Arab Academy for Science & Technology and Maritime Transport

College of Engineering & Technology

Department of Electronics and Communications Engineering

Cairo Campus

EC 341 Electromagnetic Field Theory S- 2014/2015

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Objective	*After completing this course*: the student should be able to known: - Coordinate system and vector analysis - Columns' law and Gauss' law for electrostatic - The electric field and electric flux for different types of static charges - Work done and electric energy - Solving the electrostatic boundary value problems - Currents and conductors - Static magnetic field, Biot Sevart' Law and magnetic energy - Basic equations of static fields and boundary conditions - Concept of time-varying fields and Maxwell's equations
Text Book	**William H. Hayt, Jr. and John A. Buck** "Engineering Electromagnetics", McGraw Hill, 8th Edition, 2012
References	**Nathan Ida**, "Engineering Electromagnetics", Springer-Verlag, 2nd Edition, 2012
Grading	**7th Week (30%):** ✓ 7th Exam: 30% Exam **12th Week (20%):** ✓ 12th Exam: 20% Exam **Attendance and Activities (10%)** ✓ Quiz-1 and Assign.-1 5% ✓ Quiz-2 and Assign.-2 5% **Final Exam (40%)**

Course Plan

1. Coordinates systems and vector analysis (<u>1-Lecture</u>)
- Coordinates systems - Vector operations - Vector analysis

2. Static Electric Field (<u>5-Lectures</u>)
- Column's law for electric force
- Electric field and electric flux in free-space
- Gauss' Law
- Work Done and Electrostatic Potential
- Electric Dipole
- Dielectrics and Polarization
- Boundary condition of static electric field
- Basic Equations of Static Electric Field
- Capacitance

7th Exam

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3. Electrostatic Problems (<u>2-Lectures</u>)
- Image Method
- Boundary Value Problems and Laplace and Poisson Equations

4. Currents and Conductors (<u>1-Lecture</u>)
- Ohm' Law
- Joule' Law
- Resistance
- Boundary condition of stationary currents density

12th Exam

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5. Static Magnetic Field (<u>3-Lectures</u>)
- Basic Equations of Static Magnetic Field
- Ampere's Law - Biot-Savart Law - Magnetic Vector Potential
- Boundary condition of static magnetic field
- Magnetic Force
- Inductance

6. Time-Varying Fields and Maxwell's Equations (<u>1-Lectures</u>)
- Faraday' Law
- Displacement Current
- Maxwell' Equations
- Time-Harmonic Fields
- Uniform Plane Wave

Final week 16th

Exercise and Quiz Test Plan

Problem Set No.	Week No.	Quiz Test
Problem set # 1	1st 2nd, 3rd & 4th Week	
Problem set # 2		
Problem set # 3	5th & 6th Week	1st Quiz 5th Week
Problem set # 4	8th , 9th & 10th Week	
Problem set # 5	11th, 13th & 14th Week	2nd Quiz 10th Week

1. Coordinate Systems and Vector Analysis

$$\nabla V = \frac{\partial V}{h_1 \partial u_1} \mathbf{a}_{u1} + \frac{\partial V}{h_2 \partial u_2} \mathbf{a}_{u2} + \frac{\partial V}{h_3 \partial u_3} \mathbf{a}_{u3}$$

$$\nabla^2 V = \frac{1}{h_1 h_2 h_3} \left(\frac{\partial}{\partial u1}\left(h_2 h_3 \left(\frac{1}{h_1} \frac{\partial V}{\partial u_1} \right) \right) + \frac{\partial}{\partial u2}\left(h_1 h_3 \left(\frac{1}{h_2} \frac{\partial V}{\partial u_2} \right) \right) + \frac{\partial}{\partial u3}\left(h_1 h_2 \left(\frac{1}{h_3} \frac{\partial V}{\partial u_3} \right) \right) \right)$$

$$\nabla . \overline{A} = \frac{1}{h_1 h_2 h_3} \left(\frac{\partial}{\partial u1}(h_2 h_3 A_1) + \frac{\partial}{\partial u2}(h_1 h_3 A_2) + \frac{\partial}{\partial u3}(h_1 h_2 A_3) \right)$$

$$\nabla \times \overline{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \mathbf{a}_{u1} & h_2 \mathbf{a}_{u2} & h_3 \mathbf{a}_{u3} \\ \frac{\partial}{\partial u1} & \frac{\partial}{\partial u2} & \frac{\partial}{\partial u3} \\ h_1 A_{u1} & h_2 A_{u2} & h_3 A_{u3} \end{vmatrix}$$

Divergence Theorem: $\int_{\mathbf{v}} \nabla \bullet \overline{\mathbf{A}}\, \mathbf{dv} = \oint_{\mathbf{s}} \overline{\mathbf{A}} \bullet \mathbf{ds}$

Stokes's Theorem: $\int_{\mathbf{s}} (\nabla \times \overline{\mathbf{A}}) \bullet \mathbf{ds} = \oint_{\mathbf{c}} \overline{\mathbf{A}} \bullet \mathbf{d}\ell$

Metric coefficients of Coordinate systems

	(x, y, z)	**(ρ, Φ, z)**	**(r, θ, Φ)**
$\mathbf{a}_{u1}$	$\mathbf{a}_x$	$\mathbf{a}_\rho$	$\mathbf{a}_r$
$\mathbf{a}_{u2}$	$\mathbf{a}_y$	$\mathbf{a}_\Phi$	$\mathbf{a}_\theta$
$\mathbf{a}_{u3}$	$\mathbf{a}_z$	$\mathbf{a}_z$	$\mathbf{a}_\Phi$
h_1	1	1	1
h_2	1	ρ	r
h_3	1	1	r sin θ

Transformation between Coordinate Systems

	a_ρ	a_Φ	a_z	a_r	a_θ	a_Φ
$a_x \cdot$	$\cos\Phi$	$-\sin\Phi$	0	$\sin\theta\cos\Phi$	$\cos\theta\cos\Phi$	$-\sin\Phi$
$a_y \cdot$	$\sin\Phi$	$\cos\Phi$	0	$\sin\theta\sin\Phi$	$\cos\theta\sin\Phi$	$\cos\Phi$
$a_z \cdot$	0	0	1	$\cos\theta$	$-\sin\theta$	0

Integral Forms

Integral Form	Result of Integration
$\int \frac{dx}{\sqrt{x^2+a^2}}$	$\ln(x+\sqrt{x^2+a^2})$ $\sinh^{-1}\frac{x}{a}$
$\int \frac{xdx}{\sqrt{x^2+a^2}}$	$\sqrt{x^2+a^2}$
$\int \frac{dx}{(x^2+a^2)^{3/2}}$	$\frac{x}{a^2\sqrt{x^2+a^2}}$
$\int \frac{x\ dx}{(x^2+a^2)^{3/2}}$	$\frac{-1}{\sqrt{x^2+a^2}}$
$\int \frac{dx}{(x^2+a^2)}$	$\frac{1}{a}\tan^{-1}\frac{x}{a}$
$\int \frac{xdx}{(x^2+a^2)}$	$\frac{1}{2}\ln(x^2+a^2)$
$\int \frac{dx}{x\sqrt{(x^2+a^2)}}$	$\frac{-1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$
$\int \frac{dx}{\sin\ ax}$	$\frac{1}{a}\ln\left\lvert\tan\frac{ax}{2}\right\rvert$

