Electromagnetic Field Theory

Course Description (EC341)

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Course Contents

Chapter (1): Coordinates systems and vector analysis (<u>3-Lectures</u>)

- Coordinates systems
- Vector operations
- Vector analysis

Chapter (2): Static Electric Field (<u>7-Lectures</u>)

- Column's law for electric force
- Electric field and electric flux in free-space
- Gauss' Law, and application to electrostatic field
- Work Done and Electrostatic Potential
- Electric Dipole
- Dielectrics and Polarization
- Boundary condition of static electric field
- Basic Equations of Static Electric Field
- Capacitance

Chapter (3): Electrostatic Problems (<u>3-Lectures</u>)

- Image Method
- Boundary Value Problems and Laplace and Poisson Equations

Chapter (4): Currents and Conductors (<u>2-Lectures</u>)

- Ohm' Law
- Joule' Law
- Resistance
- Boundary condition of stationary currents density

Chapter (5): Static Magnetic Field (<u>6-Lectures</u>)

- Basic Equations of Static Magnetic Field
- Ampere's Law
- Biot-Savart Law
- Magnetic Vector Potential
- Boundary condition of static magnetic field
- Magnetic Force
- Inductance

Chapter (6): Time-Varying Fields and Maxwell's Equations (5-Lectures)

- Faraday' Law
- Displacement Current
- Maxwell' Equations
- Time-Harmonic Fields
- Uniform Plane Wave

Course Plane

Chapter No.	Chapter Title	No. of Lectures
Chapter 1 1st & 2nd W	Orthogonal Coordinate Systems and Vector Analysis	3-Lecture
Chapter 2 2nd, 3rd & 4th W	Electrostatic Field in Vacuum	5-Lectures
Chapter 2 5th W	Electrostatic Field in Dielectric Media	2- Lectures
Chapter 3 6th & 8th W	Methods for the solution of Electrostatic Problems	3- Lectures
Chapter 4 8th & 9th W	Steady Electric Currents	2- Lecture
Chapter 5 9th, 10th, 11th & 13th W	The steady Magnetic Field	6- Lectures
Chapter 6 13th ,14th & 15th W	Time Varying Field & Maxwell's equations	5- Lectures

Exercise and Quiz Test Plane

Problem Set No.	Week No.	Quiz Test
Problem set # 1 1-Weeks	1st Week	
Problem set # 2 3-Weeks	2nd, 3rd & 4th Week	
Problem set # 3 2- Weeks	5th & 6th Week	1st Quiz 6th Week
Problem set # 4 3- Weeks	8th , 9th & 10th Week	
Problem set # 5 3- Week	11th, 13th & 14thW eek	2nd Quiz 11th Week

Coordinate Systems and Vector Analysis

$$\nabla V = \frac{\partial V}{h_1 \partial u_1} \mathbf{a}_{u1} + \frac{\partial V}{h_2 \partial u_2} \mathbf{a}_{u2} + \frac{\partial V}{h_3 \partial u_3} \mathbf{a}_{u3}$$

$$\nabla^2 V = \frac{1}{h_1 h_2 h_3} \left(\frac{\partial}{\partial u1}\left(h_2 h_3\left(\frac{1}{h_1}\frac{\partial V}{\partial u_1}\right)\right) + \frac{\partial}{\partial u2}\left(h_1 h_3\left(\frac{1}{h_2}\frac{\partial V}{\partial u_2}\right)\right) + \frac{\partial}{\partial u3}\left(h_1 h_2\left(\frac{1}{h_3}\frac{\partial V}{\partial u_3}\right)\right) \right)$$

$$\nabla . \overline{A} = \frac{1}{h_1 h_2 h_3} \left(\frac{\partial}{\partial u1}(h_2 h_3 A_1) + \frac{\partial}{\partial u2}(h_1 h_3 A_2) + \frac{\partial}{\partial u3}(h_1 h_2 A_3) \right)$$

$$\nabla \times \overline{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \mathbf{a}_{u1} & h_2 \mathbf{a}_{u2} & h_3 \mathbf{a}_{u3} \\ \frac{\partial}{\partial u1} & \frac{\partial}{\partial u2} & \frac{\partial}{\partial u3} \\ h_1 A_{u1} & h_2 A_{u2} & h_3 A_{u3} \end{vmatrix}$$

Divergence Theorem: $\int_{v} \nabla \bullet \overline{\mathbf{A}}\, dv = \oint_{s} \overline{\mathbf{A}} \bullet ds$

Stokes's Theorem: $\int_{s} (\nabla \times \overline{\mathbf{A}}) \bullet ds = \oint_{c} \overline{\mathbf{A}} \bullet d\ell$

Metric coefficients of Coordinate systems

	(x, y, z)	(ρ, Φ, z)	(r, θ, Φ)
$\mathbf{a}_{u1}$	$\mathbf{a}_x$	$\mathbf{a}_\rho$	$\mathbf{a}_r$
$\mathbf{a}_{u2}$	$\mathbf{a}_y$	$\mathbf{a}_\Phi$	$\mathbf{a}_\theta$
$\mathbf{a}_{u3}$	$\mathbf{a}_z$	$\mathbf{a}_z$	$\mathbf{a}_\Phi$
h_1	1	1	1
h_2	1	ρ	r
h_3	1	1	r sin θ

Transformation between Coordinate Systems

	a_ρ	a_Φ	a_z	a_r	a_θ	a_Φ
a_x ·	cosΦ	-sinΦ	0	sinθ cosΦ	cosθ cosΦ	-sinΦ
a_y ·	sinΦ	cosΦ	0	sinθ sinΦ	cosθ sinΦ	cosΦ
a_z ·	0	0	1	cosθ	-sinθ	0

Integral Forms

Integral Form	Result of Integration
$\int \frac{dx}{\sqrt{x^2+a^2}}$	$\ln(x+\sqrt{x^2+a^2})$ $\sinh^{-1}\frac{x}{a}$
$\int \frac{xdx}{\sqrt{x^2+a^2}}$	$\sqrt{x^2+a^2}$
$\int \frac{dx}{(x^2+a^2)^{3/2}}$	$\frac{x}{a^2\sqrt{x^2+a^2}}$
$\int \frac{x\ dx}{(x^2+a^2)^{3/2}}$	$\frac{-1}{\sqrt{x^2+a^2}}$
$\int \frac{dx}{(x^2+a^2)}$	$\frac{1}{a}\tan^{-1}\frac{x}{a}$
$\int \frac{xdx}{(x^2+a^2)}$	$\frac{1}{2}\ln(x^2+a^2)$
$\int \frac{dx}{x\sqrt{(x^2+a^2)}}$	$\frac{-1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$
$\int \frac{dx}{\sin\ ax}$	$\frac{1}{a}\ln\left\|\tan\frac{ax}{2}\right\|$

Differential Calculus

Function	Differentiation
sin(x)	cos(x)
cos(x)	-sin(x)
tan(x)	$\sec^2(x)$
$\sin(x)^{-1}$	$\frac{1}{\sqrt{1-x^2}}$
$\cos(x)^{-1}$	$\frac{-1}{\sqrt{1-x^2}}$
$\tan(x)^{-1}$	$\frac{1}{1+x^2}$
e^x	e^x
ln(x)	1/x
$\log_a(x)$	1/[x ln(a)]

Trigonometric relations

$\sin(x) = \sqrt{1/2(1-\cos(2x))}$
$\cos(x) = \sqrt{1/2(1+\cos(2x))}$
$\tan(x) = \frac{\sqrt{(1-\cos(2x))}}{\cos(x)}$
$\sin(2x) = 2\sin(x)\cos(x)$
$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$
$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$
$e^{\pm jx} = \cos(x) \pm j\sin(x)$

Static Electric Field

1. Coulomb's law:

$\bar{F}_2 = k \dfrac{q_1 q_2}{R^2} \bar{a}_r$, Where, $k=1/(4\pi\varepsilon_0)= 9\times10^9$, and $\bar{a}_R$ is the unit vector in the direction of the force ($\underline{a}_r = \hat{a}_r = \bar{a}_r = \bar{r}/r$). If the force is negative, it is **attraction** force, while the positive sign means **repulsion** force.

