# CHAPTER 2: DISCRETE-TIME SIGNALS AND SYSTEMS

## CHAPTER OBJECTIVES

**Upon a successful completion of this chapter, the student will be able to:**

- **Understand the properties and characterization** of **digital signals and systems.**
- **Understand the interrelations** between **CTS and DTS**.

## CHAPTER OUTLINES

### Lecture # 2

2.1 SIGNAL CLASSIFICATIONS.

2.1.1 CONTINUOUS TIME SIGNAL VERSUS DISCRETE TIME SIGNAL.

2.1.2 DIGITAL SIGNALS.

(a) Digital Signal Definition.

(b) Some Important Digital Signals.

(c) Operations on Digital Signals.

(d) Spectrum of Digital Signals.

(e) Even and Odd Digital Signals.



**2.1.3 DISCRETE SYSTEM OR DIGITAL SYSTEM.**

(a) DIGITAL SYSTEM REPRESENTATION.

(b) DIGITAL SYSTEM ANALYSIS.

(c) DIGITAL SYSTEM DESIGN.

(d) DIGITAL FILTER.

### LECTURE # 3

**2.2 INTERCONNECTIONS OF SYSTEMS**

**2.3 CHARACTERIZATION OF DIGITAL SYSTEM**

- DIFFERENCE EQUATION.
- THE IMPULSE RESPONSE.
- TRANSFER FUNCTION OF DIGITAL SYSTEM.
- THE FREQUENCY RESPONSE.

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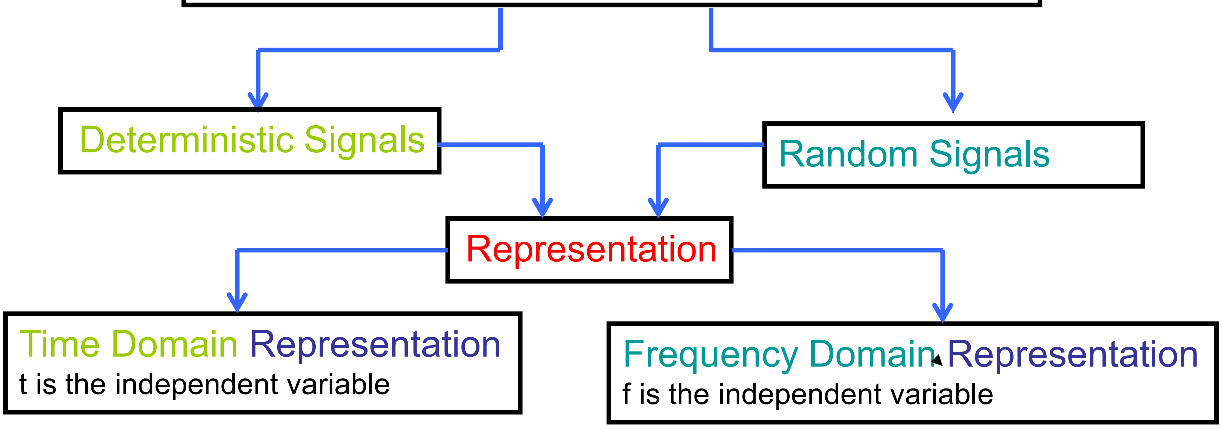
# 2.1 Signal Classifications

Signal is a representation of physical quantity or phenomenon

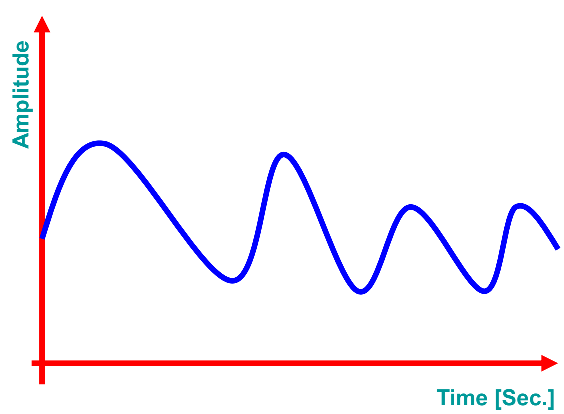





# EXAMPLES

## 1- Random Signal

Recording of **speech signals** which represents the **acoustic variation as a function of time** for a spoken word.

