Analog and Digital Signal Processing

Arab Academy For Science and Technology

Dr./ Mohamed Hassan

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Academic Year 2012/2013

Term Roadmap :

- Introduction to Signal Processing
- Differentiating and Integrating Circuits (OpAmps)
- Clipping and Clamping Circuits
- Design of analog filters
- Sinusoidal Oscillators
- Multivibrators
- Sampling and Quantization techniques of analog signals
- DACs and ADCs
- Data Acquisition Systems
- Introduction to discrete time transform and DSP
- The Z transform
- Design of Digital Filters

Text Books

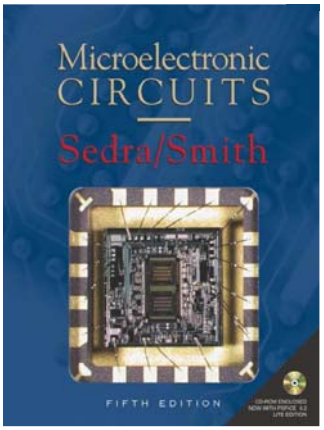
Microelectronic circuits

Adel S. Sedra

Kenneth C. Smith

Sixth Edition, copyright 2011

by Oxford University Press

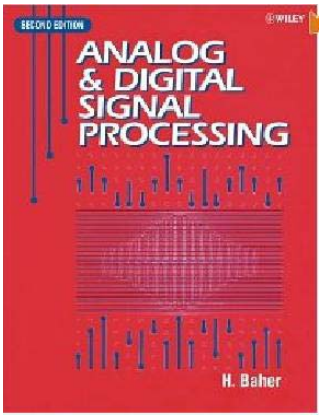


Analog and Digital Signal Processing

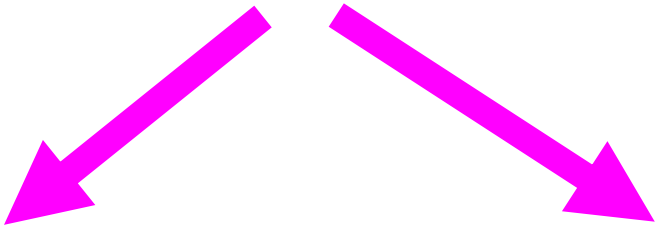
H. Baher

Second Edition, copyright 2001

by John Wiley & Sons, LTD



Analog Filters



Passive

Example:

LC filters •

Active

Example:

Op-Amp RC filters •

Filters {Sedra (Ch. 12)}

Structure of a filter

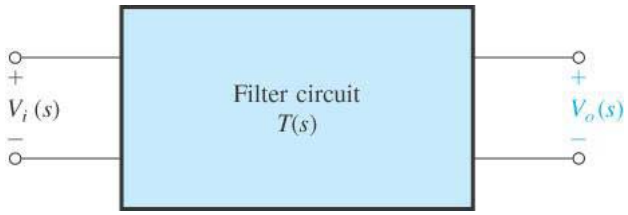


Figure 12.1 Filters studied in this chapter are linear circuits represented by the general two-port network shown.

Filter Transmission is expressed by,

$$T(s) \equiv \frac{V_o(s)}{V_i(s)}$$

$$T(j\omega) = |T(j\omega)|e^{j\phi(\omega)},$$

Magnitude of Transmission is expressed as a Gain Function $G(\omega)$,

$$G(\omega) \equiv 20\log|T(j\omega)|$$

or as an Attenuation Function $A(\omega)$,

$$A(\omega) \equiv -20\log|T(j\omega)|$$

Filter Types

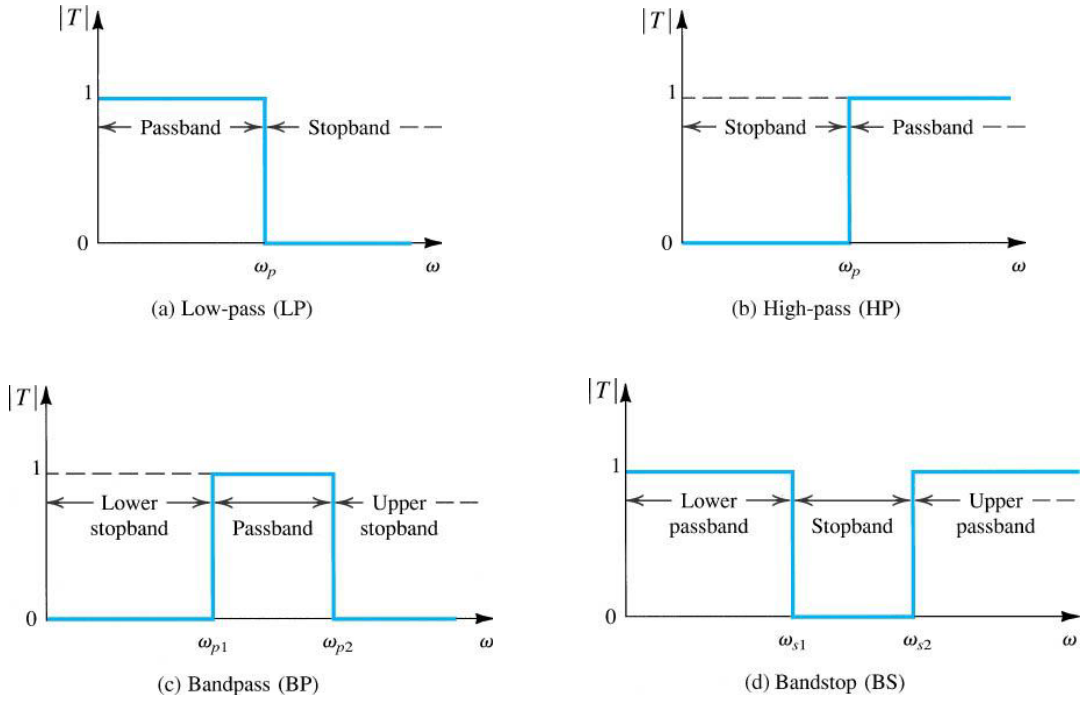


Figure 12.2 Ideal transmission characteristics of the four major filter types: **(a)** low-pass (LP), **(b)** high-pass (HP), **(c)** bandpass (BP), and **(d)** bandstop (BS).