2. Electrostatic Field and Potential (E and V):

Point	$\vec{E}_P = \dfrac{q_i}{4\pi\varepsilon_o R_i^2}\vec{R}$	$V_P = \dfrac{q_i}{4\pi\varepsilon_o R_i}$
Line	$\bar{E}_P = \dfrac{1}{4\pi\varepsilon_o}\int \dfrac{\rho}{R^3}\vec{R}\,d\ell'$	$V_P = \dfrac{1}{4\pi\varepsilon_o}\int \dfrac{\rho}{R}\,d\ell'$
Surface	$\bar{E}_P = \dfrac{1}{4\pi\varepsilon_o}\iint_s \dfrac{\rho_s}{R^3}\vec{R}\,ds'$	$V_P = \dfrac{1}{4\pi\varepsilon_o}\iint_s \dfrac{\rho_s}{R}\,ds'$
Volume	$\bar{E}_P = \dfrac{1}{4\pi\varepsilon_o}\iiint_v \dfrac{\rho_v}{R^3}\vec{R}\,dV'$	$V_P = \dfrac{1}{4\pi\varepsilon_o}\iiint_v \dfrac{\rho_v}{R}\,dV'$
$\vec{E}_p = -\nabla V_P$ and $V_p = -\int_{Ref}^{P} \vec{E}_P \bullet \partial\vec{\ell}$ The Ref. point usually have a zero potential		

3. Gauss's law:

$$\oint_s \bar{D}.\bar{n}\,ds = Q \text{ (total charge enclosed)}$$

Where;

$$Q = \iiint_v \rho_v\,dv \quad \text{volume charge}$$

$$= \iint_s \rho_s\,ds \quad \text{surface charge}$$

$$= \int \rho_\ell\,d\ell \quad \text{line charge}$$

$$\oint_s \overline{D}.\overline{n}\, ds = 4\pi r^2 D_r \text{ for sphere of radius } r$$

$$= 2\pi\rho\,\ell\, D_\rho \text{ for cylinder of radius } \rho \text{ and length } \ell$$

4. Boundary Condition of electric field (air-conductor interface):

$$E_n = \frac{\rho_s}{\varepsilon_o} \quad \text{and} \quad E_t = 0$$

5. Finite Line charge:

$$E_P = \frac{\rho_\ell}{2\pi\varepsilon_o\rho}\left(\frac{L/2}{\sqrt{\rho^2 + (L/2)^2}}\right) a_\rho$$

$$V_P = \frac{\rho_\ell}{2\pi\varepsilon_o} \ln\left(\frac{\sqrt{\rho^2 + (L/2)^2} + L/2}{\rho}\right)$$

6. Infinite Line Charge:

$$E_P = \frac{\rho_\ell}{2\pi\varepsilon_o\rho} a_\rho \quad \text{and} \quad V_P = \frac{\rho_\ell}{2\pi\varepsilon_o} \ln\left(\frac{\rho_o}{\rho}\right)$$

7. Electric Dipole:

$$V = \frac{p\cos\theta}{4\pi\varepsilon_o r^2} \quad \text{and} \quad \overline{E} = \frac{1}{4\pi\varepsilon_o r^3}(2p\cos\theta\, a_r + p\sin\theta\, a_\theta)$$

8. Basic Equations of Static Electric Field

$$\overline{\nabla}.\overline{D} = \rho \qquad \overline{\nabla} \times \overline{E} = 0$$

9. Polarization vector P:

$$\overline{P} = \overline{D} - \varepsilon_o \overline{E} = (\varepsilon_r - 1)\varepsilon_o \overline{E}$$

10. Boundary Conditions of Electric Field (dielectric-dielectric interface):

$$E_{1t} = E_{2t} \quad \text{and} \quad D_{1n} - D_{2n} = \rho_s \quad \textit{(where, } \rho_s \textit{ is free charge)}$$

11. Capacitance:

$$C = \frac{Q}{V}$$

Where;

Q is the Gauss's law ($\oint_{s} \overline{D}.\overline{n}\, ds = \iiint_{v} \rho_v \, dv = Q$)

V is the line integral of **E** along the path

$$\left(V_{ab} = -\int_{final = b}^{intial = a} \overline{E}.\overline{d\ell} = V_a - V_b \right)$$

To find the capacitance, we follow the 4-steps:

1. Assume a charge +Q and -Q on the two conductors.
2. Find E using Gauss's law or any other method.
3. Find V using line integral along E lines
4. Put Q = C V, and then Find C.

12. Electrostatic Energy:

For N-discrete charge:

$$W_e = \frac{1}{2} \sum_{k=1}^{n} Q_k \; V_k$$

Where, $V_k = \frac{1}{4\pi \varepsilon_o} \sum_{j=1}^{n} \frac{Q_j}{Rjk} \quad \text{and } j \neq k$

For continuous charge:

$$W_e = \frac{1}{2} \iiint_{v'} (\overline{D} \, . \, \overline{E}) \; d\,v' = \frac{\varepsilon}{2} \iiint_{v'} \left|\overline{E}\right|^2 \; d\,v'$$

For any capacitor configuration:

$$W_e = \frac{1}{2} C\,V^2 = \frac{1}{2} Q\,V = \frac{Q^2}{2C}$$

Surface bounded charge for any capacitor configuration:

$$\rho_{Upper} = First \;\; Conductor = \overline{P} \bullet (-a_n)$$

$$\rho_{Lower} = Second \;\; Conductor = \overline{P} \bullet (+a_n)$$

Solution of Electrostatic Problems

1.Image Method:

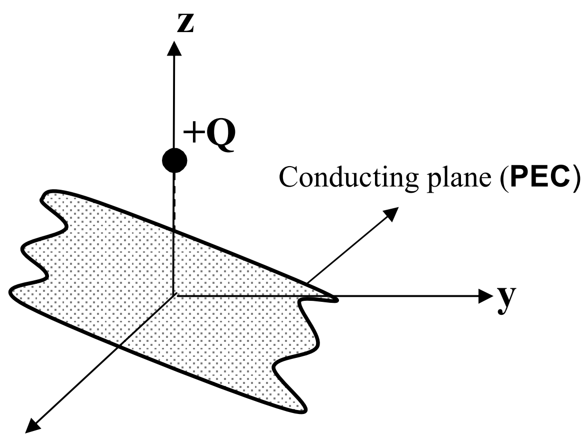


Fig.1 (a)