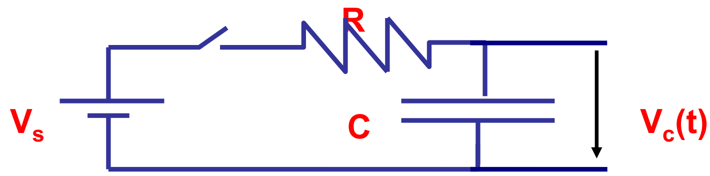


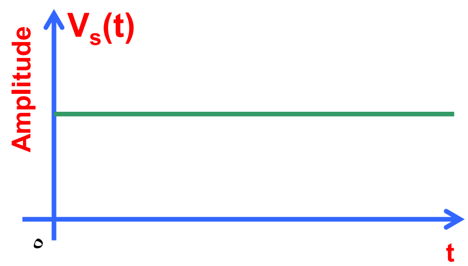
# EXAMPLES (Contd.)

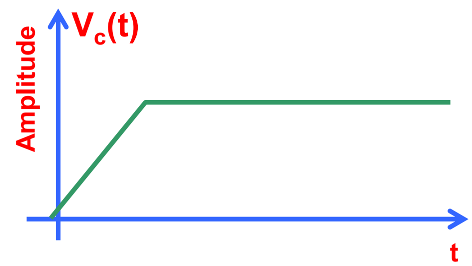
## 2- Determinstic Signal

**A-** The variation of **applied forces F** and the resulting **velocity** or **acceleration** over the time.

**B-** Voltage on RC circuit as shown in the following Figure






# Signals

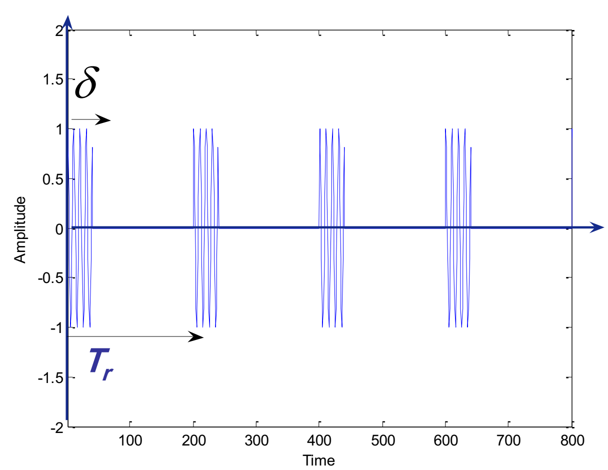
- **Mathematically** are functions of **one** or **more** variables.
- **Most of them** are functions of **one variable (t)**.

## Example

### Radar signal waveform

$$s(t)=\begin{cases} A\cos(w_c t) & 0\le t\le \delta \\ 0 & \delta \le t \le T_r \end{cases}$$

