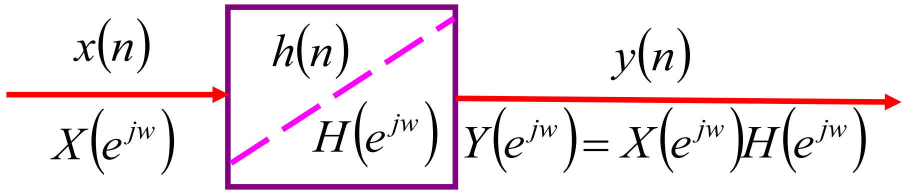
CHAPTER 4:The Z Transform

4.1 INTRODUCTION

Digital System representation using **DTFT**



The Z-transform **of a discrete –time function** ***x(n)***

$$X(z) = Z(x(n)) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

where **z** is a **complex variable**

EXAMPLE 4.1:

Find the z-transform of $x(n) = \sum_{n=0}^{\infty} a^n U(n)$

$$X(z) = \sum_{n=0}^{\infty} a^n z^{-n} = \sum_{n=0}^{\infty} (az^{-1})^n = \frac{1}{1-(az^{-1})}$$

$$= \frac{z}{z-a}$$

NOTES :

$$X(z) = \frac{z}{z-1} = \frac{B(z)}{A(z)}$$

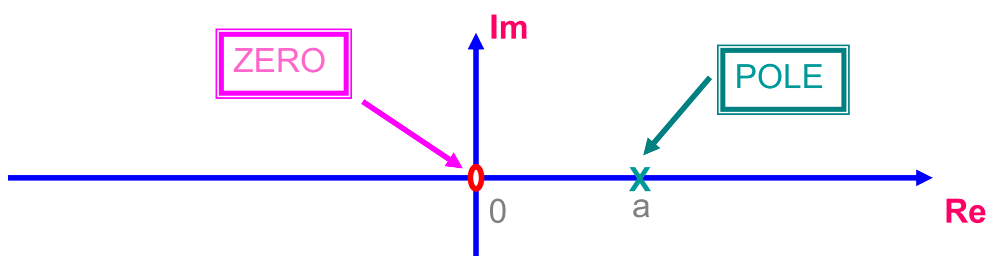
B(z) is the **numerator polynomial**

A(z) is the **denominator polynomial**

The roots of ***B(z)*** is called **the zeros of X(z)**

The roots of ***A(z)*** is called **the poles of X(z)**

- For the **previous example X(z)** has **zero at z = 0** and pole at z = a



PROPERTIES OF THE Z TRANSFORM (Contd.)

1. LINEARITY :

$$Z[a_1x_1(n) + a_2x_2(n)] = a_1X_1(z) + a_2X_2(z)$$

2. SAMPLE SHIFT :

$$Z[x(n-n_o)] = z^{-n_o}X(z)$$

3. FREQUENCY SHIFT :

$$Z[a^n x(n)] = X\left(\frac{z}{a}\right)$$

EXAMPLE 4.2 :

Let $X_1(z)=2+3z^{-1}+4z^{-2}$ and $X_2(z)=3+4z^{-1}+5z^{-2}+6z^{-3}$

Determine : $X_3(z)=X_1(z).X_2(z)$

SOLUTION :

$$X_3(z)=(2+3z^{-1}+4z^{-2})(3+4z^{-1}+5z^{-2}+6z^{-3})$$

$$=6+17z^{-1}+34z^{-2}+43z^{-3}+38z^{-4}+24z^{-5}$$

EXAMPLE 4.3 :

Let $X_1(z)=z+2+3z^{-1}$ and $X_2(z)=2z^{+3}+4z+5z^{-1}$

Determine : $X_3(z)=X_1(z).X_2(z)$

SOLUTION :

$$X_3(z)=3z^3+8z^2+17z+19z^{-1}+12z^{-2}$$

4.3 INVERSION OF THE Z TRANSFORM

The inverse z-transform of the complex function $X(z)$ is given by

$$x(n)=Z^{-1}[X(z)]=\frac{1}{2\pi j}\oint X(z)z^{n-1}dz$$

METHODS FOR OBTAINING THE INVERSE Z TRANSFORM

1- DIRECT DIVISION METHOD

2- COMPUTATIONAL METHOD

3- PARTIAL-FRACTION METHOD

4- INVERSION INTEGRAL METHOD (RESIDUE-THEOREM)

1. DIRECT DIVISION METHOD

- Express $X(z)$ into **an infinite power series in z^{-1}** .
- This method is **useful** when it is **difficult to obtain a closed form expression** for **inverse z-transform** or to find only the **first several trems of x(n).**

EXAMPLE 4.4:

Find x(k) for **k=0, 1, 2, 3, 4**, when **X(z)** is given by :

$$X(z)=\frac{10z+5}{(z-1)(z-0.2)}$$

Solution :

First, rewrite ***X(z)*** **as a ratio of polynomial in z^{-2}, as follows:**

$$X(z)=\frac{10z^{-1}+5z^{-2}}{1-1.2z^{-1}+0.2z^{-2}}$$

EXAMPLE 4.4: (Contd.)

Dividing the numerator by the denominator, we have

$$\begin{array}{r|l} & 10z^{-1}+17z^{-2}+18.4z^{-3}+18.68z^{-4} \\ \hline 1-1.2z^{-1}+0.2z^{-2} & 10z^{-1}+5z^{-2} \\ & 10z^{-1}-12z^{-2}+2z^{-3} \\ \hline & 17z^{-2}-2z^{-3} \\ & 17z^{-2}-20.4z^{-3}+3.4z^{-4} \\ \hline & 18.4z^{-3}-3.4z^{-4} \\ & 18.4z^{-3}-22.08z^{-4}+3.68z^{-5} \\ \hline & 18.68z^{-4}-3.68z^{-5} \\ & 18.68z^{-4}-22.416z^{-5}+3.736z^{-6} \end{array}$$

EXAMPLE 4.4: (Contd.)

$$X(K) = 10z^{-1} + 17z^{-2} + 18.4z^{-3} + 18.68z^{-4}$$

Solution :

$$x(0) = 0$$
$$x(1) = 10$$
$$x(2) = 17$$
$$x(3) = 18.4$$
$$x(4) = 18.68$$