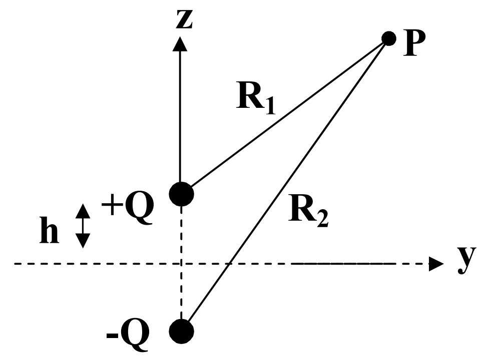


Fig.1 (b)

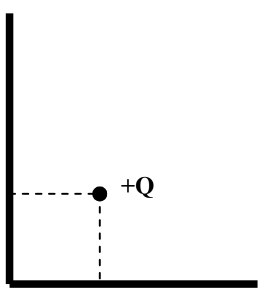


Fig.2 (a)

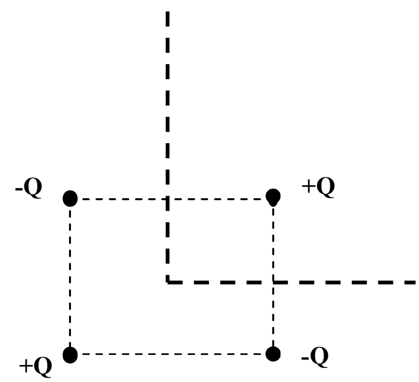


Fig.2 (b)

If the conductor is in the form of corner with an angle "α", then in this case, we have m images where,

$$m = 2n - 1 \quad \text{and} \quad n = \frac{180}{\alpha}$$

2. Boundary Value Problems

In Cartesian:

$$\nabla^2 V_P = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) V_p = 0$$

The possible solutions of $W''(u) + k_u^2 W(u) = 0$

k_u^2	k_u	W(u)	
0	0	***Linear Solution***	
		$A_o u + B_o$	
+	k	***Periodic Solution***	
		$A_1 \sin(k u) + B_1 \cos(ku)$	$C_1 e^{+jku} + C_2 e^{-jku}$
-	jk	***Decayed Solution***	
		$A_2 \sinh(ku) + B_2 \cosh(ku)$	$D_1 e^{+ku} + D_2 e^{-ku}$

In cylindrical:

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial Z^2} = 0$$

In case of **Φ**-dependence solution is given as:

$$\Phi(\Phi) = A_o \Phi + B_o$$

In case of **ρ**-dependence solution is given as:

$$\frac{d}{d\rho}\left(\rho \frac{dV(\rho)}{d\rho}\right) = 0 \qquad \Rightarrow \qquad V(\rho) = A_1 \ln(\rho) + A_2$$

In Spherical:

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) = 0$$

(a) In case of **r**-dependence only:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dV}{dr}\right) = 0 \qquad \Rightarrow \quad V(r, \theta, \Phi) \equiv V(r)$$

Then:

$$\frac{d}{dr}\left(r^2 \frac{dV}{dr}\right) = 0 \qquad \Rightarrow \qquad \left(r^2 \frac{dV}{dr}\right) = A_1$$

$$\frac{dV}{dr} = \frac{A_1}{r^2} \qquad \Rightarrow \qquad dV = \frac{A_1}{r^2} dr$$

$$V(r) = \left(\frac{-1}{r}\right) A_1 + A_2$$

The constant A_1 and A_2 are determined from the given BC's.

(b) In case of **θ**-dependence only:

$$\frac{1}{r^2 \sin\theta} \frac{d}{d\theta} \left(\sin\theta \frac{dV}{d\theta} \right) = 0 \qquad \Rightarrow \qquad \sin\theta \frac{dV}{d\theta} = A_1$$

$$\frac{dV}{d\theta} = \frac{A_1}{\sin\theta} \qquad \Rightarrow \qquad dV = \int \frac{A_1}{\sin\theta} d\theta$$

Since the integration $\int \frac{dx}{\sin ax} = \frac{1}{a} \ln \left| \tan \frac{ax}{2} \right|$, therefore, the electric potential is given as:

$$V(\theta) = A_1 \ln \left| \tan \frac{\theta}{2} \right| + A_2$$

Where, the constant A_1 and A_2 are determined from the given BC's.

Currents and Conductors

1. Ohm's law:

In any conducting medium, if an electric field is applied, a current density J is given by:

$$\mathbf{J} = \sigma \mathbf{E}$$

Where,

σ Denotes conductivity of the conductor

J Denotes current density A/m^2

$$\sigma = \frac{J}{E} = \frac{A/m^2}{V/m} = \frac{A}{V\ m} = \frac{moh\ (siemens)}{m} = \mathrm{S/m}$$

The total current passing through an arbitrary surface S of a conducting body is given by

$$I = \iint_S \bar{J}.\overline{ds}$$

2. Resistance:

$$R = \frac{V}{I} \quad \rightarrow \quad R = \frac{\int_1^2 \bar{E}.\overline{d\ell}}{\sigma \iint_s \bar{E}.\overline{ds}}$$

(a) The voltage V is applied along the coordinate u_1 while the current is passing across the surface of the plane u_2-u_3.

$$R = \int_0^L \frac{du_1}{\sigma \iint \frac{h_2 h_3}{h_1} du_2\, du_3}$$

(b) Solving Laplace Equation

1. Find the electric potential V
2. Find the electric field as $E = -\nabla V$
3. Find the current density J = σ E
4. Find the total current I = $\iint_s \bar{J}.\overline{ds}$
5. Find the resistance R as R = $\frac{V}{I}$

3. Duality Law:

There is a dual relation between the resistance and the capacitance as:

$$\overbrace{R \;\; x \;\; C}^{resistance \;\; x \; capacitance} = \underbrace{\frac{\varepsilon}{\sigma}}_{\substack{ratio\ of\ the\ constitutive\ parameters \\ of\ the\ conducting\ medium}}$$

Duality between J and D

Conductor	Dielectric
$J = \sigma E$	$D = \varepsilon E$
$\nabla . J = o$	$\nabla . D = o$
$J_{n1} = J_{n2}$	$D_{n1} = D_{n2}$
$\frac{J_{t1}}{\sigma_1} = \frac{J_{t2}}{\sigma_2}$	$\frac{D_{t1}}{\varepsilon_1} = \frac{D_{t2}}{\varepsilon_2}$
σ	ε
I	Q
$\frac{1}{R} = G$	C

Static Magnetic Field

Basic Equations of Static Magnetic Filed:

- $\nabla . \overline{B} = 0$, $\nabla \, x \, \overline{H} = \overline{J}$, and $\overline{B} = \mu \overline{H}$

Ampere's Law

- $\nabla \, x \, \overline{B} = \mu_0 \overline{J}$ Differential form
- $\oint_c \overline{B} . \overline{d\ell} = \mu_0 I$ Integral form

Biot-Savart Law for a line carrying constant current:

- $\overline{H} = \int_L \frac{I \, \overline{d\ell} \, x \, a_R}{4 \pi R^2}$

For finite Line carrying constant current:

$$\overline{H} = \frac{I}{4 \pi d} \left[\sin(\text{the angle in current direction}) - \sin(\text{the angle in oppsite current direction}) \right] \hat{a}_n$$

Magnetic Vector Potential for a line carrying constant current:

- $\overline{A} = \int_c \frac{\mu_o I \overline{d\ell}}{4\pi R}$

Loop of constant current I and radius r=b:

- $\overline{H}_{center} = \frac{I}{4b}(\pm a_z)$

Boundary Conditions:

$H_1L = H_2L \rightarrow H_{1t} = H_{2t}$
$B_{1n} = B_{2n} \rightarrow \mu_1 H_{1n} = \mu_2 H_{2n}$

In case of current existing at the boundary surface:
$H_{1t} - H_{2t} = J_s$

Magnetic Force F_m:

$$\overline{F}_{12} = \oint_c I_2 \overline{d\ell}_2 \times \overline{B}_1 \quad \text{and} \quad \overline{F}_{21} = \oint_c I_1 \overline{d\ell}_1 \times \overline{B}_2$$

Magnetostatic Energy:

$$W_m = \frac{1}{2} \iiint_{v'} (\overline{B} \cdot \overline{H}) \, dv' = \frac{\mu}{2} \iiint_{v'} \left|\overline{H}\right|^2 dv'$$

Inductance L:

To find the inductance of any geometry, we should do the following steps:

1. Choose the suitable coordinate system, and assume source current I.

2. Find the magnetic field density **B** (or **H**)

- Ampere's Law (if there is a symmetry)
- Biot-Savart Law
- Magnetic vector potential $\overline{B} = \nabla \times \overline{A}$

3. Find the magnetic flux ψ as $\psi = \iint_s \overline{B} . \overline{ds}$

4. Find the flux linkage Λ = ψ N

5. Find L as L = Λ / I

Maxwell's Equations for Time-Varying Fields

Differential form	Integral form
$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$	$\oint E.d\ell = -\iint \frac{\partial B}{\partial t}.ds$
$\nabla \times \overline{H} = J + \frac{\partial \overline{D}}{\partial t}$	$\oint H.d\ell = I + \iint \frac{\partial D}{\partial t}.ds$
$\nabla . \overline{D} = \rho$	$\oint D.ds = \iiint \rho\, dv$
$\nabla . \overline{B} = 0$	$\oint B.ds = o$
$\overline{D} = \varepsilon \overline{E}$	
$\overline{B} = \mu \overline{H}$	
$\overline{J} = \sigma \overline{E}$	

Maxwell's Equations for complex Time-Harmonic Fields

- $\nabla \times \overline{E} = -j\omega\mu \overline{H}$
- $\nabla \times \overline{H} = -j\omega\varepsilon \overline{E} + \sigma \overline{E}$ → Differential form
- $\nabla . \overline{E} = \rho/\varepsilon$
- $\nabla . \overline{H} = 0$
- $\overline{D} = \varepsilon \overline{E}$
- $\overline{B} = \mu \overline{H}$
- $\int \overline{\mathbf{E}} \bullet \overline{\mathbf{d}\ell} = -j\omega\mu \iint_{s} \overline{\mathbf{B}} \bullet \overline{\mathbf{d}}\mathbf{s}$
- $\int \overline{\mathbf{H}} \bullet \overline{\mathbf{d}\ell} = (-j\omega\mu + \sigma) \iint_{s} \overline{\mathbf{E}} \bullet \overline{\mathbf{d}}\mathbf{s}$ → Integral form
- $\iint_{s} \overline{\mathbf{E}} \bullet \overline{\mathbf{d}}\mathbf{s} = Q/\varepsilon$
- $\iint_{s} \overline{\mathbf{B}} \bullet \overline{\mathbf{d}}\mathbf{s} = 0$
- $\overline{D} = \varepsilon \overline{E}$
- $\overline{B} = \mu \overline{H}$

Problem Set #1

P.1-1 Three corners of a triangle are at **P**$_1$(0, 1, -2), **P**$_2$ (4, 1, -3), and **P**$_3$ (6, 2, 5).

a) Determine whether the triangle **P**$_1$**P**$_2$**P**$_3$ is a right triangle.

b) Find the area of the triangle.

P.1-2 Given the vector $\overline{A}$ =3**a**$_x$ + 4**a**$_y$ – 6**a**$_z$ in cartesian. Express this vector in the following coordinate systems:

a) Cylindrical coordinate system.

b) Spherical coordinate system.

P.1-3 A vector field $\overline{F}$ is expressed in spherical coordinates as:

$$\overline{F} = (25/r^2)\,\mathbf{a}_r.$$

a) Find $|\overline{F}|$ and F_x at the point **P** (-3, 4, -5).

b) Find the angle that "**F**" makes with the vector $\overline{B}$ =2**a**$_x$ - 2**a**$_y$ +**a**$_z$ at the point **P**(-3, 4, -5).

P.1-4 Given the vector field function $\overline{F} = x^2y\,\mathbf{a}_x + xy^2\,\mathbf{a}_y$, evaluate the scalar line integral $\int_C \overline{F}.\overline{d\ell}$ from the point **P**$_1$(2, 1, -1) to the point **P**$_2$ (8, 2, -1)

a) Along the parabola $x=2y^2$

b) Along the straight line joining the two points.

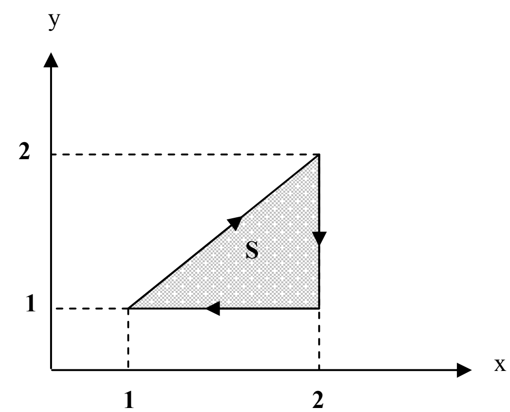
P.1-5 Given a scalar function $V=\sin(\frac{\pi}{2}x)\,\sin(\frac{\pi}{3}y)\,e^{-z}$. Find the magnitude and direction of the maximum rate of increase of V at the point **P** (1, 2, 3).

P.1-6 For the vector function $\overline{A} = \rho^2\mathbf{a}_\rho + 2z\,\mathbf{a}_z$, verify the divergence theorem for circular cylinder region enclosed by ρ=5, z=0, and z=4.

P.1-7 A vector field $\overline{A}$ **=**$\mathbf{a}_r$ $(\cos^2\Phi)/r^3$ exists in the region between two spherical shells defined by r=1 and r=2. Evaluate:

a) $\oint_S \overline{A} \cdot \overline{ds}$ b) $\int_V \nabla \cdot \overline{A}\, dv$

P.1-8 Given a vector field $\overline{A}$ **=**$3x^2y^3\, \mathbf{a}_x - x^3y^2\, \mathbf{a}_y$. Verify stokes theorem for the contour shown in figure.



HW:

P1.1, P1.3, P1.5, P1.11, P1.12, P1.15, P1.29, P3.29, P3.30, P4.5

Problem Set #2

P2-1 Two charges q_1 = +2 mc, q_2 = -5 mc are located at points (2,0,0) and (-2,0,0) respectively. Find the force acted on a third charge q_3=+3mc if it is located at (0, 0, 6). Find E at P (3, 3, 8).

P2-2 Consider a charged dielectric sphere of radius a = 0.5m and ρ_v=2mc/m^3. Find the electric field at r = 0.2 m, r = 0.5m, and r=2m. Plot the variation of the electric field versus r.