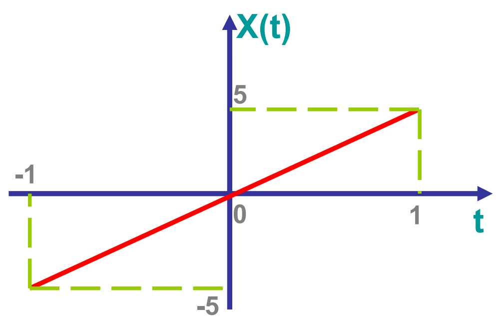
Is **periodic** with period $T_r$



## 2.1.1 CONTINUOUS VERSUS DISCRETE TIME SIGNAL

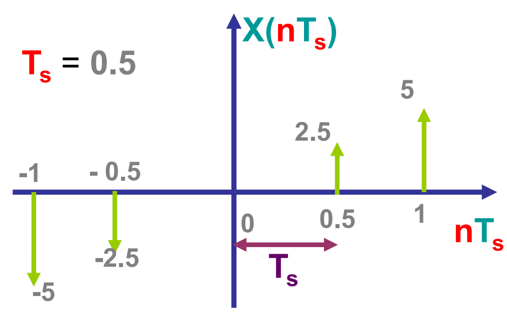
**Continuous Time Signal (CTS)**

- the **independent variable t** is **continuous**.
- the **signal values** are defined **for all t** in in the interval of interest.
- notation **x(t)**.
- represents the **phenomenon.**




**Discrete Time Signal (DTS)**

- the **independent variable t** is **discrete**.
- it takes only a discrete values **n**.
- **n** is an integer.
- notation **$x(nT_s)$, $T_s$** is the sampling time.
- it results from **sampling of (CTS).**




**$T_s$** is the **repetition period** (**sampling time**)



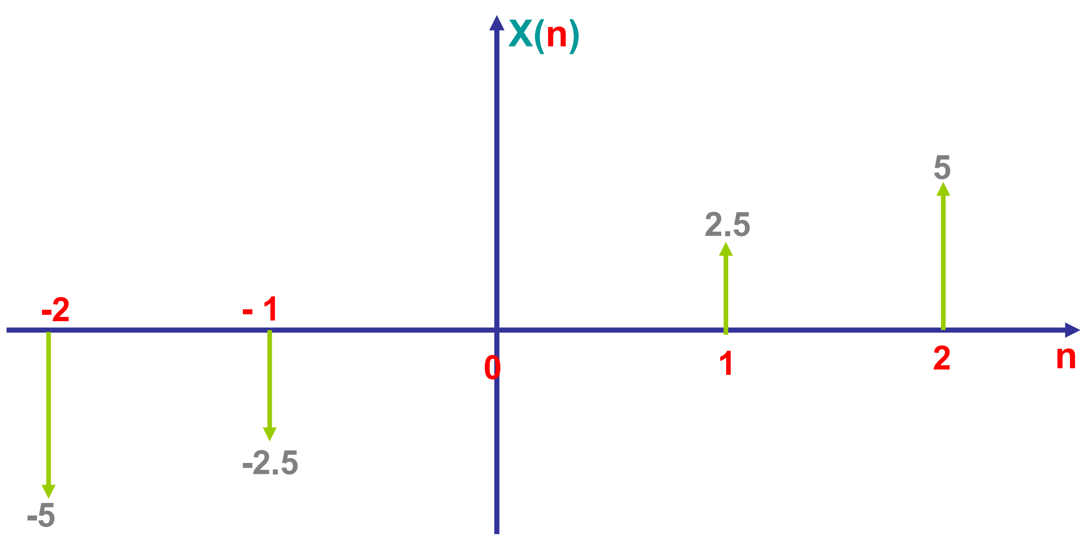
## 2.1.2 DIGITAL SIGNAL (DS)

### A. Digital Signal Definition

- **discrete time** signal is a number **sequence**.
- It is denoted by one of the following notations:

  a- **x(n)** = { **x(n)** }

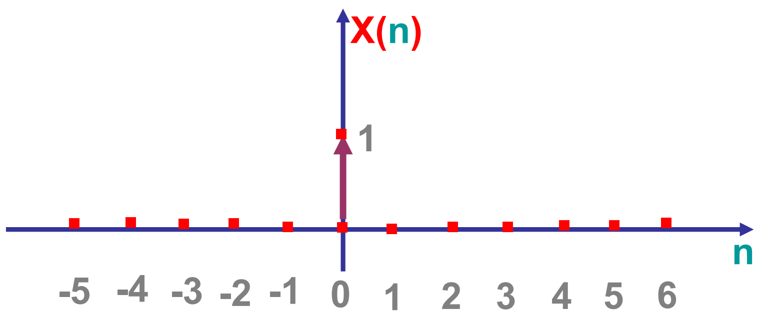
  b- **x(n)** = { …….,**x(-1), x(0), x(1), x(2),…**}