Filter Design

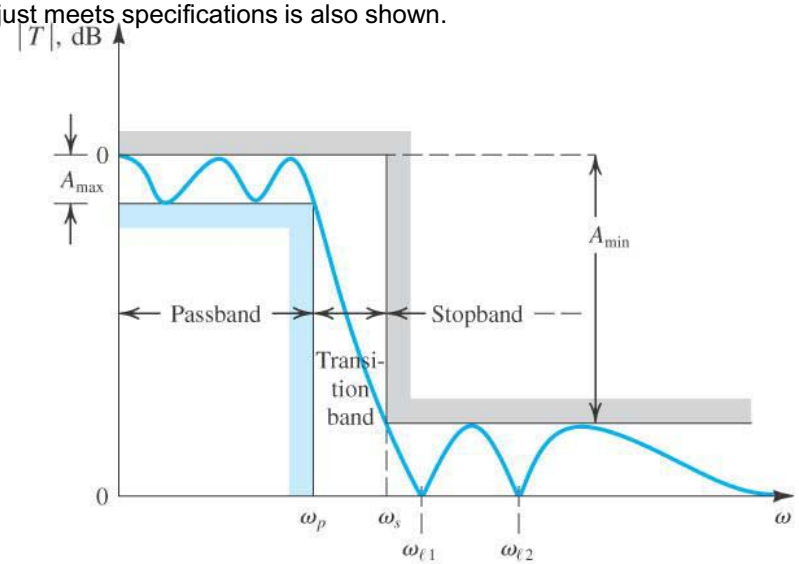
Filter Design follows the following steps:

1. Given desired specifications ($\omega_p, A_{max}, \omega_s, A_{min}$)
2. Filter Approximation, i.e. finding the Transfer Function that achieves desired specs (Cheychev, Butterworth, Elliptic, Bessel,…. etc)
3. Filter Realization (Building the filter up)

Dr. Mohamed Hassan

Transmission Characteristics of a LPF

Figure 12.3 Specification of the transmission characteristics of a low-pass filter. The magnitude response of a filter that just meets specifications is also shown.



Transmission of a LPF is specified by 4 parameters,

1. Passband edge "Ripple Bandwidth" ω_p
2. Maximum allowable variation in passband "Passband Ripple" A_{max}
3. Stopband edge ω_s
4. Minimum required stopband attenuation A_{min}

Transmission Characteristics of a BPF

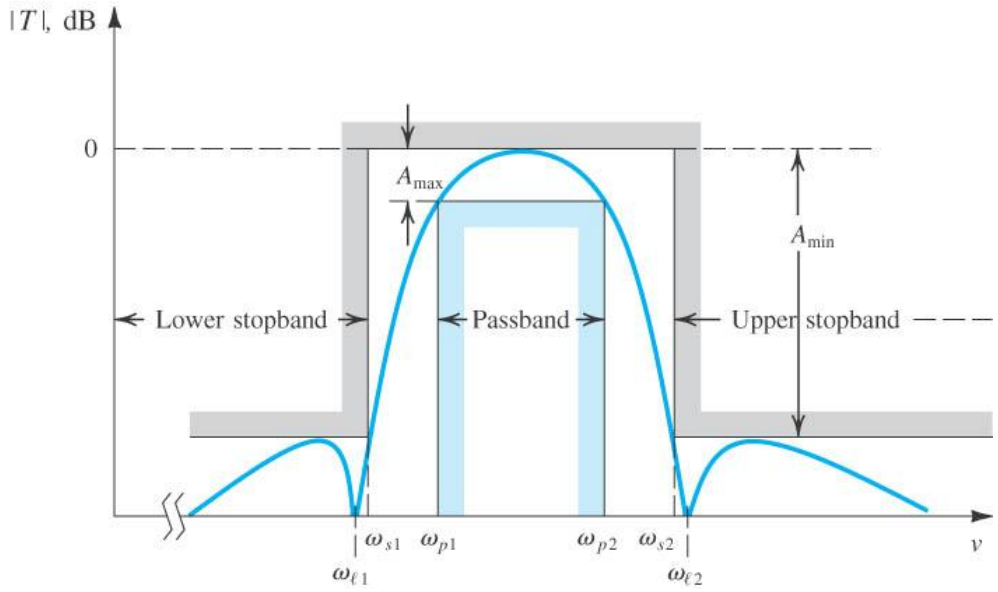
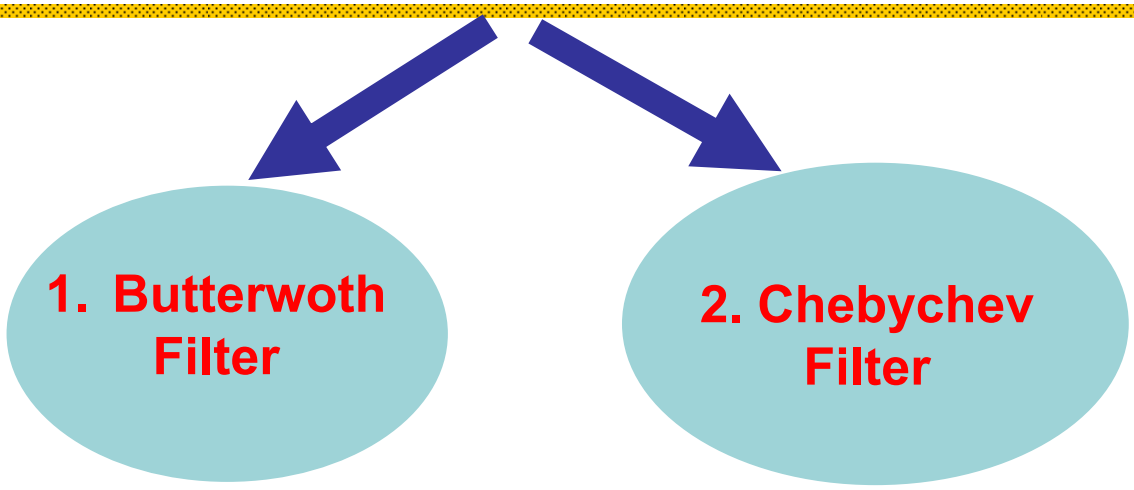