Differential Calculus

Function	Differentiation
sin(x)	cos(x)
cos(x)	-sin(x)
tan(x)	$\sec^2(x)$
$\sin(x)^{-1}$	$\frac{1}{\sqrt{1-x^2}}$
$\cos(x)^{-1}$	$\frac{-1}{\sqrt{1-x^2}}$
$\tan(x)^{-1}$	$\frac{1}{1+x^2}$
e^x	e^x
ln(x)	1/x
$\log_a(x)$	1/[x ln(a)]

Trigonometric relations

$\sin(x) = \sqrt{1/2(1-\cos(2x))}$
$\cos(x) = \sqrt{1/2(1+\cos(2x))}$
$\tan(x) = \frac{\sqrt{(1-\cos(2x))}}{\cos(x)}$
$\sin(2x) = 2\sin(x)\cos(x)$
$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$
$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$
$e^{\pm jx} = \cos(x) \pm j\sin(x)$

2. Static Electric Field

1. Coulomb's law:

$\bar{F}_2 = k \frac{q_1 q_2}{R^2} \bar{a}_r$, Where, $k=1/(4\pi\varepsilon_0)= 9\times10^9$, and $\bar{a}_R$ is the unit vector in the direction of the force ($\underline{a}_r = \hat{a}_r = \bar{a}_r = \bar{r}/r$). If the force is negative, it is **attraction** force, while the positive sign means **repulsion** force.

2. Electrostatic Field and Potential (E and V):

Point	$\vec{E}_P = \frac{q_i}{4\pi\varepsilon_o R_i^2} \vec{R}$	$V_P = \frac{q_i}{4\pi\varepsilon_o R_i}$
Line	$\bar{E}_P = \frac{1}{4\pi\varepsilon_o} \int \frac{\rho}{R^3} \vec{R}\, d\ell'$	$V_P = \frac{1}{4\pi\varepsilon_o} \int \frac{\rho}{R} d\ell'$
Surface	$\bar{E}_P = \frac{1}{4\pi\varepsilon_o} \iint_s \frac{\rho_s}{R^3} \vec{R}\, ds'$	$V_P = \frac{1}{4\pi\varepsilon_o} \iint_s \frac{\rho_s}{R} ds'$
Volume	$\bar{E}_P = \frac{1}{4\pi\varepsilon_o} \iiint_v \frac{\rho_v}{R^3} \vec{R}\, dV'$	$V_P = \frac{1}{4\pi\varepsilon_o} \iiint_v \frac{\rho_v}{R} dV'$
$\vec{E}_p = -\nabla V_P$ and $V_p = -\int_{Ref}^{P} \vec{E}_P \bullet \partial\vec{\ell}$ The Ref. point usually have a zero potential		

3. Gauss's law:

$$\oint_s \bar{D}.\bar{n}\, ds = Q \text{ (total charge enclosed)}$$

Where;

$$Q = \iiint_v \rho_v\, dv \quad \text{volume charge}$$
$$= \iint_s \rho_s\, ds \quad \text{surface charge}$$
$$= \int \rho_\ell\, d\ell \quad \text{line charge}$$

$$\oint_S \overline{D}.\overline{n}\, ds = 4\pi r^2 D_r \text{ for sphere of radius } r$$

$$= 2\pi\rho\,\ell\, D_\rho \text{ for cylinder of radius } \rho \text{ and length } \ell$$

4. Boundary Condition of electric field (air-conductor interface):

$E_n = \dfrac{\rho_s}{\varepsilon_o}$ and $E_t = 0$

5. Finite Line charge:

$$E_P = \frac{\rho_\ell}{2\pi\varepsilon_o\rho}\left(\frac{L/2}{\sqrt{\rho^2 + (L/2)^2}}\right)\mathbf{a}_\rho$$

$$V_P = \frac{\rho_\ell}{2\pi\varepsilon_o}\ln\left(\frac{\sqrt{\rho^2 + (L/2)^2} + L/2}{\rho}\right)$$

6. Infinite Line Charge:

$E_P = \dfrac{\rho_\ell}{2\pi\varepsilon_o\rho}\mathbf{a}_\rho$ and $V_P = \dfrac{\rho_\ell}{2\pi\varepsilon_o}\ln\left(\dfrac{\rho_o}{\rho}\right)$

7. Electric Dipole:

$V = \dfrac{p\cos\theta}{4\pi\varepsilon_o r^2}$ and $\overline{E} = \dfrac{1}{4\pi\varepsilon_o r^3}(2p\cos\theta\,\mathbf{a}_r + p\sin\theta\,\mathbf{a}_\theta)$

8. Basic Equations of Static Electric Field

$\overline{\nabla}.\overline{D} = \rho$ $\overline{\nabla}\times\overline{E} = 0$

9. Polarization vector P:

$$\overline{P} = \overline{D} - \varepsilon_o\overline{E} = (\varepsilon_r - 1)\varepsilon_o\overline{E}$$

10. Boundary Conditions of Electric Field (dielectric-dielectric interface):

$E_{1t} = E_{2t}$ and $D_{1n} - D_{2n} = \rho_s$ *(where, ρ_s is free charge)*

11. Capacitance:

$C = \dfrac{Q}{V}$

Where;

Q is the Gauss's law ($\oint_{s} \overline{D}.\overline{n}\, ds = \iiint_{v} \rho_v\, dv = Q$)

V is the line integral of **E** along the path

($V_{ab} = -\int_{final=b}^{intial=a} \overline{E}.\overline{d\ell}$ = V_a - V_b)

To find the capacitance, we follow the 4-steps:

1. Assume a charge +Q and -Q on the two conductors.

2. Find E using Gauss's law or any other method.

3. Find V using line integral along E lines

4. Put Q = C V, and then Find C.

12. Electrostatic Energy:

For N-discrete charge:

$$W_e = \frac{1}{2} \sum_{k=1}^{n} Q_k\ V_k$$

Where, $V_k = \frac{1}{4\pi \varepsilon_o} \sum_{j=1}^{n} \frac{Q_j}{Rjk}$ and $j \neq k$

For continuous charge:

$$W_e = \frac{1}{2} \iiint_{v'} (\overline{D}\ .\ \overline{E})\ d v' = \frac{\varepsilon}{2} \iiint_{v'} \left|\overline{E}\right|^2\ d v'$$

For any capacitor configuration:

$$W_e = \frac{1}{2} C V^2 = \frac{1}{2} Q V = \frac{Q^2}{2C}$$

Surface bounded charge for any capacitor configuration:

$$\rho_{Upper} = First\ \ Conductor = \overline{P} \bullet (-a_n)$$

$$\rho_{Lower} = Second\ \ Conductor = \overline{P} \bullet (+a_n)$$

3. Solution of Electrostatic Problems

1.Image Method:

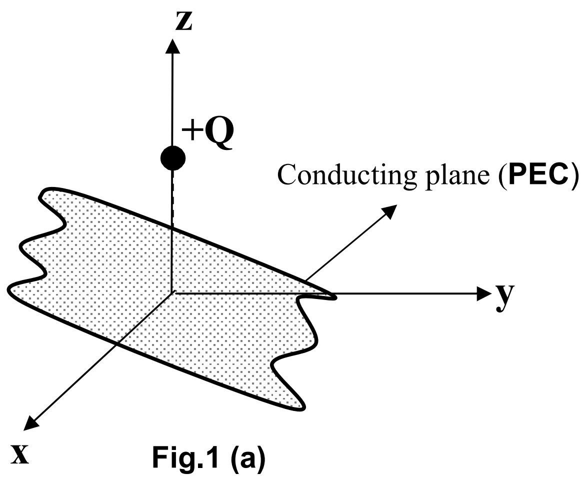


Fig.1 (a)

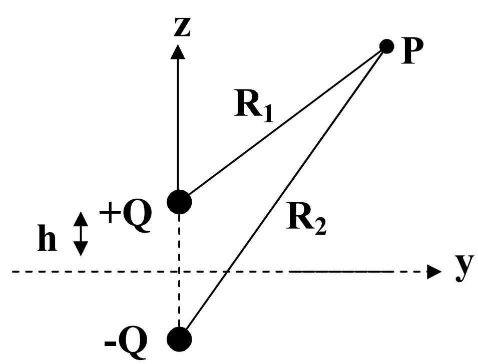


Fig.1 (b)

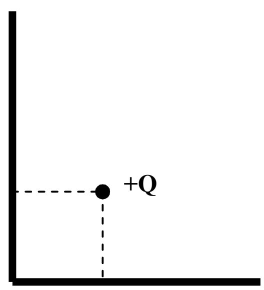


Fig.2 (a)

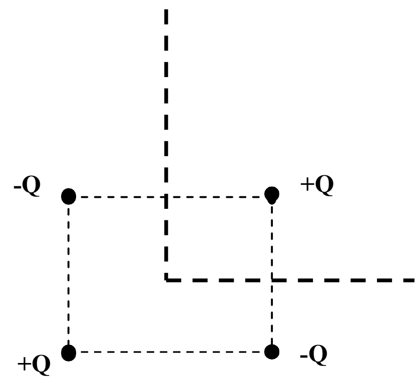


Fig.2 (b)

If the conductor is in the form of corner with an angle "α", then in this case, we have m images where,

$$m = 2n - 1 \text{ and } n = \frac{180}{\alpha}$$

2. Boundary Value Problems

In Cartesian:

$$\nabla^2 V_P = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) V_p = 0$$

The possible solutions of $W''(u) + k_u^2 W(u) = 0$

k_u^2	k_u	W(u)	
0	0	***Linear Solution*** **$A_o u + B_o$**	
+	k	***Periodic Solution***	
		$A_1 \sin(k u) + B_1 \cos(ku)$	$C_1 e^{+jku} + C_2 e^{-jku}$
-	jk	***Decayed Solution***	
		$A_2 \sinh(ku) + B_2 \cosh(ku)$	**$D_1 e^{+ku} + D_2 e^{-ku}$**

<u>In cylindrical:</u>

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial\varphi^2} + \frac{\partial^2 V}{\partial Z^2} = 0$$

In case of **Φ**-dependence solution is given as:

$$\Phi(\Phi) = A_o \Phi + B_o$$

In case of **ρ**-dependence solution is given as:

$$\frac{d}{d\rho}\left(\rho\frac{dV(\rho)}{d\rho}\right) = 0 \quad \Rightarrow \quad V(\rho) = A_1 \ln(\rho) + A_2$$

<u>In Spherical:</u>

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) = 0$$

<u>(a) In case of **r**-dependence only:</u>

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = 0 \quad \Rightarrow \quad V(r, \theta, \Phi) \equiv V(r)$$

Then:

$$\frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = 0 \quad \Rightarrow \quad \left(r^2\frac{dV}{dr}\right) = A_1$$

$$\frac{dV}{dr} = \frac{A_1}{r^2} \quad \Rightarrow \quad dV = \frac{A_1}{r^2} dr$$

$$V(r) = \left(\frac{-1}{r}\right) A_1 + A_2$$

The constant A_1 and A_2 are determined from the given BC.

(b) In case of **θ**-dependence only:

$$\frac{1}{r^2 \sin\theta} \frac{d}{d\theta}\left(\sin\theta \frac{dV}{d\theta} \right) = 0 \qquad \Rightarrow \qquad \sin\theta \frac{dV}{d\theta} = A_1$$

$$\frac{dV}{d\theta} = \frac{A_1}{\sin\theta} \qquad \Rightarrow \qquad dV = \int \frac{A_1}{\sin\theta} d\theta$$

Since the integration $\int \frac{dx}{\sin ax} = \frac{1}{a} \ln \left| \tan \frac{ax}{2} \right|$, therefore, the electric potential is given as:

$$V(\theta) = A_1 \ln \left| \tan \frac{\theta}{2} \right| + A_2$$

Where, the constant A_1 and A_2 are determined from the given BC

4. Currents and Conductors

1. Ohm's law:

In any conducting medium, if an electric field is applied, a current density J is given by:

$$\mathbf{J} = \sigma \mathbf{E}$$

Where,

σ Denotes conductivity of the conductor

J Denotes current density A/m^2

$$\sigma = \frac{J}{E} = \frac{A/m^2}{V/m} = \frac{A}{V\ m} = \frac{moh\ (siemens)}{m} = \mathbf{S/m}$$

The total current passing through an arbitrary surface S of a conducting body is given by

$$I = \iint_s \overline{\mathbf{J}}\cdot\overline{\mathbf{ds}}$$

2. Resistance:

$$R = \frac{V}{I} \quad \rightarrow \quad R = \frac{\int_1^2 \overline{E}\cdot\overline{d\ell}}{\sigma \iint_s \overline{E}\cdot\overline{ds}}$$

(a) The voltage V is applied along the coordinate u_1 while the current is passing across the surface of the plane u_2-u_3.

$$R = \int_o^L \frac{du_1}{\sigma \iint \frac{h_2 h_3}{h_1} du_2\, du_3}$$

(b) Solving Laplace Equation

1. Find the electric potential V
2. Find the electric field as $E = -\nabla V$
3. Find the current density $J = \sigma E$
4. Find the total current $I = \iint_s \overline{J}\cdot\overline{ds}$
5. Find the resistance R as $R = \frac{V}{I}$

3. Duality Law:

There is a dual relation between the resistance and the capacitance as:

$$\overbrace{R \;\; x \;\; C}^{\textit{resistance x capacitance}} = \underbrace{\frac{\varepsilon}{\sigma}}_{\substack{\textit{ratio of the constitutive parameters} \\ \textit{of the conducting medium}}}$$