# B. SOME IMPORTANT DIGITAL SIGNALS

1. **Unit impulse** (unit sample sequence)

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

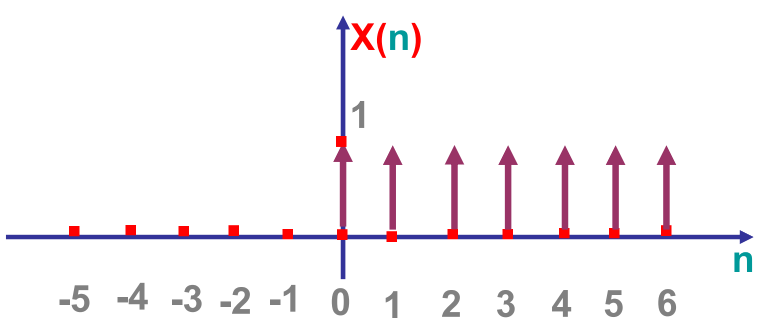





# SOME IMPORTANT DIGITAL SIGNALS (Contd.)

2. **Unit Step** (unit sample sequence)

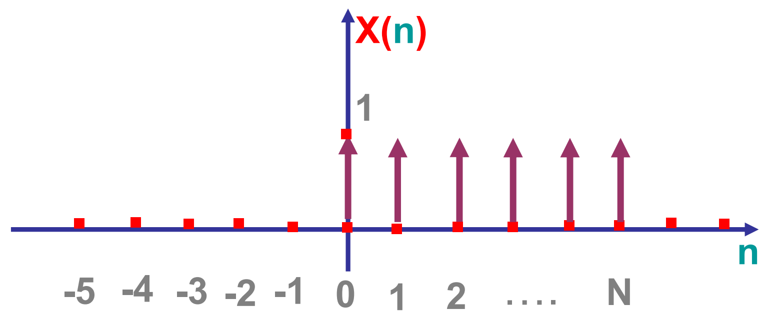
$$U(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n \prec 0 \end{cases}$$