Figure 12.4 Transmission specifications for a bandpass filter. The magnitude response of a filter that just meets specifications is also shown. Note that this particular filter has a monotonically decreasing transmission in the passband on both sides of the peak frequency.

Filter Realization Functions



1. Butterworth Filter

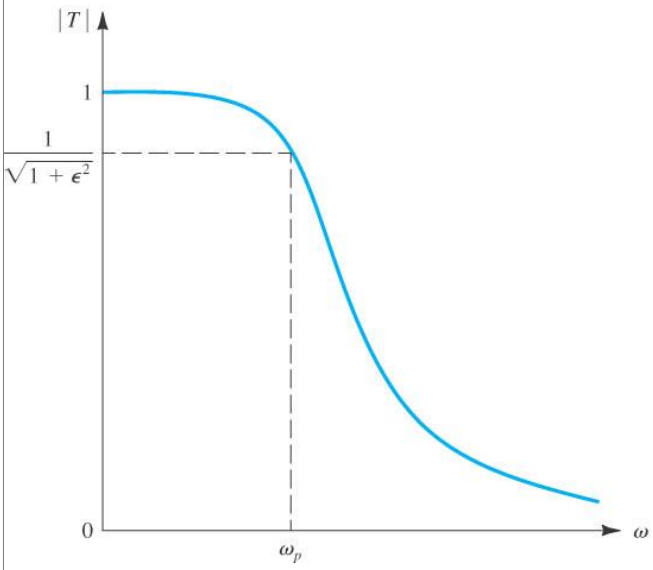


Fig. 12.8 Magnitude response of Butterworth filter

$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}}}$$

$$@\,\omega=\omega_p \Rightarrow |T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2}}$$

$$\therefore A_{\max} = 20\log\sqrt{1+\varepsilon^2} \qquad \varepsilon = \sqrt{10^{A_{\max}/10}-1}$$

$$@\,edge\ \text{of stopband},\ \omega=\omega_s, atten.is$$

$$A(\omega_s) = -20\log\left[1/\sqrt{1+\varepsilon^2(\omega_s/\omega_p)^{2N}}\right]$$

$$= 10\log\left[1+\varepsilon^2(\omega_s/\omega_p)^{2N}\right] \Rightarrow N$$

$$T(s) = \frac{k\omega_o^N}{(s-p_1)(s-p_2).....(s-p_N)}$$

Butterworth Order "N"

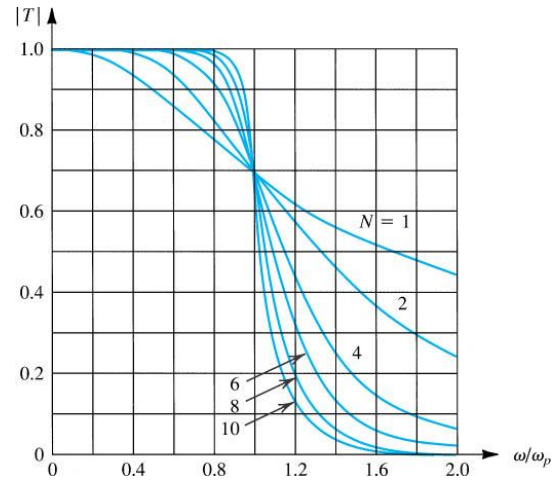
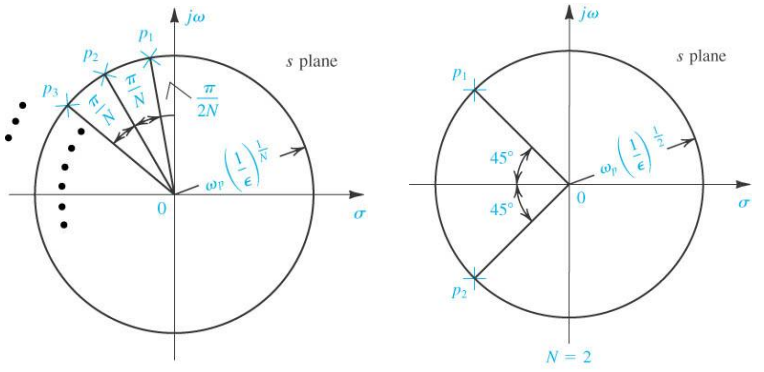
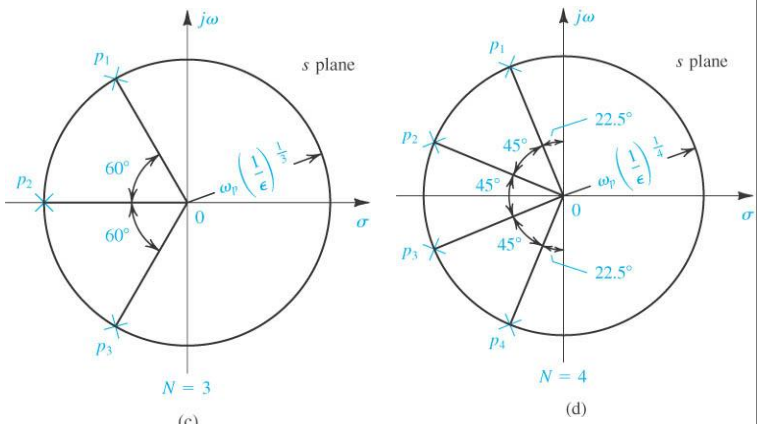