P2-3 A coaxial line has an inner conductor of a radius “a” and outer conductor of a radius “b”. The inner conductor is charged by +ρ_ℓ while the outer is charged by -ρ_ℓ. Find electric field as function of **ρ**, and then find the potential difference between outer and inner conductors.

P2-4 A circular ring has a radius a=2m lies in z=0 plane with its center at the origin. If ρ_ℓ =10 nc/m. Find a point charge at the origin which could produce the same electric field at P (0, 0, 5).

P2-5 Two uniform infinite line charges ρ_ℓ= 4 nc/m lies at x = 0, y=$\pm$4 and parallel to z – axis. Find the field at (4, 0, z).

P2-6 Show that the electric field is zero inside conducting sphere charged by Q=5 nc and has radius 5=5 m. Find E at r=10 m.

P2-7 Find the work don-e to move Q = 5 μc from the origin to the point P (2, $^{\pi}/_{4}$, $^{\pi}/_{2}$) in an electric field: $\overline{E} = 5\, e^{-r/4}\, a_r + \frac{10}{r \sin\theta} a_\phi$ V/m.

P2-8 Prove that the potential of a single infinite line charge (consider a reference point at $\rho=\rho_o$ has a zero potential) $V = \frac{\rho_\ell}{2\pi\varepsilon_0} \ln(\frac{\rho_0}{\rho})$.

P2-9 Three uniform finite line charges + ρ_{ℓ_1} , ρ_{ℓ_2}, and ρ_{ℓ_3} each of length "L" forming an equilateral triangle. Assuming that $\rho_{\ell_1}=2\rho_{\ell_2}= 2\rho_{\ell_3}$. Determine **E** at the center of the triangle.

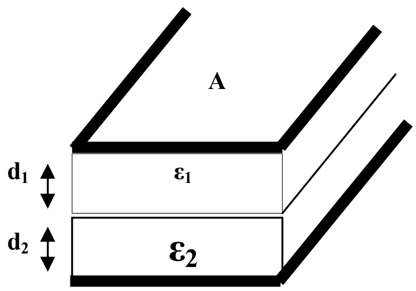
<u>HW</u>:

P2.11, P2.18, P.2.22, P2.26, P3.2, P3.9, P3.15, P3.20, P4.2, P4.6, P4.8, P4.16, P4.33, P4.35

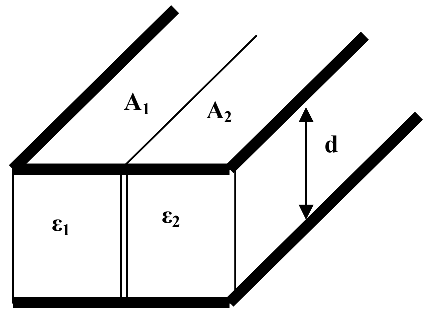
<u>Problem Set#3</u>

P.3.1 Find the capacitance of the following structures:

(a) Two parallel plates:

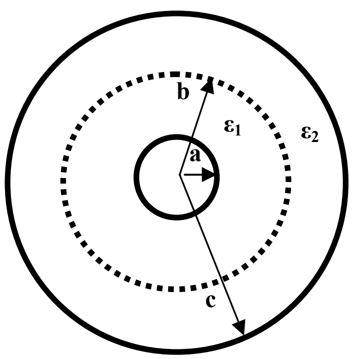


(i)

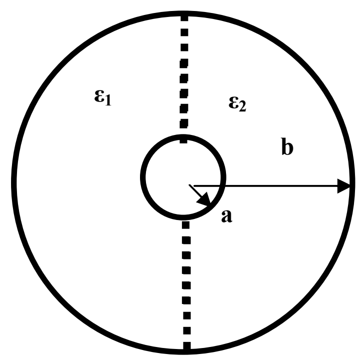


(ii)

(b)Coaxial lines:



(i)

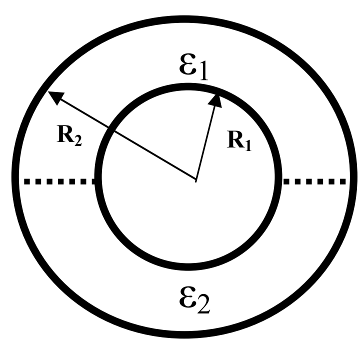


(ii)

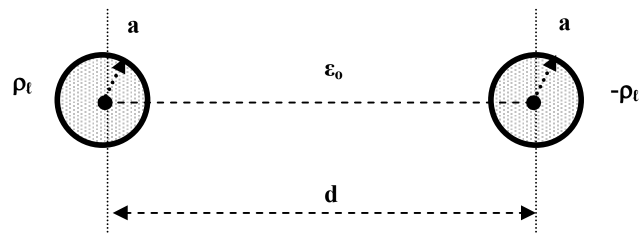
(c) Concentric spheres:

$R_1 = a$

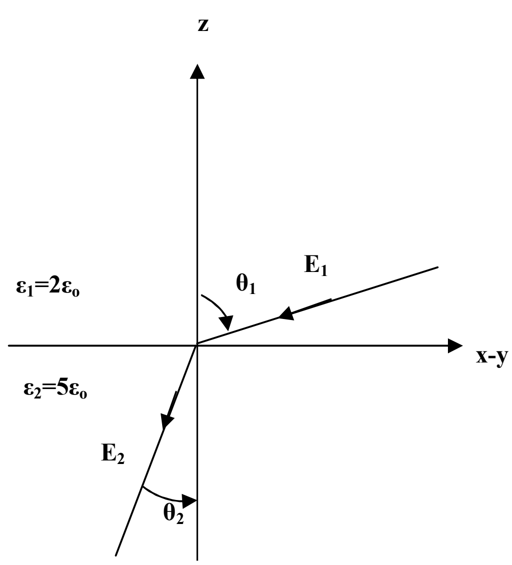
$R_2 = b$



P.3.2. Find the capacitance between two identical and long cylindrical conductors of radius **"a"** as shown in the following figure. These conductors are separated by air, and the distance between their centers is **"d"**.



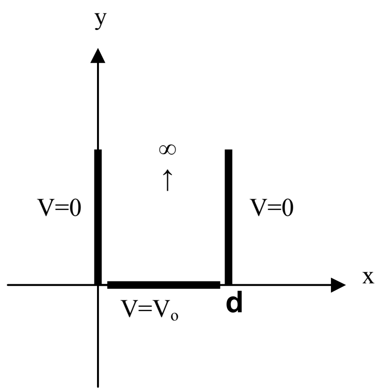
P.3.3. Given that $\bar{E}_1 = 2\,\mathbf{a}_x - 3\,\mathbf{a}_y + 5\,\mathbf{a}_z$ v/m as shown in the following figure. Find $\mathbf{D_2}$ and the angles θ_1 and θ_2.