## SOME IMPORTANT DIGITAL SIGNALS (Contd.)

### 3. Rectangular Signal

$$x(n)=\begin{cases}1 & 0\le n\le N\\ 0 & otherwise\end{cases}$$

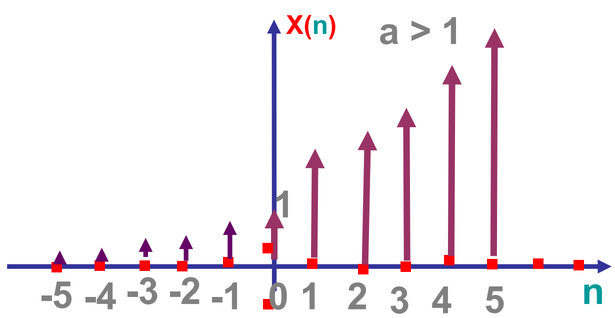


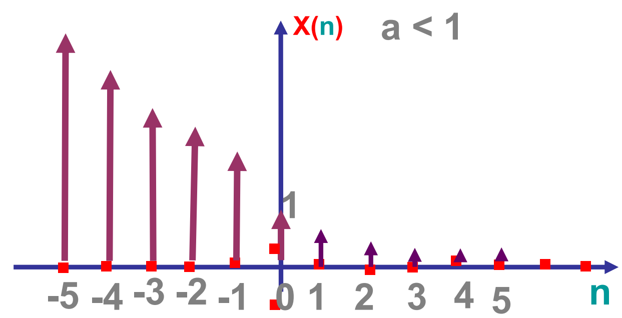



## SOME IMPORTANT DIGITAL SIGNALS (Contd.)

### 4. Real value exponential

$$x(n)=a^n \quad \forall\, a\in\Re$$

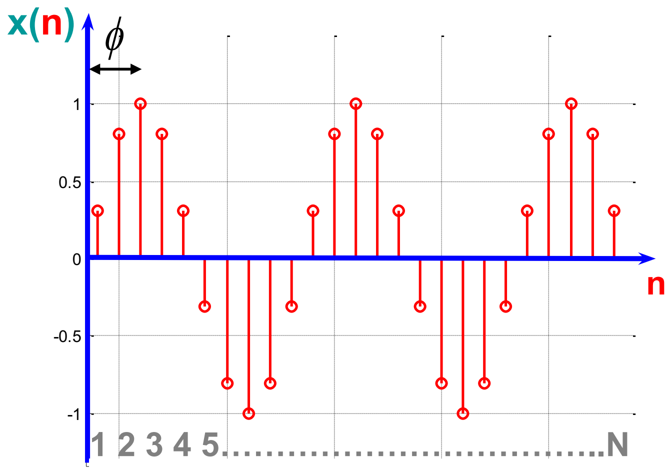


# SOME IMPORTANT DIGITAL SIGNALS (Contd.)

## 5. Sinusoidal Signal

$$x(n) = \cos(w_o n + \phi)$$

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# C. OPERATIONS ON SIGNALS

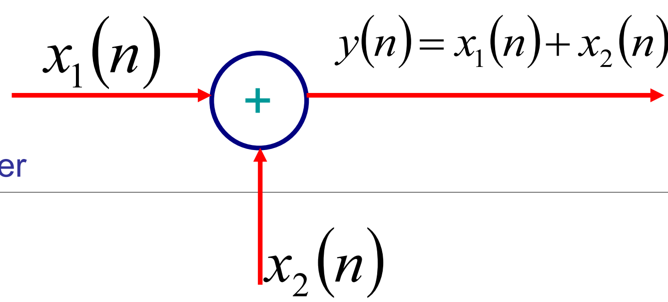
## 1. Signal Addition

- This is **sample –to-sample addition** given by :

$$y(n) = x_1(n) + x_2(n)$$

**Or**

$$\{y(n)\} = \{x_1(n) + x_2(n)\}$$




+ Adder

# OPERATIONS ON SIGNALS (Contd.)

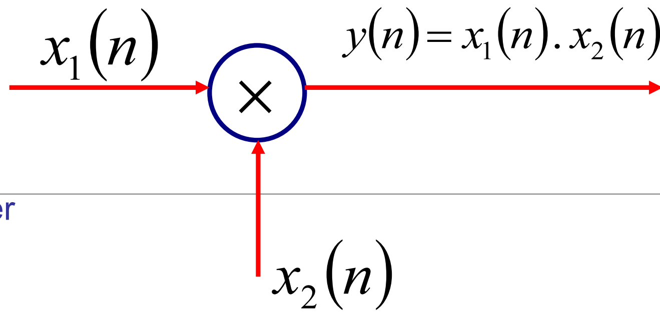
## 2. Signal Multiplication

- This is **sample –to-sample multiplication** given by :