Fig. 12.9 Magnitude response for Butterworth filters of various orders with $\varepsilon = 1$. As order increases, response approaches ideal brick-wall type of transmission.

Fig. 12.10 Graphical construction to determe poles of N^th order Butterworth filter. All poles lie in left half of *s* plane on a circle of radius $\omega_0 = \omega_p(1/\varepsilon)^{1/N}$, where ε is the passband deviation parameter ($\varepsilon = \sqrt{10^{A_{max}/10} - 1}$)



(a) General case **(b) *N* = 2,**

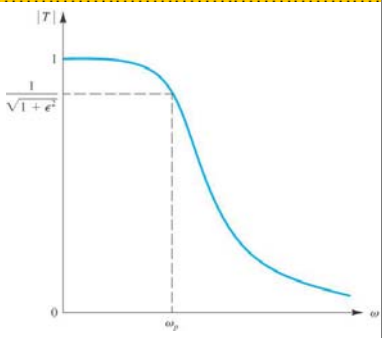


(c) *N* = 3, **(d) *N* = 4.**

Procedure to find Butterworth T.F. meeting certain specs:

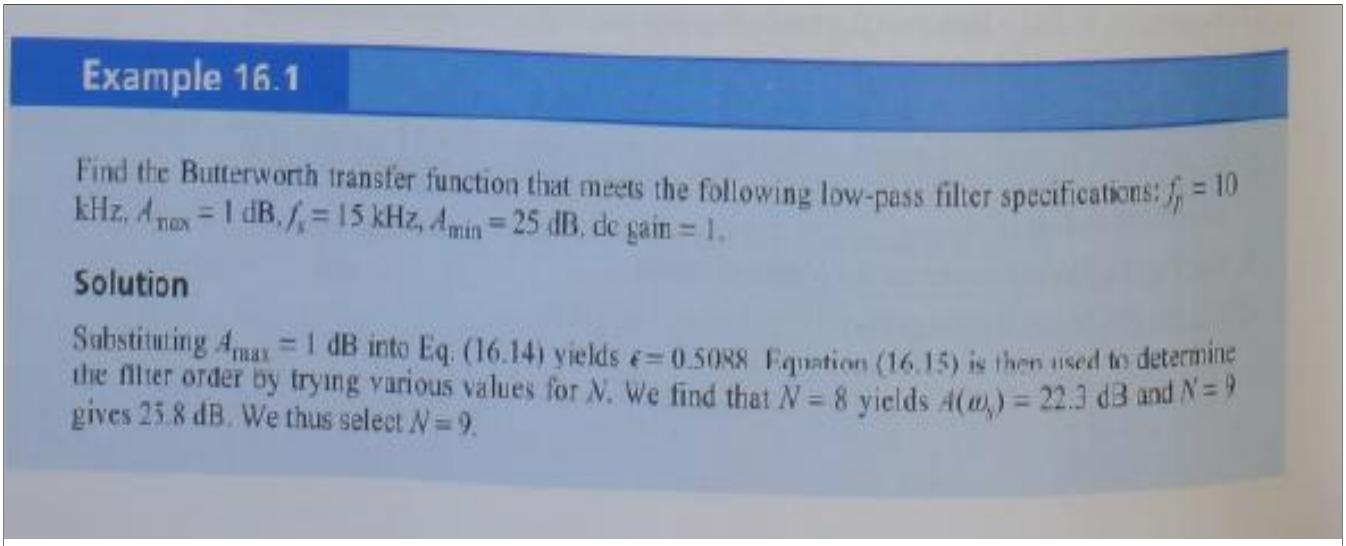
Given:

1. A_{max} 2. A_{min}
3. F_p 4. f_s



1. Determine ε using eqn.. $\varepsilon = \sqrt{10^{A_{\max}/10} - 1}$
2. Determine filter order "N" s.t. {$A(\omega_s) \geq A_{min}$ $A(\omega_s) = 10\log\left[1 + \varepsilon^2 \left(\omega_s / \omega_p\right)^{2N}\right] \Rightarrow N$
3. Determine the filter "N" natural modes (poles)
4. Determine filter transfer function T(s)

$$T(s) = \frac{k\omega_o^N}{(s - p_1)(s - p_2).....(s - p_N)}$$



Example 16.1

Find the Butterworth transfer function that meets the following low-pass filter specifications: $f_p = 10$ kHz, $A_{max} = 1$ dB, $f_s = 15$ kHz, $A_{min} = 25$ dB, dc gain = 1.

Solution

Substituting $A_{max} = 1$ dB into Eq. (16.14) yields $\epsilon = 0.5088$. Equation (16.15) is then used to determine the filter order by trying various values for N. We find that $N = 8$ yields $A(\omega_s) = 22.3$ dB and $N = 9$ gives 25.8 dB. We thus select $N = 9$.