Duality between J and D

Conductor	Dielectric
$J = \sigma E$	$D = \varepsilon E$
$\nabla . J = o$	$\nabla . D = o$
$J_{n1} = J_{n2}$	$D_{n1} = D_{n2}$
$\frac{J_{t1}}{\sigma_1} = \frac{J_{t2}}{\sigma_2}$	$\frac{D_{t1}}{\varepsilon_1} = \frac{D_{t2}}{\varepsilon_2}$
σ	ε
I	Q
$\frac{1}{R} = G$	C

5. Static Magnetic Field

Basic Equations of Static Magnetic Filed:

- $\nabla . \overline{B} = 0$, $\nabla \times \overline{H} = \overline{J}$, and $\overline{B} = \mu \overline{H}$

Ampere's Law

- $\nabla \times \overline{B} = \mu_o \overline{J}$ Differential form
- $\oint_c \overline{B} . \overline{d\ell} = \mu_o I$ Integral form

Biot-Savart Law for a line carrying constant current:

- $\overline{H} = \int_L \frac{I\, \overline{d\ell} \times a_R}{4 \pi R^2}$

For finite Line carrying constant current:

$$\overline{H} = \frac{I}{4 \pi d} \left[\sin(\text{the angle in current direction}) - \sin(\text{the angle in oppsite current direction}) \right] \hat{a}_n$$

Magnetic Vector Potential for a line carrying constant current:

- $\overline{A} = \int_c \frac{\mu_o I\, \overline{d\ell}}{4 \pi R}$

Loop of constant current I and radius r=b:

- $\overline{H}_{center} = \frac{I}{4b}(\pm \mathbf{a_z})$

Boundary Conditions:

$H_1 L = H_2 L \rightarrow H_{1t} = H_{2t}$
$B_{1n} = B_{2n} \rightarrow \mu_1 H_{1n} = \mu_2 H_{2n}$

In case of current existing at the boundary surface:
$H_{1t} - H_{2t} = J_s$

Magnetic Force F_m:

$$\overline{\mathbf{F}}_{12} = \oint_c I_2 \, \overline{\mathbf{d}\ell}_2 \times \overline{\mathbf{B}}_1 \quad \text{and} \quad \overline{\mathbf{F}}_{21} = \oint_c I_1 \, \overline{\mathbf{d}\ell}_1 \times \overline{\mathbf{B}}_2$$

Magnetostatic Energy:

$$W_m = \frac{1}{2} \iiint_{v'} (\overline{B} \cdot \overline{H}) \, dv' = \frac{\mu}{2} \iiint_{v'} |\overline{H}|^2 \, dv'$$

Inductance L:

To find the inductance of any geometry, we should do the following steps:

1. Choose the suitable coordinate system, and assume source current I.

2. Find the magnetic field density **B** (or **H**)

- Ampere's Law (if there is a symmetry)
- Biot-Savart Law
- Magnetic vector potential $\overline{B} = \nabla \times \overline{A}$

3. Find the magnetic flux ψ as $\psi = \iint_s \overline{B} \cdot \overline{ds}$

4. Find the flux linkage $\Lambda = \psi N$

5. Find L as $L = \Lambda / I$

6. Maxwell's Equations for Time-Varying Fields

Differential form	Integral form
$\nabla \times \overline{E} = -\dfrac{\partial \overline{B}}{\partial t}$	$\oint E.d\ell = -\iint \dfrac{\partial B}{\partial t}.ds$
$\nabla \times \overline{H} = J + \dfrac{\partial \overline{D}}{\partial t}$	$\oint H.d\ell = I + \iint \dfrac{\partial D}{\partial t}.ds$
$\nabla . \overline{D} = \rho$	$\oint D.ds = \iiint \rho\, dv$
$\nabla . \overline{B} = 0$	$\oint B.ds = o$
$\overline{D} = \varepsilon \overline{E}$	
$\overline{B} = \mu \overline{H}$	
$\overline{J} = \sigma \overline{E}$	

Maxwell's Equations for complex Time-Harmonic Fields

- $\nabla\, x\, \overline{E} = -j\omega\mu \overline{H}$
- $\nabla\, x\, \overline{H} = -j\omega\varepsilon \overline{E} + \sigma \overline{E}$ → Differential form
- $\nabla . \overline{E} = \rho/\varepsilon$
- $\nabla . \overline{H} = 0$
- $\overline{D} = \varepsilon \overline{E}$
- $\overline{B} = \mu \overline{H}$
- $\int \overline{\mathbf{E}} \bullet \overline{\mathbf{d}\ell} = -j\omega\mu \iint_{\mathbf{s}} \overline{\mathbf{B}} \bullet \overline{\mathbf{d}}\mathbf{s}$
- $\int \overline{\mathbf{H}} \bullet \overline{\mathbf{d}\ell} = (-j\omega\mu + \sigma) \iint_{\mathbf{s}} \overline{\mathbf{E}} \bullet \overline{\mathbf{d}}\mathbf{s}$ → Integral form
- $\iint_{\mathbf{s}} \overline{\mathbf{E}} \bullet \overline{\mathbf{d}}\mathbf{s} = \mathbf{Q}/\varepsilon$
- $\iint_{\mathbf{s}} \overline{\mathbf{B}} \bullet \overline{\mathbf{d}}\mathbf{s} = \mathbf{0}$
- $\overline{D} = \varepsilon \overline{E}$
- $\overline{B} = \mu \overline{H}$

Problem Set #1

P.1-1 Three corners of a triangle are at $\mathbf{P}_1$ (0, 1, -2), $\mathbf{P}_2$ (4, 1, -3), and $\mathbf{P}_3$ (6, 2, 5).

a) Determine whether the triangle $\mathbf{P}_1\mathbf{P}_2\mathbf{P}_3$ is a right triangle.

b) Find the area of the triangle.

P.1-2 Given the vector $\overline{A} = 3\mathbf{a}_x + 4\mathbf{a}_y - 6\mathbf{a}_z$ in cartesian. Express this vector in the following coordinate systems:

a) Cylindrical coordinate system.

b) Spherical coordinate system.

P.1-3 A vector field $\overline{F}$ is expressed in spherical coordinates as:

$$\overline{F} = (25/r^2)\ \mathbf{a}_r.$$

a) Find $|\overline{F}|$ and F_x at the point **P** (-3, 4, -5).

b) Find the angle that "**F**" makes with the vector $\overline{B} = 2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z$ at the point **P**.

P.1-4 Given the vector field function $\overline{F} = x^2y\ \mathbf{a}_x + xy^2\ \mathbf{a}_y$, evaluate the scalar line integral $\int_C \overline{F}.\overline{d\ell}$ from the point $\mathbf{P}_1$(2, 1, -1) to the point $\mathbf{P}_2$ (8, 2, -1)

a) Along the parabola $x=2y^2$

b) Along the straight line joining the two points.