$$y(n) = x_1(n) . x_2(n)$$

**Or**

$$\{y(n)\} = \{x_1(n) . x_2(n)\}$$




× Multiplier

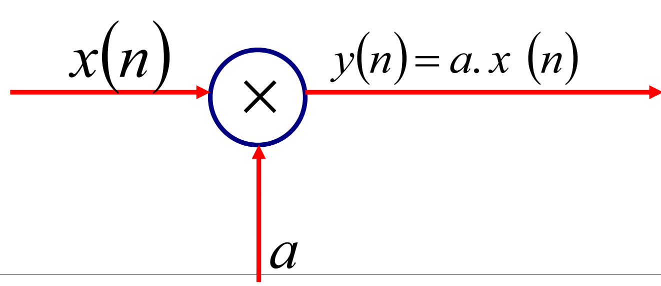


# OPERATIONS ON SIGNALS (Contd.)

## 3. Scaling

- Each sample is **multiplied** by a **scale or constant a**

$$y(n) = a\,x(n)$$

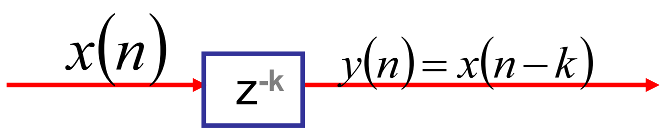


# OPERATIONS ON SIGNALS (Contd.)

## 4. Shifting or Delay

- Each sample is **delayed** by a k **samples**

$$y(n) = x(n-k)$$






# OPERATIONS ON SIGNALS (Contd.)

## 5. Sample Summation

- It adds all sample values of ***x(n)*** between $n_1$ and $n_2$

$$y(n) = \sum_{n=n_1}^{n_2} x(n) = x(n_1) + x(n_1+1) \ldots\ldots\ldots + x(n_2)$$

## 6. Sample Product

- It multiplies all sample values of ***x(n)*** between $n_1$ and $n_2$

$$y(n) = \prod_{n=n_1}^{n_2} x(n) = x(n_1) \times x(n_1+1) \times \ldots\ldots\ldots \times x(n_2)$$

## 7. Time Reversal

- The signal ***y(n) = x (-n)*** is obtained by reflecting ***x (n)*** about ***n=0.***

# D. Spectrum of Digital Signals

- For the **DTS** *x(n)*, the **Discrete Time Fourier Transform (DTFT)** is given by :

$$x\left(e^{jw}\right) = \sum_{n=-\infty}^{\infty} x(n) e^{-jwn}$$

- The Inverse Discrete Time Fourier Transform **(IDTFT)** is given by :

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} x\left(e^{jw}\right) e^{jwn} dw$$

- Details of this topics are introduced in Chapter 3

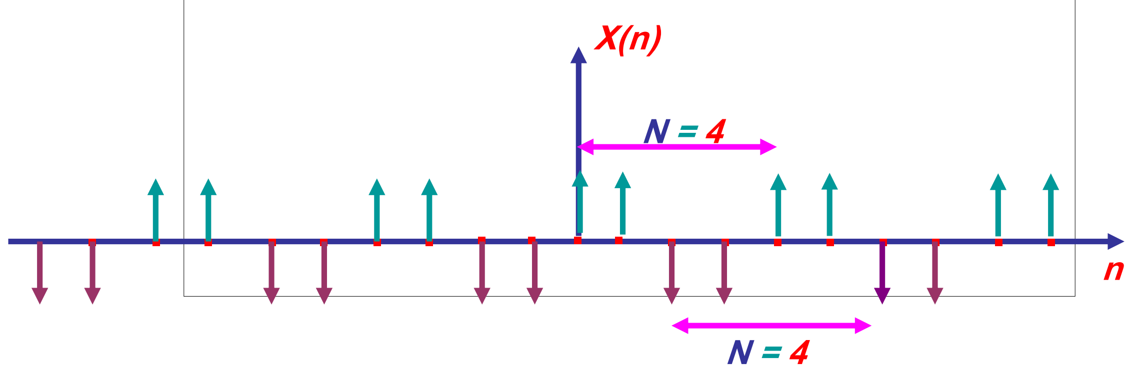


# Periodic Signals

***x(n)*** is **periodic** with period ***N***, where **N** is a **positive integer if**

$$x(n) = x(n+N)$$

***N*** is called the **fundamental period**



# F. Even and Odd Signals

**x(n)** is **even** if it is **symmetric around** **y-axis** → $x(n) = x(-n)$

**x(n)** is **odd** if it is **symmetric around** **the origin** → $x(n) = -x(-n)$

**Examples**

**1-** $x(n) = U(n)$ **neither even nor odd**

**2-** $$x(n) = \begin{cases} \frac{1}{2} & n \succ 0 \\ 1 & n = 0 \\ \frac{1}{2} & n \prec 0 \end{cases}$$ **even signal**