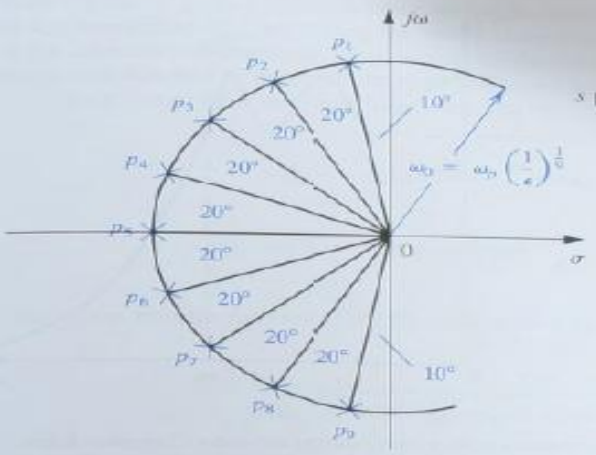


Figure 16.11 Poles of the ninth-order Butterworth filter of Example 16.1

Figure 16.11 shows the graphical construction for determining the poles. The poles all have the same frequency $\omega_0 = \omega_p(1/\epsilon)^{1/N} = 2\pi \times 10 \times 10^3 (1/0.5088)^{1/9} = 6.773 \times 10^4$ rad/s. The first pole p_1 is given by

$$p_1 = \omega_0(-\cos 80° + j \sin 80°) = \omega_0(-0.1736 + j0.9848)$$

Combining p_1 with its complex conjugate p_9 yields the factor $(s^2 + s0.3472\omega_0 + \omega_0^2)$ in the denominator of the transfer function. The same can be done for the other complex poles, and the complete transfer function is obtained using Eq. (16.16),

$$T(s) = \frac{\omega_0^9}{(s + \omega_0)(s^2 + s1.8794\omega_0 + \omega_0^2)(s^2 + s1.5321\omega_0 + \omega_0^2)} \times \frac{1}{(s^2 + s\omega_0 + \omega_0^2)(s^2 + s0.3472\omega_0 + \omega_0^2)} \tag{16.17}$$

2. Chebychev Filter

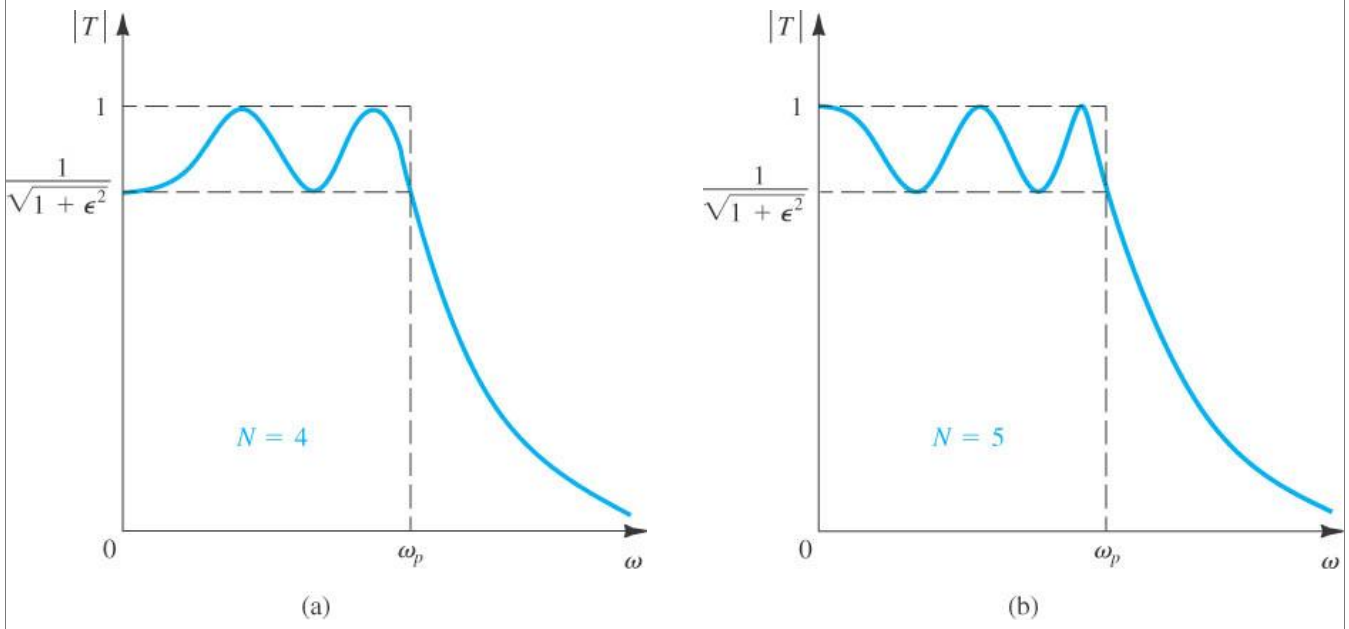


Figure 12.12 Sketches of the transmission characteristics of representative (b) odd-order Chebyshev filters (a) even-order

Chebychev Filter design equations

$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2 \cos^2\left[N\cos^{-1}(\omega/\omega_p)\right]}} \quad \text{.....for } \omega \le \omega_p$$