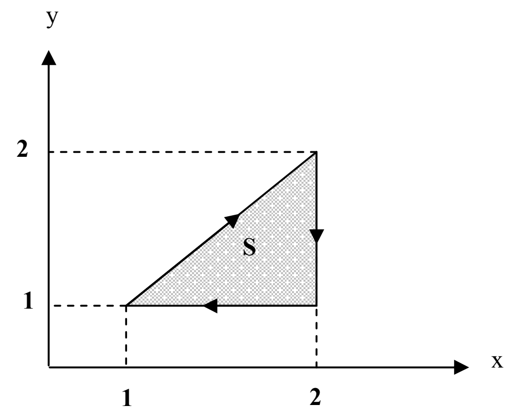
P.1-5 Given a scalar function $V=\sin(\frac{\pi}{2}x)\ \sin(\frac{\pi}{3}y)\ e^{-z}$. Find the magnitude and direction of the maximum rate of increase of V at the point **P** (1, 2, 3).

P.1-6 For the vector function $\overline{A} = \rho^2\mathbf{a}_\rho + 2z\ \mathbf{a}_z$, verify the divergence theorem for circular cylinder region enclosed by ρ=5, z=0, and z=4.

P.1-7 A vector field $\overline{A} = \mathbf{a}_r\ (\cos^2\Phi)/r^3$ exists in the region between two spherical shells defined by r=1 and r=2. Evaluate:

a) $\oint_S \overline{A}\ .\ \overline{ds}$ b) $\int_V \nabla \cdot \overline{A}\ dv$

P.1-8 Given a vector field $\overline{A}$ =$3x^2y^3$ $\mathbf{a}_x$ - x^3y^2 $\mathbf{a}_y$. Verify stokes theorem for the contour shown in figure.



HW#1:

(a) For the vector function $\mathbf{A=2\rho^2 a_\rho + 8z a_z}$, verify the <u>divergence theorem</u> for cylinder region enclosed by ρ=4, z=0, and z=5

(b) Given a vector function $\mathbf{F = x^2y\ a_x + xy^2\ a_y}$, evaluate <u>the scalar line integral</u> $\int_C \overline{F}.d\overline{\ell}$ from the point $P_1(2, \pi/6, 1)$ to the point $P_2(4, \pi/3, 4)$ along the <u>parabola $x=2y^2$</u>

(c) Given a vector of flux density $\mathbf{D= \varepsilon\ (xyza_x+yz^2a_y+x^2yza_z)}$ C/m^2. Find $\nabla\times\mathbf{E}$

HW#2:

(a) Given a vector $\mathbf{A}=4a_x+2a_y+3a_z$. Transform this vector into cylindrical coordinates

(b) Given a vector of flux density $\mathbf{D}=2xya_x+x^2a_y$ C/m^2. Verify the <u>divergence theorem</u> within a parallelepiped formed by planes $0\leq x \leq2$, $0\leq y \leq3$, and $0\leq z \leq4$

(c) Calculate the flux density vector **D** at the point (1, 1, 0). What is the divergence of the flux density vector in this case?

HW#3:

(a) Given a vector filed $\mathbf{A=\rho^3/k_1\ a_\rho + k_2\ \rho\ z\ a_z}$, verify the divergence theorem for cylinder region enclosed by $0\leq \rho \leq2$, $0\leq z \leq4$, and $0\leq \varphi \leq2\pi$

(b) Given a vector field $\mathbf{F = x^2y\ a_x + xy^2\ a_y}$, evaluate the scalar line integral $\int_{path} \overline{F}.d\overline{\ell}$ from the point $P_1(4, 45^o, 30^o)$ to the point $P_2(8, 120^o, 45^o)$ along the path $x^2=4y-2$

HW#4:

(a) Given a vector field $\mathbf{F= (r^2\cos(\Theta))/k_1\ a_r+ k_1\ r\ \sin(\varphi/2)\ a_\Theta + k^2_1\cos(\Theta)\ a_\varphi}$, verify the divergence theorem for a sphere defined by $0 \leq r \leq R_o$, $0 \leq \varphi \leq 2\pi$, and $0 \leq \Theta \leq \pi$

(b) Given a vector field $\mathbf{A = k_2\ \rho z^2\ a_\rho + z\ \rho^2\ /k_2\ a_z}$, evaluate the scalar line integral $\int_{path} \overline{A}.d\overline{\ell}$ from the point $P_1(2, 45^o, 4)$ to the point $P_2(4, 45^o, 8)$ along the shortest path

HW:

P1.1, P1.3, P1.5, P1.11, P1.12, P1.15, P1.29, P3.29, P3.30, P4.5

Problem Set #2

P2-1 Two charges $q_1 = +2$ mc, $q_2 = -5$ mc are located at points (2,0,0) and (-2,0,0) respectively. Find the force acted on a third charge q_3=+3mc if it is located at (0, 0, 6). Find E at P (3, 3, 8).

P2-2 Consider a charged dielectric sphere of radius a = 0.5m and ρ_v=2mc/m^3. Find the electric field at r = 0.2 m, r = 0.5m, and r=2m. Plot the variation of the electric field versus r.

P2-3 A coaxial line has an inner conductor of a radius "a" and outer conductor of a radius "b". The inner conductor is charged by $+\rho_\ell$ while the outer is charged by $-\rho_\ell$. Find electric field as function of **ρ**, and then find the potential difference between outer and inner conductors.

P2-4 A circular ring has a radius a=2m lies in z=0 plane with its center at the origin. If ρ_ℓ =10 nc/m. Find a point charge at the origin which could produce the same electric field at P (0, 0, 5).

P2-5 Two uniform infinite line charges ρ_ℓ= 4 nc/m lies at x = 0, y=$\pm$4 and parallel to z – axis. Find the field at (4, 0, z).

P2-6 Show that the electric field is zero inside conducting sphere charged by Q=5 nc and has radius 5=5 m. Find E at r=10 m.

P2-7 Find the work don-e to move Q = 5 μc from the origin to the point P (2, $^{\pi}/_{4}$, $^{\pi}/_{2}$) in an electric field: $\bar{E} = 5\, e^{-r/4} a_r + \frac{10}{r \sin\theta} a_\phi$ V/m.

P2-8 Prove that the potential of a single infinite line charge (consider a reference point at $\rho=\rho_o$ has a zero potential) $V = \frac{\rho_\ell}{2\pi\varepsilon_0} \ln(\frac{\rho_o}{\rho})$.

P2-9 Three uniform finite line charges + ρ_{ℓ_1}, ρ_{ℓ_2}, and ρ_{ℓ_3} each of length "L" forming an equilateral triangle. Assuming that $\rho_{\ell_1} = 2\rho_{\ell_2} = 2\rho_{\ell_3}$. Determine **E** at the center of the triangle.

HW#1:

A point charge $\mathbf{Q_1}$ (10mc) is located inside a closed room at the point $\mathbf{P_1}$ (0, 0, z_o=5.0) as shown in **Fig.1.** This room having an internal uniform electric field vector given by $\mathbf{E}=5\mathbf{a}_\rho+8\mathbf{a}_z$ **(V/m)** with an angle $\varphi=45^o$. Find the electric force **F** acting on the given charge $\mathbf{Q_1}$, the electric field $\mathbf{E_{P0}}$ at the point $\mathbf{P_0}$, and the work done to move $\mathbf{Q_1}$ from the point $\mathbf{P_1}$ to the point $\mathbf{P_2}$ along the given path (assume x_o=3.0 and y_o=4.0). If an additional charge $\mathbf{Q_2}$ (5mc) is located at the point $\mathbf{P_0}$, find the electric force acting on the charge $\mathbf{Q_1}$ and the potential $\mathbf{V_1}$ the point $\mathbf{P_1}$ in this case.