**3-** $$x(n) = \begin{cases} \frac{1}{2} & n \succ 0 \\ 1 & n = 0 \\ \frac{-1}{2} & n \prec 0 \end{cases}$$ **odd signal**

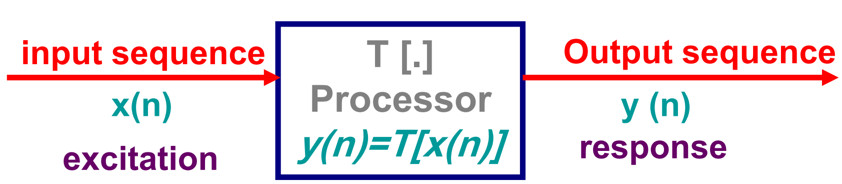


# 2.1.3 Discrete System or Digital system

## a. Representation of Discrete –Time system or discrete system

is **mathematically** described as an an **operator or transformation** ***T [.]***

- It takes an **input sequence x(n)**, or **excitation** and transform it into another sequence called the **output sequence** or the **response y (n)** .




## b. Discrete System Analysis

- It is the process of **determining the response** of that system **to a given excitation**

## 2.1.3 Discrete System or Digital system (Contd.)

### c. Discrete System Design

- It is the process of **synthesizing the system parameters** that satisfy **the input output specification.**

### d. Digital Filter

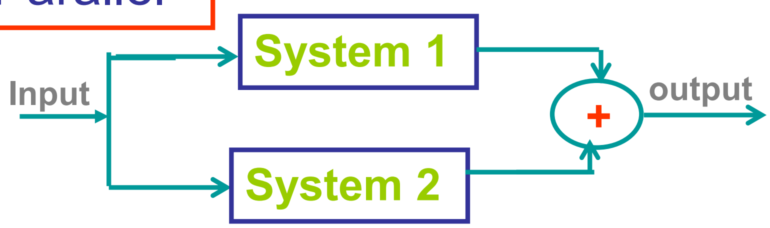
- It is a digital system **that can be used to filter discrete –time signal.**