$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2 \cosh^2\left[N\cosh^{-1}(\omega/\omega_p)\right]}} \quad \text{.....for } \omega \ge \omega_p$$

$$@\,\omega = \omega_p \Rightarrow |T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2}} \qquad \varepsilon = \sqrt{10^{A_{\max}/10}-1}$$

$$\therefore A_{\max} = 20\log\sqrt{1+\varepsilon^2} \Rightarrow \text{.....}$$

$@\,edge$ of stopband, $\omega = \omega_s, atten.is$

$$A(\omega_s) = 10\log\left[1+\varepsilon^2\cosh^2\left(N\cosh^{-1}(\omega_s/\omega_p)\right)\right] \Rightarrow N$$

$$p_k = -\omega_p \sin\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right) + j\omega_p \cos\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right) \text{.....} k = 1,2,\text{.....},N$$

$$T(s) = \frac{k\omega_p^N}{\varepsilon 2^{N-1}(s-p_1)(s-p_2)\text{.....}(s-p_N)}$$

2. Chebychev Filter (cnt'd): Design equations

$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2 \cos^2\left[N\cos^{-1}(\omega/\omega_p)\right]}} \quad \text{.....for } \omega \le \omega_p$$

$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2 \cosh^2\left[N\cosh^{-1}(\omega/\omega_p)\right]}} \quad \text{.....for } \omega \ge \omega_p$$

$$@\,\omega = \omega_p \Rightarrow |T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2}} \qquad \varepsilon = \sqrt{10^{A_{\max}/10}-1}$$

$$\therefore A_{\max} = 20\log\sqrt{1+\varepsilon^2} \Rightarrow \text{.....}$$

$$p_k = -\omega_p \sin\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right) + j\omega_p \cos\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right) \text{.....} k = 1,2,\text{.....},N$$

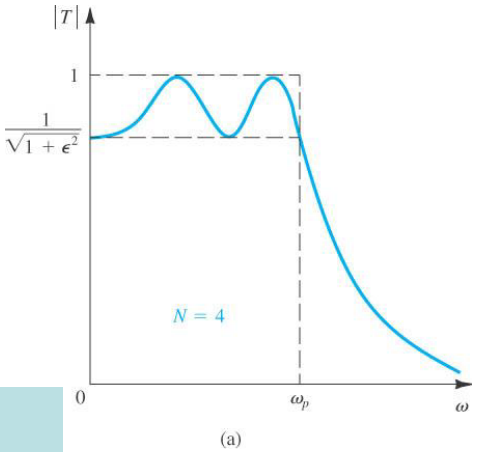
$$T(s) = \frac{k\omega_p^N}{\varepsilon 2^{N-1}(s-p_1)(s-p_2)\text{.....}(s-p_N)}$$

$@\,edge$ of stopband, $\omega = \omega_s, atten.is$

$$A(\omega_s) = 10\log\left[1+\varepsilon^2\cosh^2\left(N\cosh^{-1}(\omega_s/\omega_p)\right)\right] \Rightarrow N$$

2. Chebychev Filter (cont'd)

Procedure to find Chebychev Filter T.F. meeting certain specs:



(a)

Given:

A_{max} 2. A_{min} .)
3. F_p 4. f_s

$$\varepsilon = \sqrt{10^{A_{max}/10} - 1}$$

@ *edge* of stopband, $\omega = \omega_s$, *atten. is*

$$A(\omega_s) = 10\log\left[1 + \varepsilon^2 \cosh^2\left(N \cosh^{-1}(\omega_s / \omega_p)\right)\right] \Rightarrow N$$

1. Determine ε using eqn..
2. Determine filter order "N" s.t. $\{A(\omega_s) \geq A_{min}$
3. Determine the filter "N" natural modes (poles)
4. Determine filter transfer function T(s)

$$T(s) = \frac{k\omega_p^N}{\varepsilon 2^{N-1}(s - p_1)(s - p_2).....(s - p_N)}$$

Example 16.2

Find the Chebyshev transfer function that meets the same low-pass filter specifications given in Example 16.1: namely, $f_p = 10$ kHz, $A_{max} = 1$ dB, $f_s = 15$ kHz, $A_{min} = 25$ dB, dc gain = 1.

Solution

Substituting $A_{max} = 1$ dB into Eq. (16.21) yields $\epsilon = 0.5088$. By trying various values for N in Eq. (16.22) we find that $N = 4$ yields $A(\omega_s) = 21.6$ dB and $N = 5$ provides 29.9 dB. We thus select $N = 5$. Recall that we required a ninth-order Butterworth filter to meet the same specifications in Example 16.1.

The poles are obtained by substituting in Eq. (16.23) as

$$p_1, p_5 = \omega_p(-0.0895 \pm j0.9901)$$

$$p_2, p_4 = \omega_p(-0.2342 \pm j0.6119)$$

$$p_3 = \omega_p(-0.2895)$$

The transfer function is obtained by substituting these values in Eq. (16.24) as

$$T(s) = \frac{\omega_p^5}{8.1408(s + 0.2895\omega_p)(s^2 + s0.4684\omega_p + 0.4293\omega_p^2)} \times \frac{1}{s^2 + s0.1789\omega_p + 0.9883\omega_p^2}$$

where $\omega_p = 2\pi \times 10^4$ rad/s.