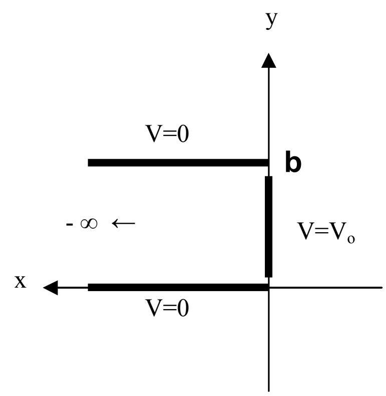


<u>HW</u>:
P6.5, P6.7, P6.10, P6.12, P6.13, P6.16, P6.19, 6.31

Problem Set #4

P.4.1 Find the potential **V** inside the following drawn wells:

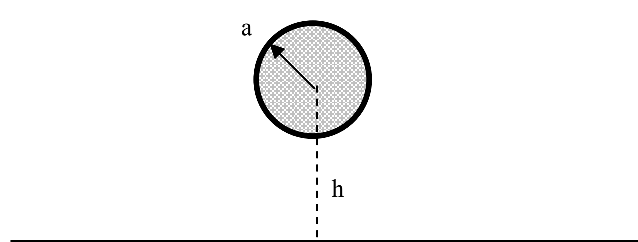




P.4.2 Find V between the two planes $\phi = 0$ and $\phi = \frac{\pi}{4}$ if the potential V = 0 at $\phi = 0$ and V = V_o at $\phi = \frac{\pi}{4}$.

P.4.3 Find V between two coaxial cones if V = o at $\theta = \frac{\pi}{4}$ V = V_o at $\theta = \frac{\pi}{10}$

P.4.4 A straight conducting wire of radius "a" is parallel to and at height "h" from the surface of earth as shown in the following figure. Assuming the earth is perfectly electric conducting; determine the capacitance and the force per unit length between the wire and the earth.



P.5 Find the resistance of the different configurations given in **P.3.1** of problem set#3, assume lossy media. [Hint: use duality equation $\mathbf{R}\times\mathbf{C} = \frac{\varepsilon}{\sigma}$].

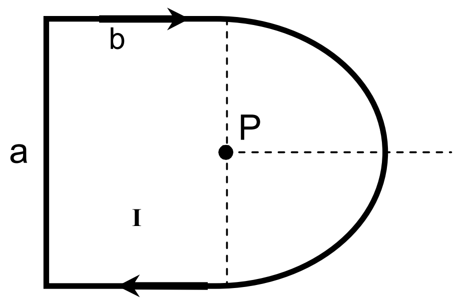
P.4.6 Find the resistance of only one of the different configurations given in **P.3.1** of problem set#3, assume lossy media. [Hint: use Laplace equation].

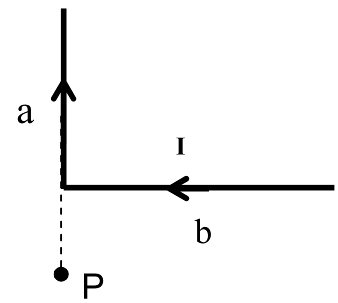
HW:

P7.6, P7.10, P7.15,

Problem Set #5

P.5.1 Find the magnetic field at point P due to the stationary currents going through the following wire configurations:





P.5.2 Find the flux inside a rectangle of dimensions a x b near a straight wire with current I as shown in **Fig.1**.

P.5.3 Find the magnetic flux density **B** of a wire carrying constant current $\mathbf{I_1}$ at the point **P** as shown in **Fig.2**. Assume $\mathbf{I_1}$**=0.5 mA, d=0.3m**, and the loop radius **b=0.05m**.

P.5.4 Find the inductance of the following magnetic coil shown in **Fig.3**.

P.5.5 Consider a transmission line of two long parallel conducting wires of radius "a" as shown in **Fig.4**. Assume that the two wires are located in x-z plane and they carry equal currents in opposite direction. Find the internal inductance, mutual (external) inductance and the magnetic force.

Fig. 1

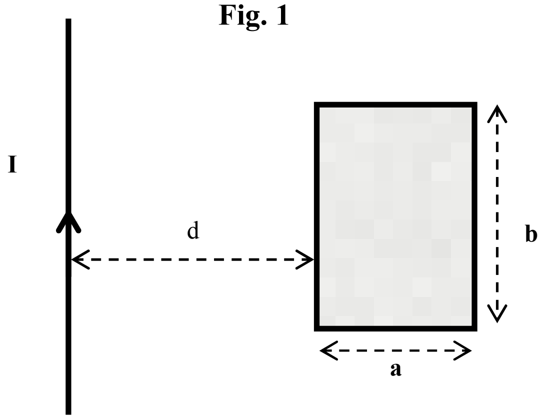


Fig. 2

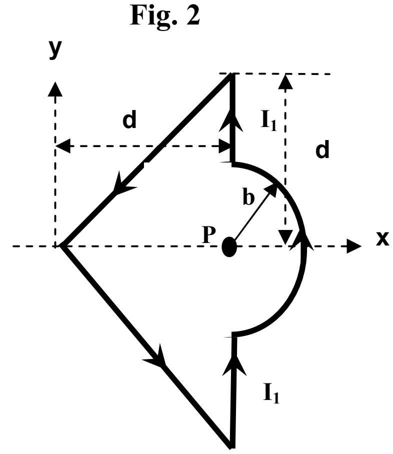


Fig. 3

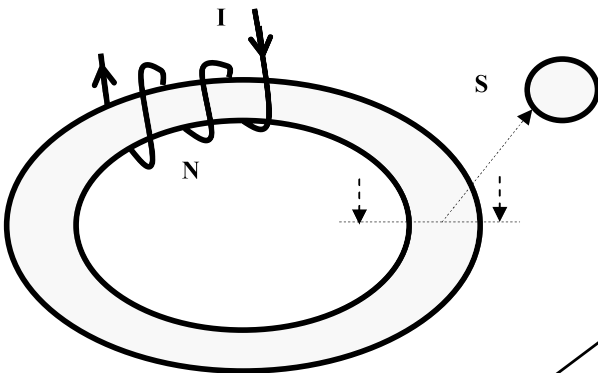
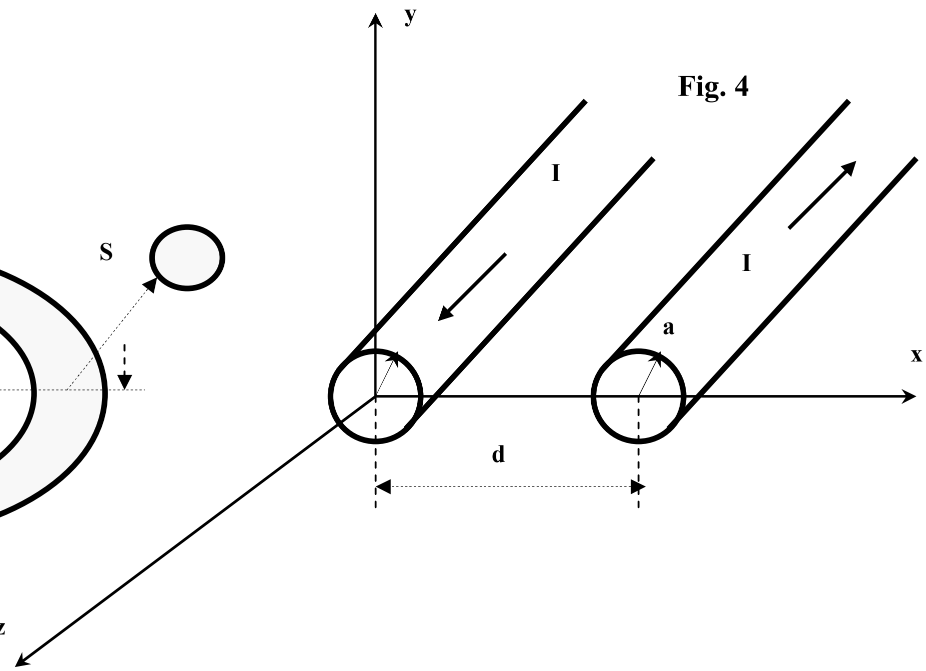


Fig. 4



P.5.6 Given $\overline{E}$(t, z) = E_o sin (ω t – βz)$\mathbf{a}_x$, find $\overline{D}$ and $\overline{\mathrm{H}}$ in the free space.