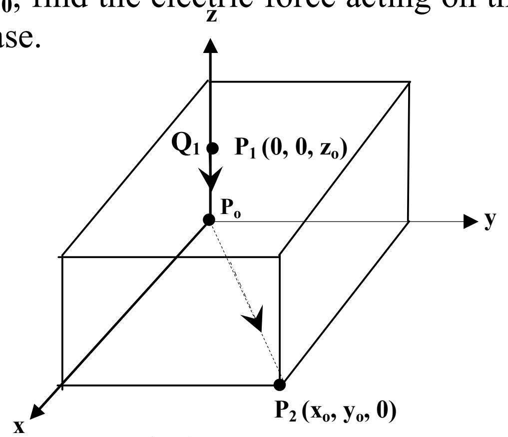


Fig.1

HW#2:

Three point charges Q_1=10 nc, Q_2=-5 nc, and Q_3=-10 nc are located in space at the points P_1 (x_o,0,0), P_2 (0,y_o,0), and P_3 (0,0,z_o) respectively. Find the following:

(a) The electric force acting on Q_3 assume x_o=0, and y_o=z_o=1.0

(b) The electric field and potential at the point P (x_o, 2y_o, 3z_o)

(c) The electric energy $\mathbf{W_e}$

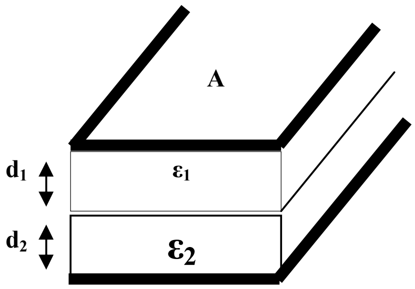
HW:

P2.11, P2.18, P.2.22, P2.26, P3.2, P3.9, P3.15, P3.20, P4.2, P4.6, P4.8, P4.16, P4.33, P4.35

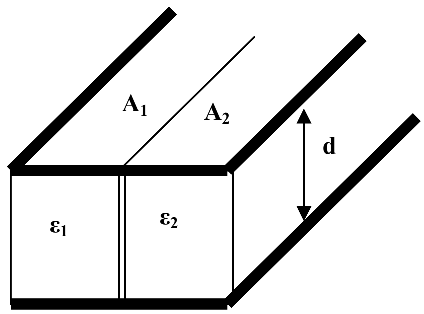
Problem Set#3

P.3.1 Find the capacitance of the following structures:

(a) Two parallel plates:

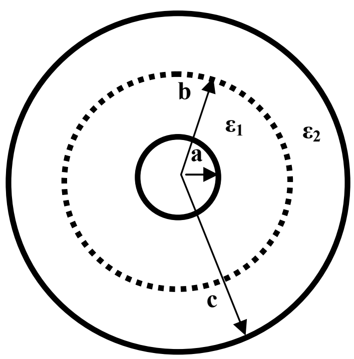


(i)

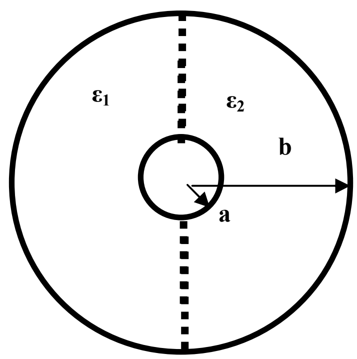


(ii)

(b)Coaxial lines:



(i)

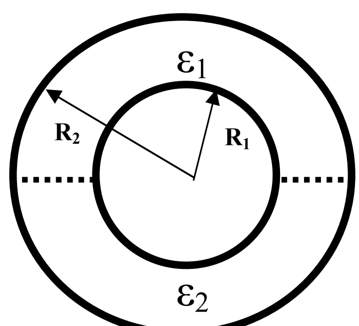


(ii)

(c) Concentric spheres:

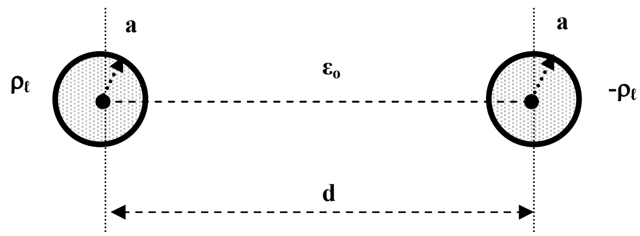
$R_1 = a$

$R_2 = b$



P.3.2. Find the capacitance between two identical and long cylindrical conductors of radius **"a"** as shown in the following figure.

These conductors are separated by air, and the distance between their centers is **"d"**.



P.3.3. Given that $\overline{E}_1 = 2\,\mathbf{a}_x - 3\,\mathbf{a}_y + 5\,\mathbf{a}_z$ v/m as shown in Fig.1. Find $\mathbf{D_2}$ and the angles θ_1 and θ_2.

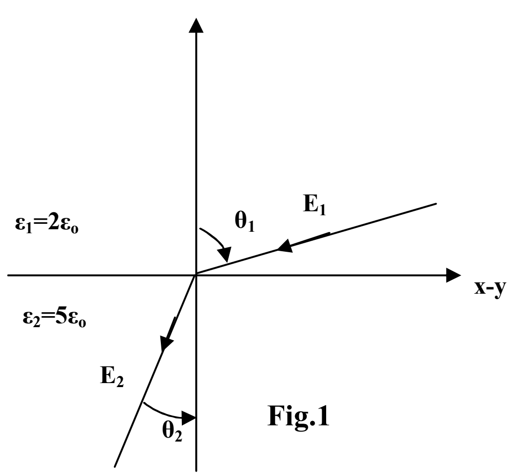


Fig.1

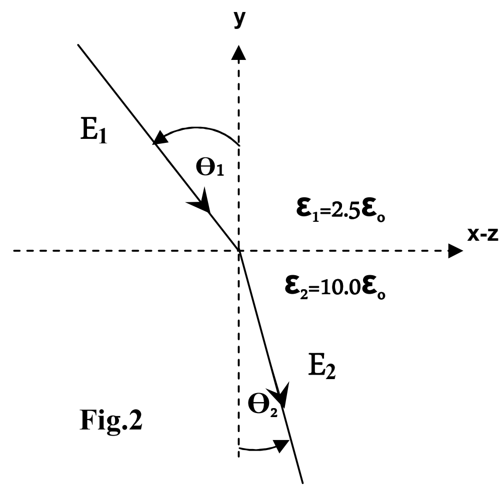


Fig.2

HW#1:

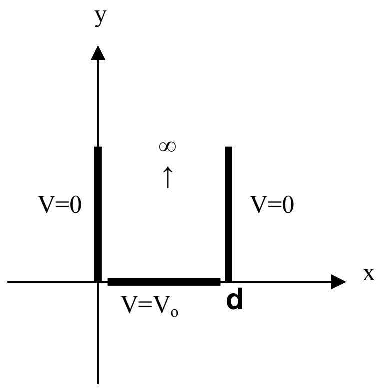
The cross section of two lossless dielectric volumes infinitely extended in y-z plane is shown in **Fig.2**. The electric field vector in the first dielectric medium is given as $\mathbf{E_1 = 6a_\rho + 4a_z}$ with an angle $\mathbf{\Phi = 60^o}$. The first and second electric field intensities make angles $\mathbf{\Theta_1}$ and $\mathbf{\Theta_2}$ with respect the normal respectively(x-direction). Using the boundary condition of the electric field, find $\mathbf{D_2}$, $\mathbf{\Theta_1}$ **&** $\mathbf{\Theta_2}$

HW:

P6.5, P6.7, P6.10, P6.12, P6.13, P6.16, P6.19, 6.31

Problem Set #4

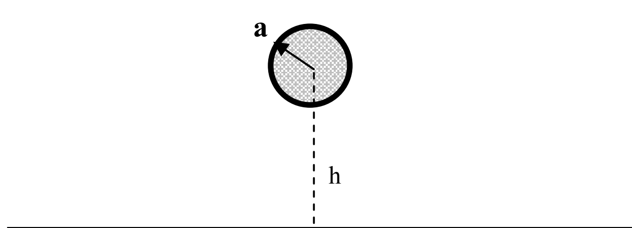
P.4.1 Find the potential **V** inside the following drawn wells:



P.4.2 Find V between the two planes $\phi = 0$ and $\phi = \frac{\pi}{4}$ if the potential V = 0 at $\phi = 0$ and V = V_o at $\phi = \frac{\pi}{4}$.

P.4.3 Find V between two coaxial cones if V = o at $\theta = \frac{\pi}{4}$ V = V_o at $\theta = \frac{\pi}{10}$

P.4.4 A straight conducting wire of radius "a" is parallel to and at height "h" from the surface of earth as shown in the following figure. Assuming the earth is perfectly electric conducting; determine the capacitance and the force per unit length between the wire and the earth.



P.4.5 Find the resistance of the different configurations given in **P.3.1** of problem set#3, assume lossy media. [Hint: use duality equation $\mathbf{R}\times\mathbf{C}=\frac{\varepsilon}{\sigma}$].

P.4.6 Find the resistance of only one of the different configurations given in **P.3.1** of problem set#3, assume lossy media. [Hint: use Laplace equation].

HW#1

Two infinite, identical and isolated conducting planes are located in x-z plane as in shown in **Fig. 1**. The first conducting plane makes an angle of 10^o with z-axis and 20^o with x-axis, and it has a zero potential. The second conducting plane makes an angle of 55^o with the first plane and it has a constant potential $V=V_o$=8.0 volt. Solve the Laplace equation to find the following: **(2-mark each)**

1. The electric potential $\mathbf{V_P}$ at any point between the conducting planes

2. The electric field intensity $\mathbf{E_P}$ at any point between the conducting planes

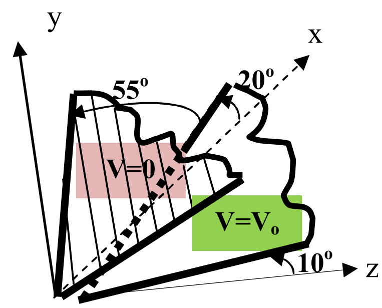


Fig.1

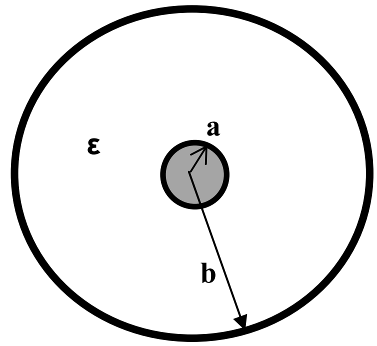


Fig.2

HW#2

A cross section view of a cylindrical capacitor having radii **"a"** and **"b"** is shown in **Fig.2**. The inner conductor is kept at constant potential $V=V_o$=10.0 volt while the outer conductor is grounded. The space between the inner and outer conductors is filled with a lossless dielectric material. Assume: **b=15a**, $\boldsymbol{\varepsilon_r}$**=6.8** and **a**=1.0 mm. Solve the Laplace equation to find the following: **(8-mark)**

1. The electric field intensity, the polarization vector and the capacitance in each region

2. The inner and outer induced surface charge densities: $\boldsymbol{\rho_{Sa}}$ and $\boldsymbol{\rho_{Sb}}$

HW#3

A point charge Q=16.0 nc is located at the point P (x_o=6.0, 0) in the region between two infinite conducting ground planes form a right angle as shown in **Fig. 3**. Using the image method, find the following:

(2-mark each)

1. The induced surface charge densities on the ground planes: $\boldsymbol{\rho_{G1}}$ and $\boldsymbol{\rho_{G2}}$

2. The electric force between the point charge and the ground planes: $\mathbf{F_{QG}}$

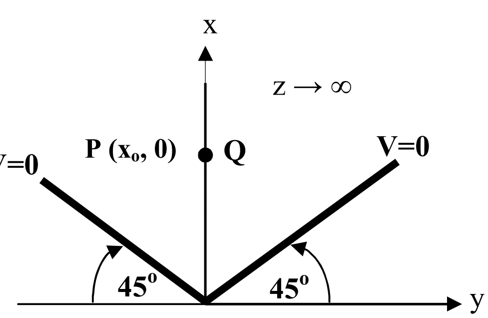


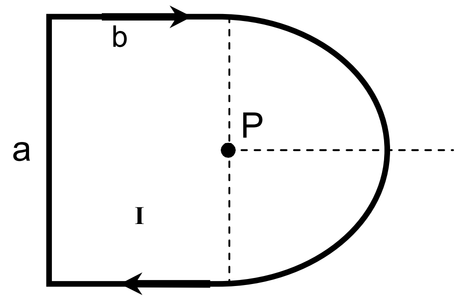
Fig.3

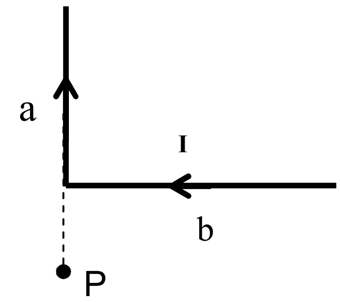
HW:

P7.6, P7.10, P7.15

Problem Set #5

P.5.1 Find the magnetic field at point P due to the stationary currents going through the following wire configurations:





P.5.2 Find the flux inside a rectangle of dimensions a x b near a straight wire with current I as shown in **Fig.1**.

P.5.3 Find the magnetic flux density **B** of a wire carrying constant current $\mathbf{I_1}$ at the point **P** as shown in **Fig.2**. Assume $\mathbf{I_1}$**=0.5 mA, d=0.3m**, and the loop radius **b=0.05m**.

P.5.4 Find the inductance of the following magnetic coil shown in **Fig.3**.

