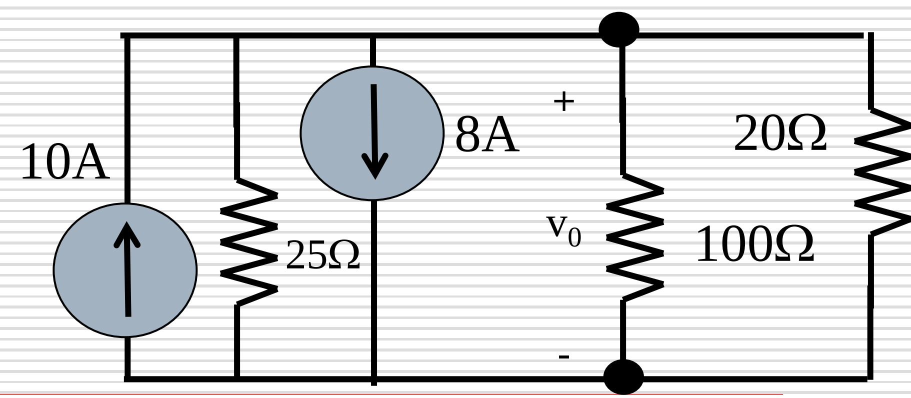


Source Transformation Example

- Find v_0

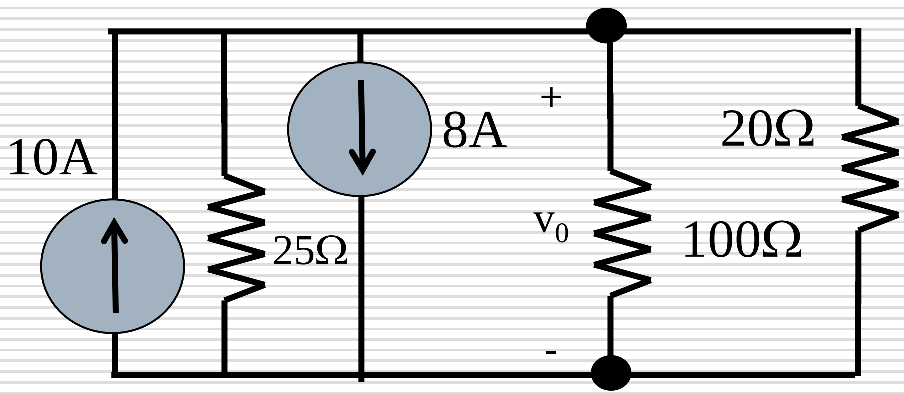


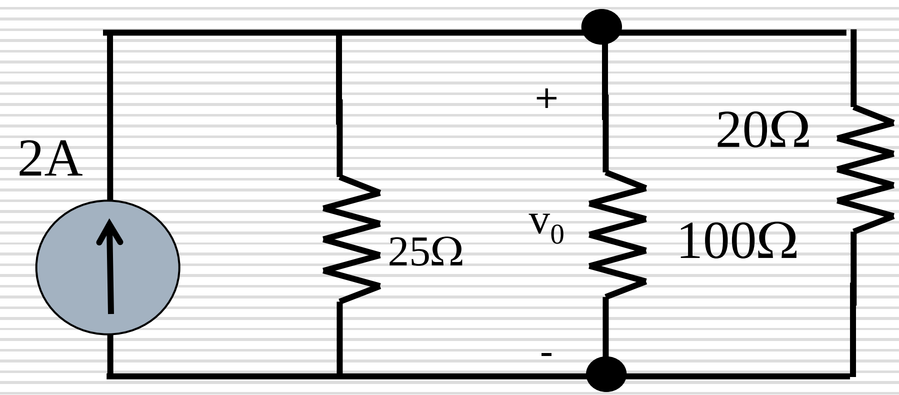


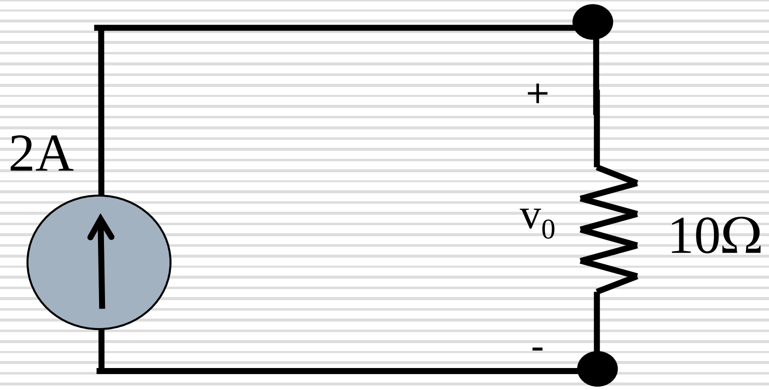


2.7 Source Transformation

Example : Find V_0



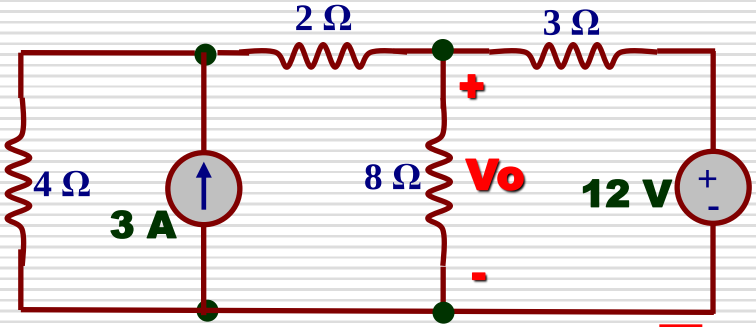




$v_0 = 20V$

Example 2-3

In the circuit shown in Fig., use the ST to find vO



Solution