2. Chebychev Filter (cnt'd): Design equations

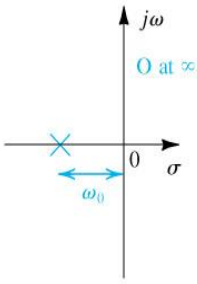
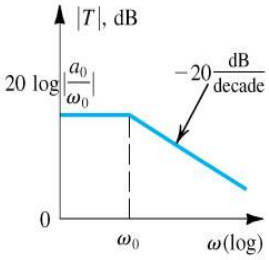
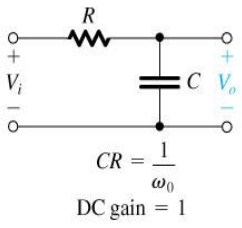
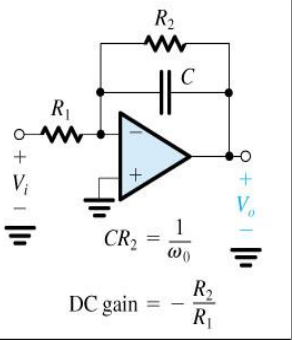
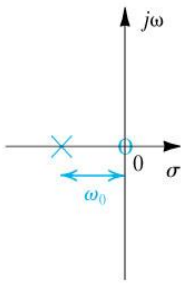
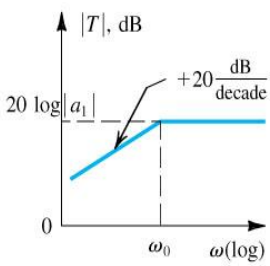
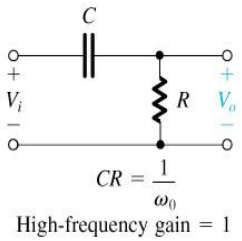
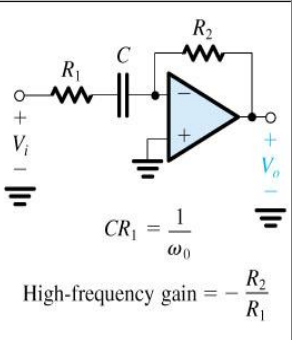
Chebychev Filter *more efficient* than Butterworth one:

Thus, for same order "N" & same A_{max}, Chebychev provides greater stopband attenuation than Butterworth filter

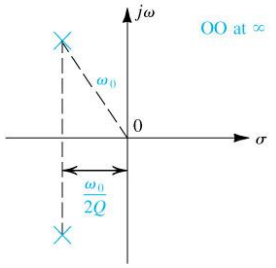
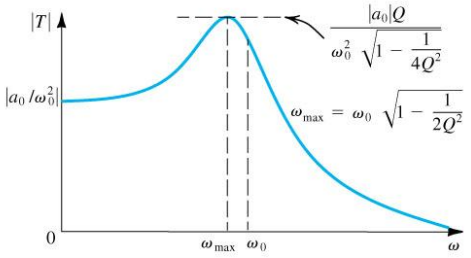
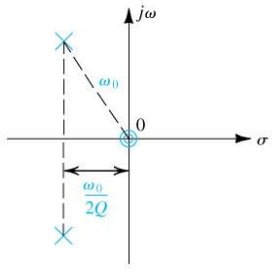
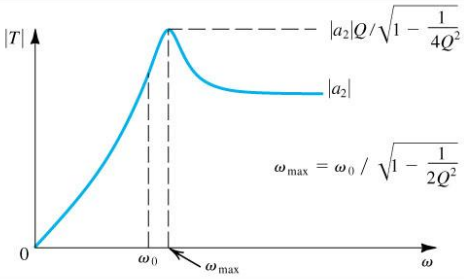
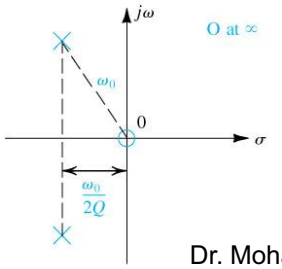
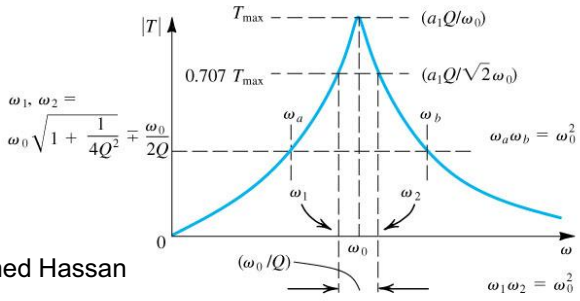
i.e. to meet identical specs,

$$N_{Chebychev} < N_{Butterworth}$$

First Order Filters {T.F., B.P., Passive/active Realization}

Filter Type and $T(s)$	s-Plane Singularities	Bode Plot for $\lvert T\rvert$	Passive Realization	Op Amp–RC Realization
(a) Low pass (LP) $T(s) = \frac{a_0}{s+\omega_0}$				
(b) High pass (HP) $T(s) = \frac{a_1 s}{s+\omega_0}$				

Second Order Filters {T.F., B.P.}

Filter Type and $T(s)$	s-Plane Singularities	$\lvert T\rvert$
(a) Low pass (LP) $T(s) = \dfrac{a_0}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ DC gain $= \dfrac{a_0}{\omega_0^2}$		 figure_text|T| |a0|Q / ω0² √(1 − 1/4Q²) |a0/ω0²| ωmax = ω0 √(1 − 1/2Q²) 0 ωmax ω0 ω
(b) High pass (HP) $T(s) = \dfrac{a_2 s^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ High-frequency gain $= a_2$		 figure_text|T| |a2|Q/√(1 − 1/4Q²) |a2| ωmax = ω0 / √(1 − 1/2Q²) 0 ω0 ωmax ω
(c) Bandpass (BP) $T(s) = \dfrac{a_1 s}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ Center-frequency gain $= \dfrac{a_1 Q}{\omega_0}$		 figure_text|T| Tmax (a1Q/ω0) 0.707 Tmax (a1Q/√2ω0) ω1, ω2 = ω0 √(1 + 1/4Q²) ∓ ω0/2Q ωa ωb ωaωb = ω0² ω1 ω2 0 ω0 ω (ω0/Q) ω1ω2 = ω0²

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Second Order Passive Filter Realizations