## 2.2. INTERCONNECTIONS OF SYSTEMS

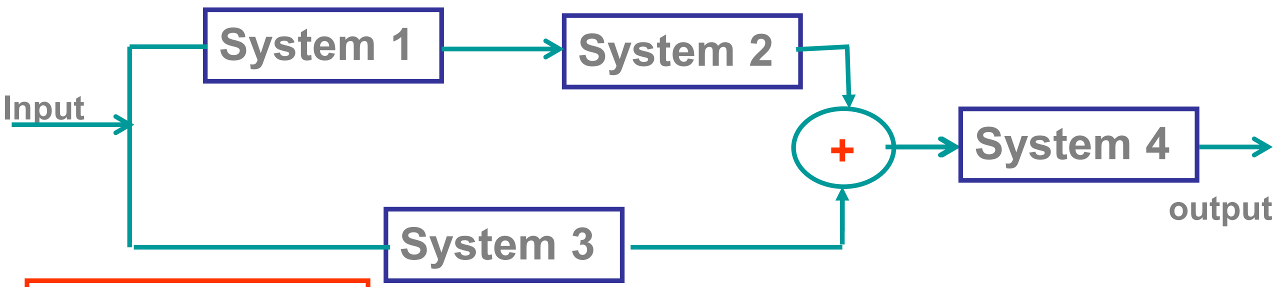
### A. Series or Cascade


### B. Parallel

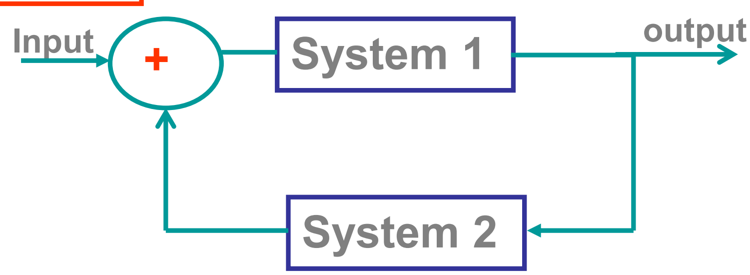


# INTERCONNECTIONS OF SYSTEMS (Contd.)

C. Series parallel




d. Feed Back






# 2.4. CHARACTERIZATION OF DIGITAL SYSTEM

- Any **digital system** can be **characterized** by one of the following:

1. **Difference Equation**

2. **Impulse Response**

3. **Transfer Function**

4. **Frequency Response**

## 1. Difference Equation

• A discrete **system** can be described by a **linear constant coefficient difference equation** of the form:

$$\sum_{i=0}^{N} a_i y(n-i) = \sum_{i=0}^{M} b_i x(n-i)$$

•This equation describes **a non-recursive approach** for computing **the output** values.

**OR**

$$y(n) = \sum_{i=0}^{M} b_i x(n-i) - \sum_{i=1}^{N} a_i y(n-i)$$

•This equation describes **a recursive approach** for computing **the output** values.



## 1. Difference Equation (Contd.)

### NON RECURSIVE DIGITAL FILTER

• This is also called Finite Impulse Response (**FIR**) filter.
• For time invariant (**FIR**), the **difference equation** has the form:

$$y(n) = \sum_{i=0}^{M} b_i x(n-i)$$

***M*** is called the **order** of the filter and

$\{b_i\}_0^M$ are the constant coefficients of the filter

## 1. Difference Equation (Contd.)

### RECURSIVE DIGITAL FILTER

- This is also called Infinite Impulse Response (**IIR**) filter

$$y(n) = \sum_{i=0}^{M} b_i x(n-i) - \sum_{i=1}^{N} a_i y(n-i)$$

***M*** is called the **order** of the **all zero.**

***N*** is called the order of the **all pole** .



## 2. The Impulse Response

If the **excitation** to the filter is the unit impulse **δ(n)**

The **response *y(n)*** is said to be the **impulse response**
i.e., the **response** to the **impulse excitation.**

## 1. For **Non Recursive (FIR)**

$$h(n) = \sum_{i=0}^{M} b_i \delta(n-i)$$

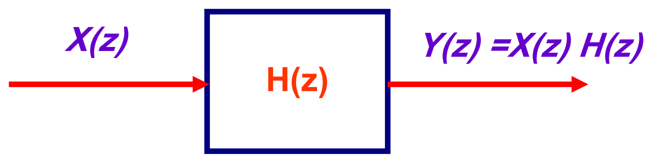

## 2. For **Recursive (IIR)**

$$h(n) = \sum_{i=0}^{M} b_i \delta(n-i) - \sum_{i=1}^{N} a_i h(n-i)$$

## 3. Transfer Function of Digital System

The **transfer function** or the **system function** ***H(z)***

$$H(z)=\frac{Y(z)}{X(z)}$$




**X(z)** and ***Y(z)*** are the Z Transform of ***x(n)*** and ***y(n)***

$$Z\{y(n)\}\Rightarrow Y(z)$$

$$Z\{x(n)\}\Rightarrow X(z)$$

$$Z\{y(n-i)\}\Rightarrow Z^{-i}\,Y(z)$$



## 4. The Frequency Response

- The **discrete-time Fourier transform** of an impulse response is called the **frequency response or the transfer function.**

$$H(e^{jw})=\sum_{-\infty}^{\infty}h(n)e^{-jwn}$$