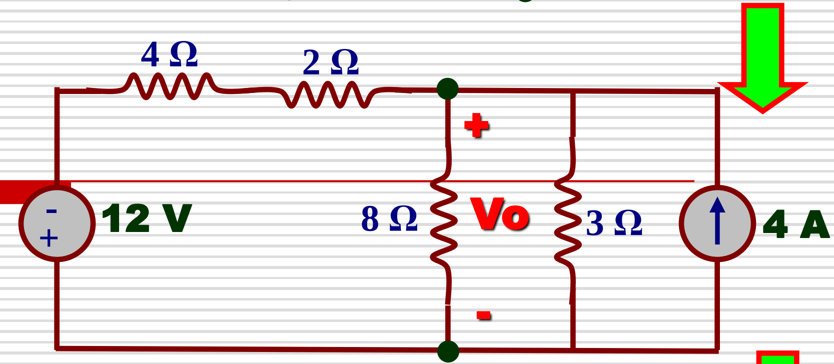
3 A, 4-Ω (‖) ⟶ 12 V, 4-Ω (Series)

12 V, 3-Ω (Series) ⟶ 4 A, 3-Ω (‖)

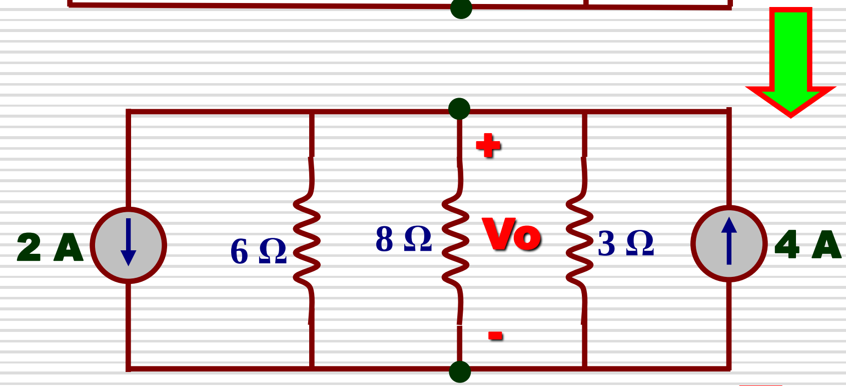
4 Ω + 2 Ω = 6 Ω

12 V, 6-Ω (Series) ⟶ 2 A, 6-Ω (‖)

6 Ω ‖ 3 Ω = 6 * 3 / (6+3) = 2 Ω

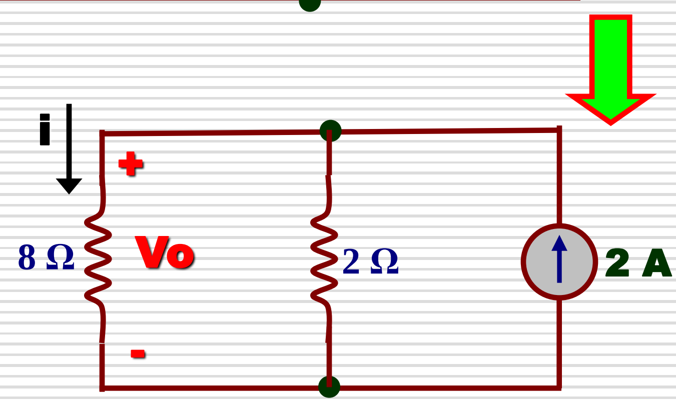


Also, current sources in parallel can be (combined) added together according to the current direction, in our case here