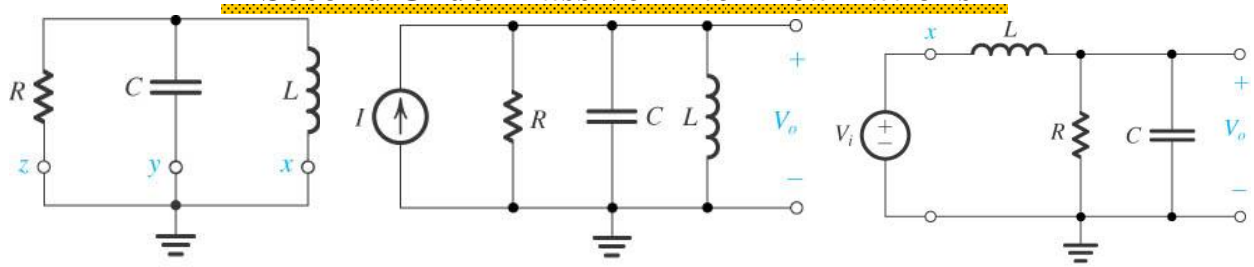
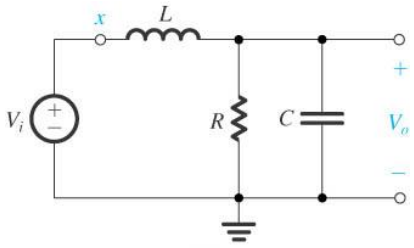
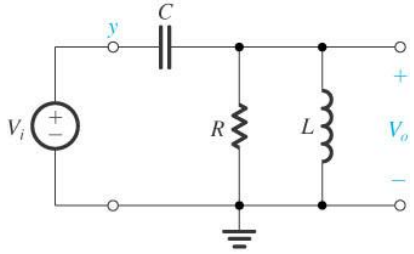


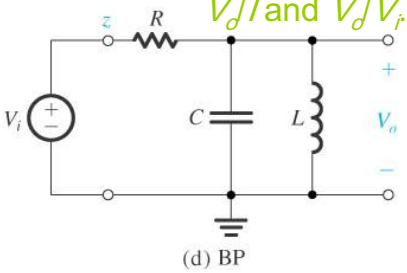
Figure 12.17 **(a)** Second-order parallel LCR resonator. **(b, c)** Two ways of exciting the resonator of (a) without changing its *natural structure:* resonator poles are those poles of V_o/I and V_o/V_i.

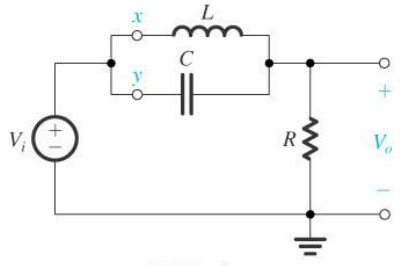
Figure 12.18

Realization of various **Second-order Passive** filter functions using LCR resonator of Fig. 12.17(b)









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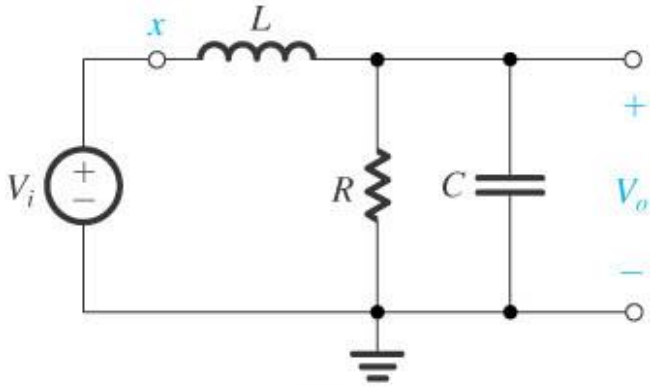
Second Order Passive Filter Realizations (cont'd)

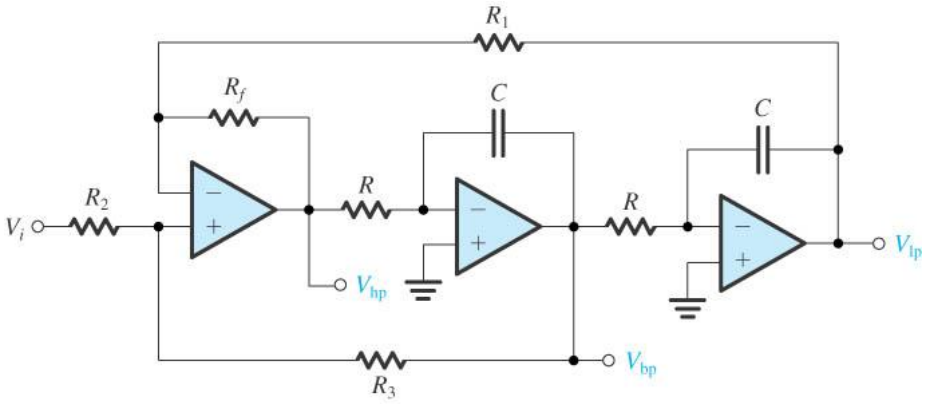


(b) LP

$$T(s)=\frac{1/LC}{s^2+s(1/RC)+(1/LC)}=\frac{a_o}{s^2+s\frac{\omega_o}{Q}+\omega_o{}^2}\Rightarrow$$

2 **Equations in 3 unknowns** $\left\{\begin{array}{l} B=\dfrac{\omega_o}{Q}=\dfrac{1}{RC} \\ \omega_o=\dfrac{1}{LC} \end{array}\right\}$

Second Order Filters Active Realizations



(a)

Thank you

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