P.5.5 Consider a transmission line of two long parallel conducting wires of radius "a" as shown in **Fig.4**. Assume that the two wires are located in x-z plane and they carry equal currents in opposite direction. Find the internal inductance, mutual (external) inductance and the magnetic force.

Fig. 1

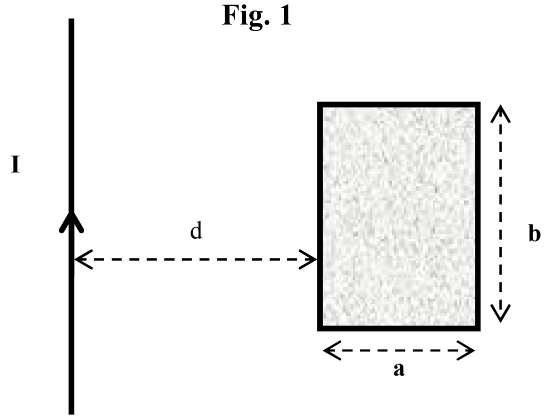


Fig. 2

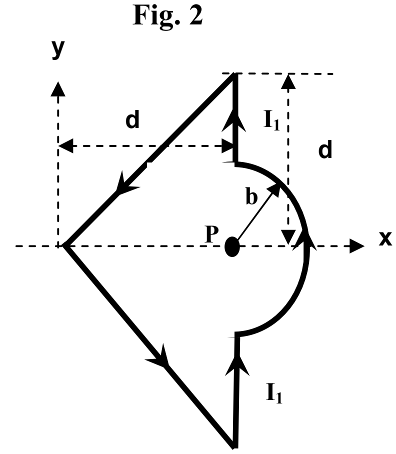


Fig. 3

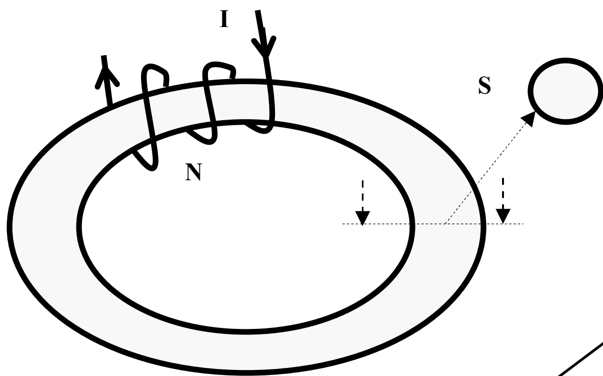
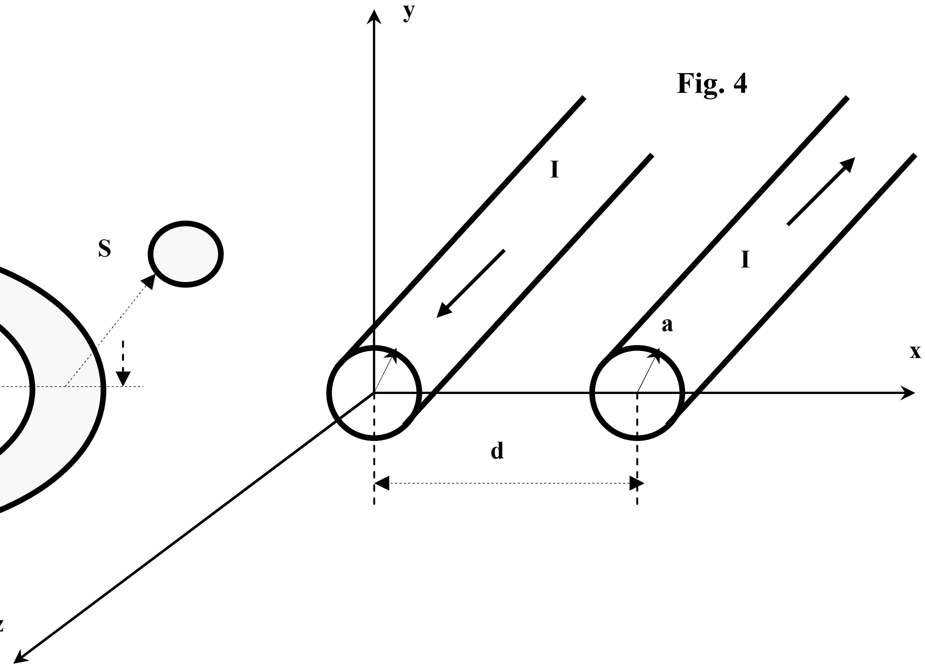


Fig. 4



P.5.6 Given $\overline{E}(t, z) = E_o \sin(\omega t - \beta z)\mathbf{a}_x$, find $\overline{D}$ and $\overline{H}$ in the free space. Sketch $\overline{E}$ and $\overline{H}$ at t = 0.

P.5.7 An electric field component in y-direction propagates in the free-space along the z-direction. The electric field intensity has an instantaneous form given by:

$$\mathbf{E}(t, z) = \mathrm{Re}\left(\mathbf{E_o}\, e^{j[\omega_o t - (\pi/3) z]}\, \mathbf{a_y}\right)$$

Find **D** (t, z) and **B** (t, z).

HW#1:

(a) A cross sectional view of a coaxial line having radii "a" and "b" is shown in **Fig.1**. The space between the inner and outer conductors is divided into three equally regions each with different lossless magnetic material. Assume that **b=10a** (**a**=2.0 mm), μ_{r1}**=400**, μ_{r2}**=250**, and stationary current **I**=I_o $\mathbf{a_z}$ (I_o=100mA) is going through the inner conductor. You may neglect the outer conductor thickness, find the following**:**

1. The total magnetostatic energy **"W_m"** per unit length.

2. The equivalent magnetic circuit of a unit length ℓ of the coaxial line (b)Find the magnetic flux density **B** of a wire carrying constant current $\mathbf{I_2}$ at the point **P** as shown in **Fig.2**. Assume $\mathbf{I_2=0.5\ mA}$ and **b=0.2m (5-Mark)**

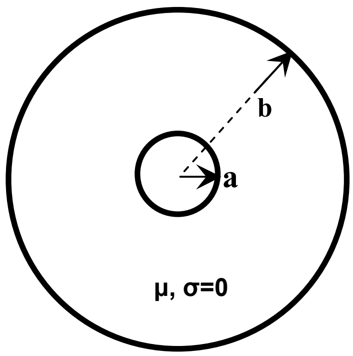


Fig.1

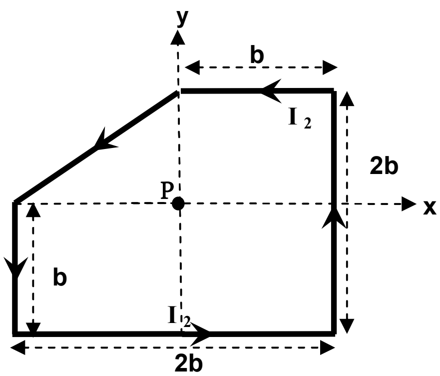


Fig.2

HW:

P8.5, P8.12, P8.25, P8.27, P8.36, P8.41, P10.15, P10.18, P10.26