4 A (up) + 2 A (down) ⟶ 2 A (up)

$$i = \frac{2}{2+8}(2\ A) = 0.4\ A$$

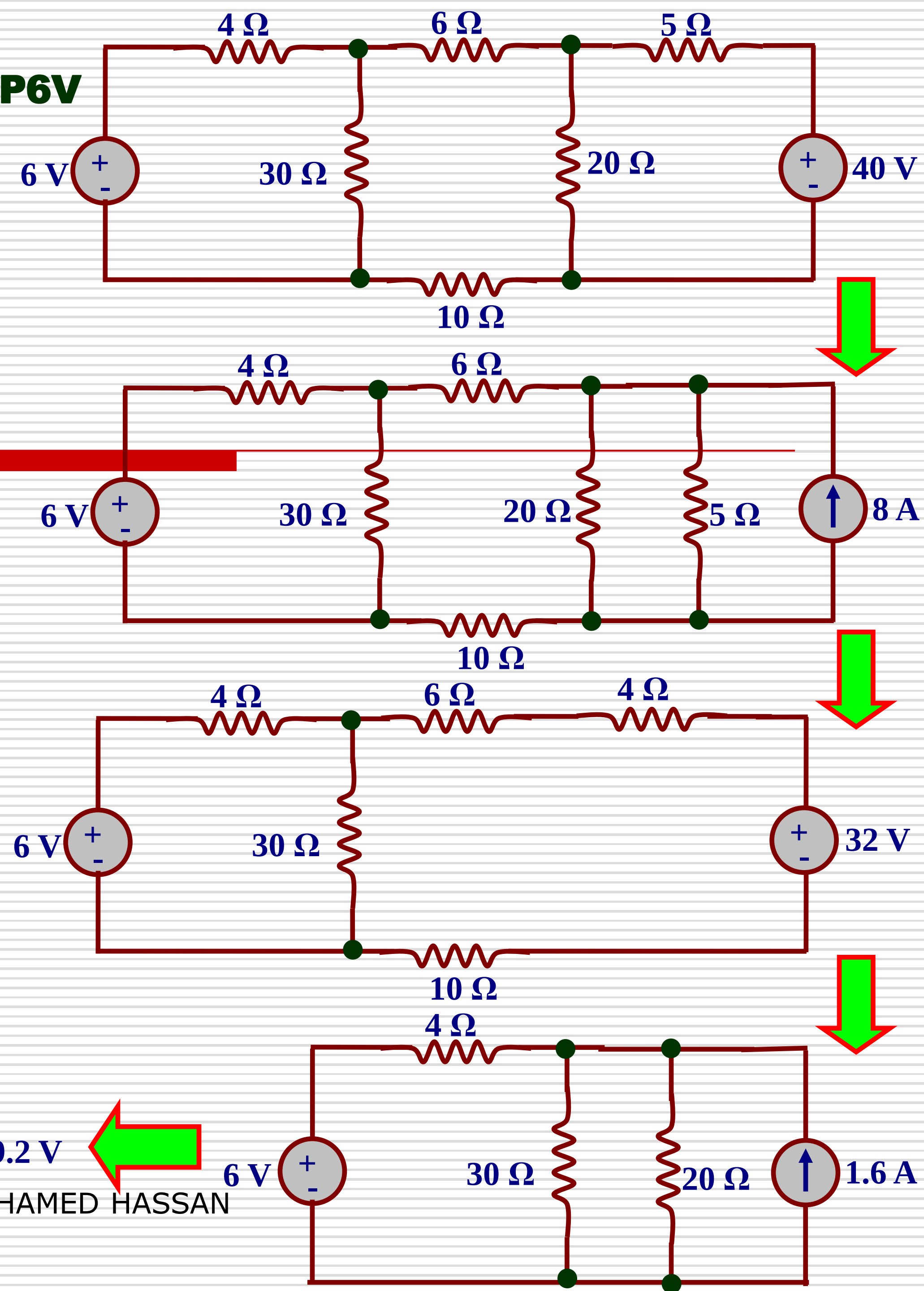


vO = 8 * i = 8 * 0.4 = 3.2 V

Example 2.4

In the circuit shown in Fig., calculate P6V

Solution



40 V, 5-Ω (Series) → 8 A, 5-Ω (∥)

5 Ω ∥ 20 Ω = 5 * 20 / (5+20) = 4 Ω

8 A, 4-Ω (∥) → 32 V, 4-Ω (Series)

6 Ω + 4 Ω + 10 Ω = 20 Ω

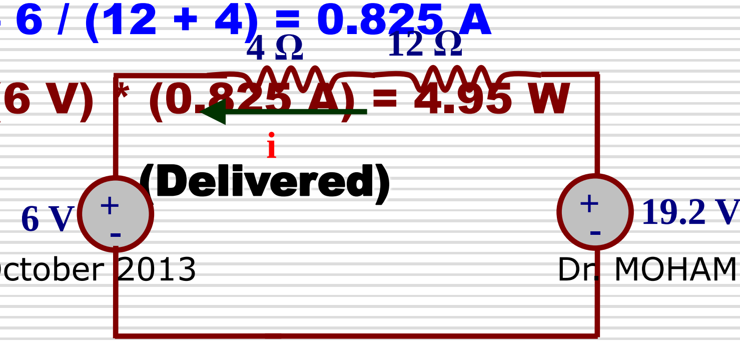
32 V, 20-Ω (Series) → 1.6 A, 20-Ω (∥)

30 Ω ∥ 20 Ω = 30 * 20 / (30+20) = 12 Ω