Sketch $\overline{\mathrm{E}}$ and $\overline{\mathrm{H}}$ at t = 0.

P.5.7 An electric field component in y-direction propagates in the free-space along the z-direction. The electric field intensity has an instantaneous form given by:

$$\mathbf{E\ (t, z) = Re\ (E_o} e^{j[\omega_o t - (\pi/3) z]}\ \mathbf{a_y)}$$

Find **D** (t, z) and **B** (t, z).

<u>**HW**</u>**:**

P8.5, P8.9, P8.12, P8.25, P8.27, P8.36, P8.41, P10.15, P10.18, P10.26

COLLEGE OF ENGINEERING & TECHNOLOGY

Department: Electronics and Communications Engineering

Instructor: Dr. Hussein Hamed Ghouz

Course Title: Electromagnetics

Course No.: EC341 **Marks: 30**

Date: Tue., May, 3, 2011 **Time: 60 Min**

Answer the following questions: (Version A)

Question No. 1

(a) Given Two vectors $\mathbf{A}=4a_x+3a_y+5a_z$ and $\mathbf{B}=5a_\rho+4a_z$ with an angle $\mathbf{\Phi=36.87^o}$. Find the cross and dot product in cylindrical coordinate. Find the angle between **A** and **B**

(b) Given a vector of flux density $\mathbf{D}=xy^2a_x+4xya_y$ C/m^2. Verify the divergence theorem within a parallelepiped formed by planes $0\le x\le 2$, $0\le y\le 3$, and $0\le z\le 4$.

(c) Find the curl of the vector **D**

Question No. 2

Three point charges $Q_1=10$ nc, $Q_2=-5$ nc, and $Q_3=-10$ nc are located in space at the points P_1 $(x_o,0,0)$, P_2 $(0,y_o,0)$, and P_3 $(0,0,z_o)$ respectively. Find the following:

(a) The electric force acting on Q_3 assume $x_o=0$, and $y_o=z_o=1.0$

(b) The electric field and potential at the point P $(x_o, 2y_o, 3z_o)$

(c) The electric energy $\mathbf{W_e}$

Question No. 3

Given a Charged Disk of radius b = 50 cm, and charge density ρ_s = 50 nc/m^2 is located in x-y plane as shown in the following figure. Find the following:

(a) The electric field at the point P

(b) The electric field at the point P if $b\to\infty$

(c) Verify the results of part (b) using **Gauss' Law**

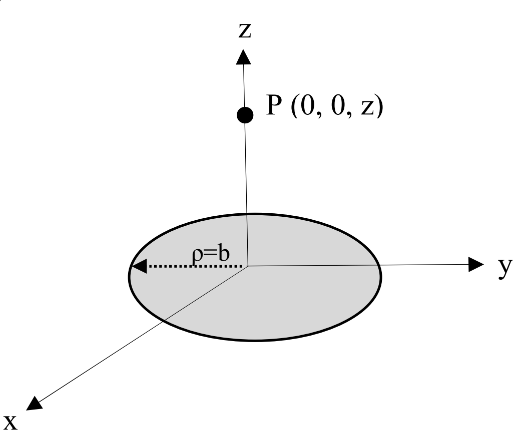
Formula Sheet

$$\oiint_S (\nabla\times\mathbf{D})\bullet d\mathbf{s}=\int_c \mathbf{D}\bullet d\ell \qquad \oiint_S \mathbf{D}\bullet d\mathbf{s}=\iiint_V \nabla\bullet\mathbf{D}\ dV \qquad \nabla\bullet\mathbf{D}=\frac{\partial D_x}{\partial x}+\frac{\partial D_y}{\partial y}+\frac{\partial D_z}{\partial z}$$

$$\nabla\times\mathbf{D}=\left(\frac{\partial D_z}{\partial y}-\frac{\partial D_y}{\partial z}\right)a_x-\left(\frac{\partial D_z}{\partial x}-\frac{\partial D_x}{\partial z}\right)a_y+\left(\frac{\partial D_y}{\partial x}-\frac{\partial D_x}{\partial y}\right)a_z$$

$$\int\frac{dx}{(x^2+a^2)^{3/2}}=\frac{x}{a^2\sqrt{x^2+a^2}}$$

$$\int\frac{x\,dx}{(x^2+a^2)^{3/2}}=\frac{-1}{\sqrt{x^2+a^2}}$$



GoodLuck

COLLEGE OF ENGINEERING & TECHNOLOGY

Department : Electronics and Communications Engineering

Instructor : Dr. Hussein Hamed Ghouz

Course Title : Electromagnetics

Course No.: EC341 *Marks: 20*

Date : Sun., May, 29, 2011 *Time: 60 Min*

Answer the following questions: (Version_A)

(a) Given a *cylindrical capacitor* ($b=2.7a$, $V_o=5$ & $\varepsilon_2=4\varepsilon_1$) shown in Fig. 1. Using Gauss' law, find the following:

1. The electric field vector E and the polarization vector P in each region
2. The electric energy W_e, and the bounded charge densities on inner and outer conductors
3. The capacitance in each region

(b) Given two, isolated and infinite conducting planes form a sector of an angle $\alpha=60^o$ as shown in Fig.2. One of the conducting planes is kept at constant potential $V_o=10$ volt while the other is grounded. Solve the Laplace equation to find the following:

1. The potential distribution V between the conducting planes
2. The electric field distribution E between the conducting planes
3. The capacitance

(c) Given two ideal dielectric regions as shown in Fig.3. The electric field in the first region is $E_1= 4a_\rho+2a_z$ with an angle $\Phi=30^o$. Using the boundary condition of the electric field, find D_1, D_2, α_1 & α_2

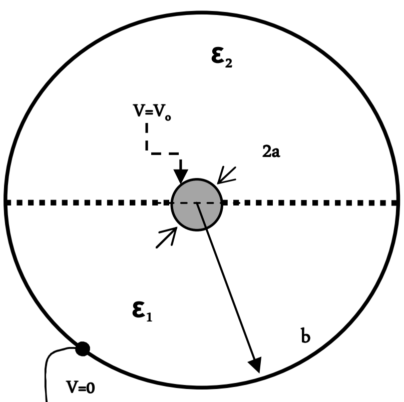


Fig.1

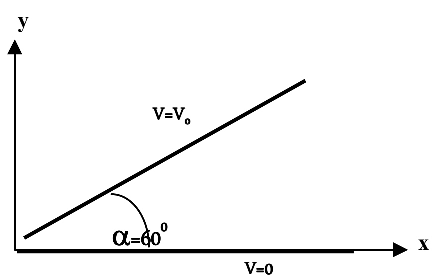


Fig.2

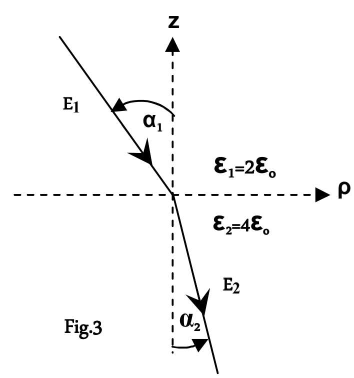


Fig.3

$$\nabla V = \mathbf{A} = \frac{\partial V}{h_1 \partial u_1}\mathbf{a}_{u_1} + \frac{\partial V}{h_2 \partial u_2}\mathbf{a}_{u2} + \frac{\partial V}{h_3 \partial u_3}\mathbf{a}_{u_3}$$

$$\nabla^2 V = \frac{1}{h_1 h_2 h_3}\left(\frac{\partial}{\partial u1}(h_2 h_3 A_1) + \frac{\partial}{\partial u2}(h_1 h_3 A_2) + \frac{\partial}{\partial u3}(h_1 h_2 A_3)\right)$$

$$W_e = \frac{\varepsilon}{2}\iiint_{v'} (\bar{E}\cdot\bar{E})\, dv'$$

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COLLEGE OF ENGINEERING & TECHNOLOGY