1.6 A, 12-Ω (∥) → 19.2 V, 12-Ω (Series)

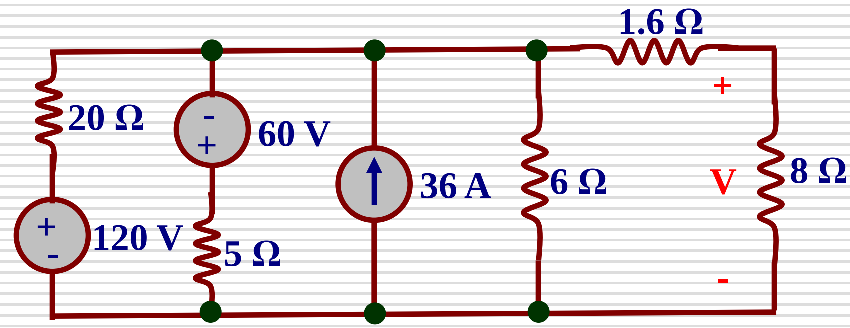
i = 19.2 – 6 / (12 + 4) = 0.825 A

P6V = + (6 V) * (0.825 A) = 4.95 W (Delivered)



Example 2.5

In the circuit shown in Fig., use the ST to calculate (a) V and (b) P120V

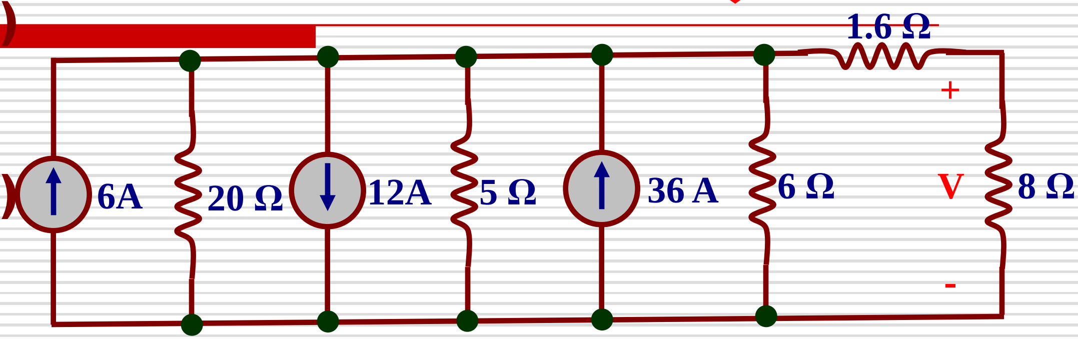


Solution

(a)

120 V, 20-Ω (Series)

→ **6 A, 20-Ω (‖)**



60 V, 5-Ω (Series)

→ **12 A, 5-Ω (‖)**

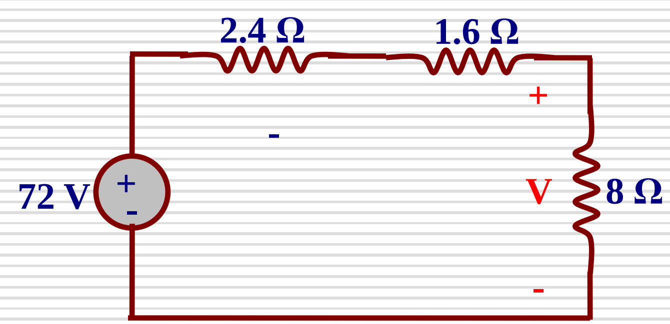
20 Ω ‖ 5 Ω ‖ 6 Ω = 2.4 Ω

6A (up) + 12A (down) + 36A (up)

→ **30 (up)**

30 A, 2.4-Ω (‖)

→ **72 V, 2.4-Ω (Series)**



$$V = \frac{72}{2.4 + 1.6 + 8}(8) = 48\ V$$

(b)

60 V, 5-Ω (Series)

→ **12 A, 5-Ω (║)**

5 Ω ║ 6 Ω ║ (8 Ω + 1.6 Ω) = 2.124 Ω

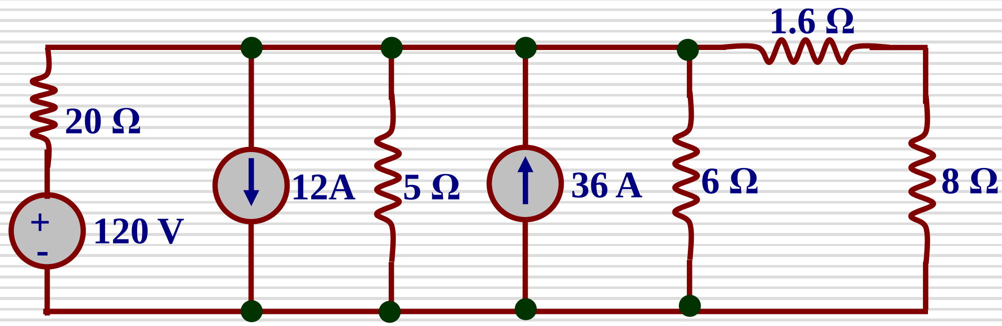
36A (up) + 12A (down) = 24 (up)

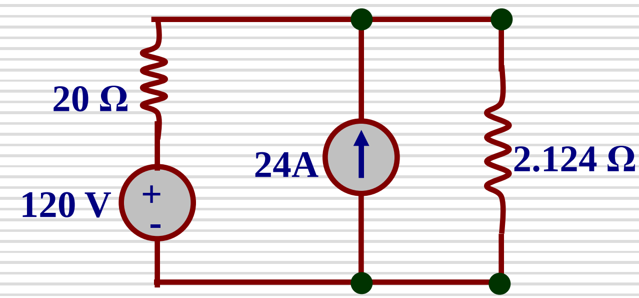
24 A, 2.124-Ω (║)

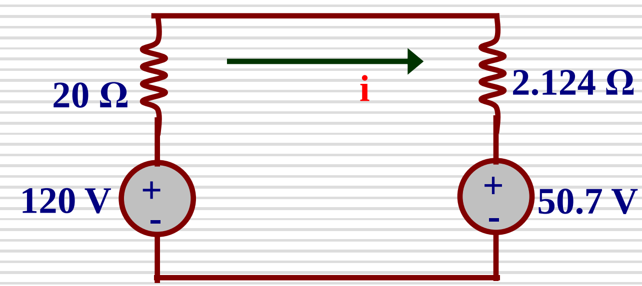
→ **50.7 V, 2.124-Ω (Series)**

$$i = \frac{120 - 50.7}{20 + 2.124} = 3.12\ A$$

P120V = - 120 (3.12) = 374.4 W (Delivered)

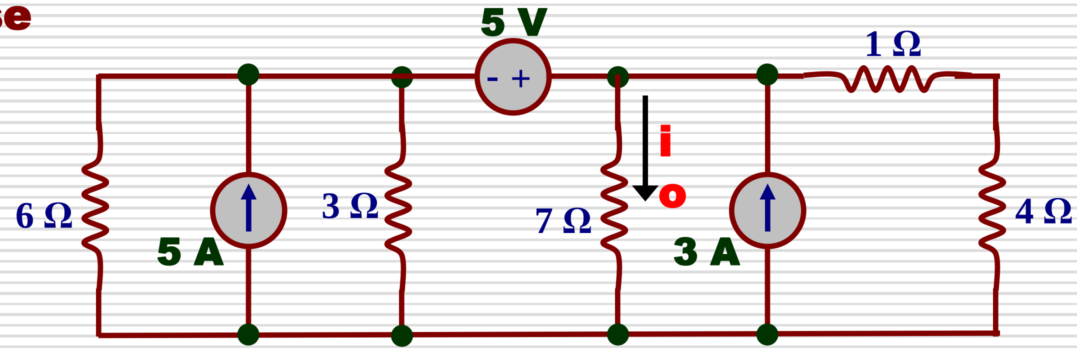






Example 2.6

In the circuit shown in Fig., use the ST to calculate iO



Answer

iO = 1.78 A