Department: Electronics and Communications Engineering
Lecturer: Associate Prof. Dr. Abd-El-Hamid Gafer
Associate Prof. Dr. Hussein Hamed Ghouz
Course: Electromagnetics-I

Course Code: EC341 | Time : 120 Min
Date : Thr. 30, June, 2011 | Total Marks : 40

Answer All Question

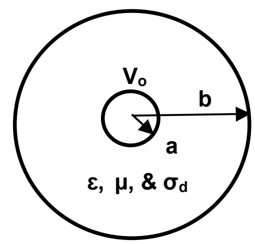
Question No. 1 (8 Mark)

(a) A lossy material (**ε=4.6ε_o, σ_d=3x10^{-2} & μ=μ_o**) is used to fill the space between the inner and outer conductors of a coaxial cable as shown in the figure. The inner conductor is kept with a positive voltage **V_o=5** volt and the outer conductor is grounded. Assume, the thickness of the outer conductor can be neglected, find the following**:**

1. The capacitance **C** and the inductance **L** per unit length
2. The conductance **G=1/R** per unit length
3. Draw the equivalent circuit

(b) Find the coaxial cable radii ratio to achieve characteristic impedance

$$Z_o = \sqrt{\frac{L}{C}} = 50\ \Omega$$

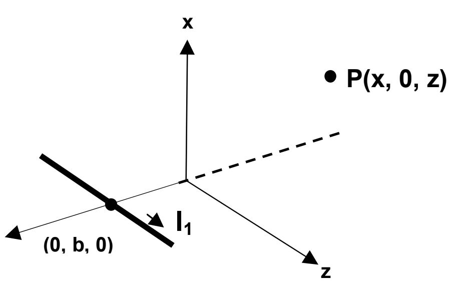


Question No.2 (10 Mark):

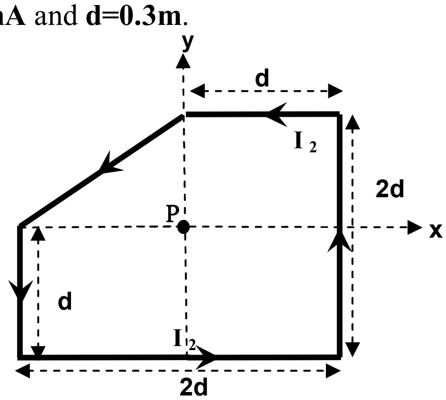
(a) Consider a finite line of a length **ℓ=0.5m** carrying a stationary current **I_1=100** mA as shown in the figure. The line is located symmetrically on y-axis at **b=0.5m** and parallel to-z-axis. Find the magnetic field intensity at the point **P(x, 0, z)**

(b) Find the magnetic flux density **B** of a line conductor configuration carrying constant current **I_2** at the point **P** as shown in the figure. Assume **I_2=50 mA** and **d=0.3m**.

(a)



(b)



Question No.3 (8 Mark)

(**a**) Write down the set of Maxwell's equations in differential form for Complex Exponential Time-Harmonic fields

(**b**) A Time-Harmonic electric field having an **x-component** propagates in a general lossless meduim (ε=4.6ε_o& μ=$\mu_o\mu_r$) along the **z-direction**. The electric field intensity has an instantaneous form given by:

$$\mathbf{E}\,(t, z) = 10^{+03} \sin\left(2\pi \times 10^{+07} t - \frac{\pi}{4} z + \frac{\pi}{3}\right) \mathbf{a}_x$$

Find **D** (t, z), **B** (t, z) and μ_r

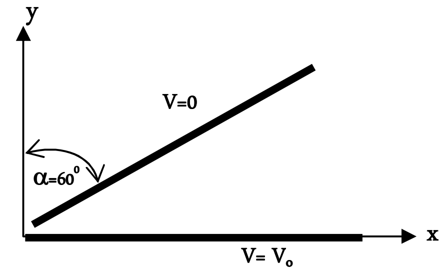
P.T.O.

Question No.4 (10 Mark)

(a) Given two isolated and infinite conducting planes form a sector of an angle **α=60°** as shown in figure. One of the conducting planes is kept at constant potential $\mathbf{V_o}$ while the other is grounded. Solve the Laplace equation to find the potential and the electric field distributions between the conducting planes (Hint: V is function of the angle ϕ)

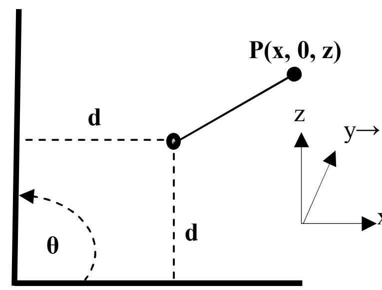
$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial Z^2}$$

$$\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{\partial V}{\rho \partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z$$



(b) Given a corner shape of infinite copper conducting planes having a zero potential and an angle **θ=90°** as shown in figure. Assume a point charge Q= 20 nC is located at the mid-point between the two conductors (x=d, z=d, d=0.5m). Use the image method to find the following**:**

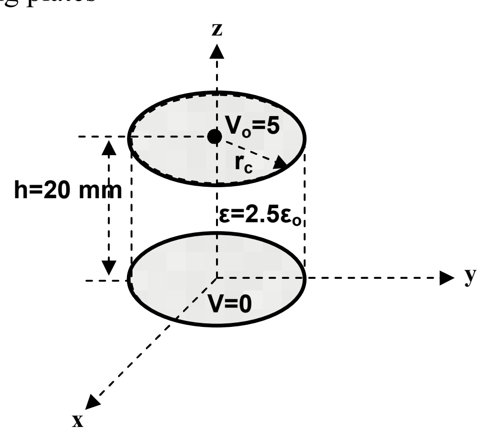
1. The total number of images to replace the ground planes
2. The electric potential and electric field at observation point P(x, 0, z)



Question No.5 (4 Mark)

Two identical circular conducting plates each of radius $\mathbf{r_c}$=2.5mm form a parallel plate capacitor as shown in figure. Find the following:

1. The electric field intensity, electric flux density, and polarization vectors
2. The capacitance
3. The bounded surface charge on upper and lower conducting plates



Constants

$1/(4\pi\varepsilon_o) = 9\text{x}10^9$ m/F
$\varepsilon_o = 10^{-09}/(36\text{x}\pi)$ F/m
$\mu_o = 4\pi\text{x}10^{-07}$ H/m

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