Heat transfer through a fluid is by convection in the presence of bulk fluid motion and by conduction in the absence of it.
Experience shows that convection heat transfer strongly depends on the fluid properties dynamic viscosity $\mu$, thermal conductivity $k$, density $\rho$, and specific heat $C_p$, as well as the fluid velocity $v$. It also depends on the geometry and the roughness of the solid surface, in addition to the type of fluid flow (such as being streamlined or turbulent).

Thus, we expect the convection heat transfer relations to be rather complex.
\[ q_{\text{conv}} = h(T_s - T_\infty) \quad \text{(W/m}^2) \]

\[ \dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) \quad \text{(W)} \]
An implication of the no-slip and the no-temperature jump conditions is that heat transfer from the solid surface to the fluid layer adjacent to the surface is by pure conduction,

\[ q_{\text{conv}} = q_{\text{cond}} = -k_{\text{fluid}} \left. \frac{\partial T}{\partial y} \right|_{y=0} \quad (\text{W/m}^2) \]

\[ h = \frac{-k_{\text{fluid}}(\partial T/\partial y)_{y=0}}{T_s - T_\infty} \quad (\text{W/m}^2 \cdot ^\circ\text{C}) \]
A Nusselt number of $\text{Nu} = 1$ for a fluid layer represents heat transfer across the layer by pure conduction.
CLASSIFICATION OF FLUID FLOWS

• Viscous versus Inviscid Flow

• Internal versus External Flow

• Compressible versus Incompressible Flow

• Laminar versus Turbulent Flow
CLASSIFICATION OF FLUID FLOWS

- Natural (or Unforced) versus Forced Flow

- Steady versus Unsteady (Transient) Flow

- One-, Two-, and Three-Dimensional Flows
Surface Shear Stress

\[ \tau_s = \mu \frac{\partial u}{\partial y} \bigg|_{y=0} \quad (N/m^2) \]

\[ \tau_s = C_f \frac{\rho \nu^2}{2} \quad (N/m^2) \]
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THERMAL BOUNDARY LAYER
Prandtl Number

The relative thickness of the velocity and the thermal boundary layers is best described by the dimensionless parameter Prandtl number, defined as

\[ Pr = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k} \]

Pr about 1 for gases
Pr \(\ll 1\) for liquid metals
Pr \(\gg 1\) for oil
Chapter 1: 
Introduction
CONVECTION COEFFICIENTS

\[ \text{Nu} = C \text{Re}^m \text{Pr}^n \]

where \( m \) and \( n \) are constant exponents (usually between 0 and 1), and the value of the constant \( C \) depends on geometry.

Sometimes more complex relations are used for better accuracy.
The fluid properties are usually evaluated at the so-called film temperature, defined as

\[ T_f = \frac{T_s + T_\infty}{2} \]
PARALLEL FLOW OVER FLAT PLATES

\[ \text{Re}_x = \frac{\rho \nu x}{\mu} = \frac{\nu x}{\nu} \]

\[ \text{Re}_{cr} = \frac{\rho \nu x_{cr}}{\mu} = 5 \times 10^5 \]

**Laminar:** \( \delta_{v,x} = \frac{5x}{\text{Re}_x^{1/2}} \) and \( C_{f,x} = \frac{0.664}{\text{Re}_x^{1/2}}, \quad \text{Re}_x < 5 \times 10^5 \)

**Turbulent:** \( \delta_{v,x} = \frac{0.382x}{\text{Re}_x^{1/5}} \) and \( C_{f,x} = \frac{0.0592}{\text{Re}_x^{1/5}}, \quad 5 \times 10^5 \leq \text{Re}_x \leq 10^7 \)
PARALLEL FLOW OVER FLAT PLATES

**Turbulent:** \[ \text{Nu}_x = \frac{h_x x}{k} = 0.0296 \text{Re}_x^{0.8} \text{Pr}^{1/3} \]
\[ 0.6 \leq \text{Pr} \leq 60 \]
\[ 5 \times 10^5 \leq \text{Re}_x \leq 10^7 \]

**Laminar:** \[ \text{Nu}_x = \frac{h_x x}{k} = 0.332 \text{Re}_x^{0.5} \text{Pr}^{1/3} \]
\[ \text{Pr} > 0.60 \]

**Laminar:** \[ \text{Nu} = \frac{hL}{k} = 0.664 \text{Re}_L^{0.5} \text{Pr}^{1/3} \]
\[ \text{Re}_L < 5 \times 10^5 \]

**Turbulent:** \[ \text{Nu} = \frac{hL}{k} = 0.037 \text{Re}_L^{0.8} \text{Pr}^{1/3} \]
\[ 0.6 \leq \text{Pr} \leq 60 \]
\[ 5 \times 10^5 \leq \text{Re}_L \leq 10^7 \]
Uniform Heat Flux

Laminar: \( \text{Nu}_x = 0.453 \text{ Re}_x^{0.5} \text{ Pr}^{1/3} \)

Turbulent: \( \text{Nu}_x = 0.0308 \text{ Re}_x^{0.8} \text{ Pr}^{1/3} \)
Example

Engine oil at 60°C flows over the upper surface of a 5-m-long flat plate whose temperature is 20°C with a velocity of 2 m/s. Determine the rate of heat transfer per unit width of the entire plate.

\[ \rho = 876 \text{ kg/m}^3 \quad \text{Pr} = 2870 \]
\[ k = 0.144 \text{ W/m \cdot °C} \quad \nu = 242 \times 10^{-6} \text{ m}^2/\text{s} \]

\[ \text{Re}_L = \frac{\nu L}{\nu} = \frac{(2 \text{ m/s})(5 \text{ m})}{0.242 \times 10^{-5} \text{ m}^2/\text{s}} = 4.13 \times 10^4 \]

\[ \text{Nu} = \frac{hL}{k} = 0.664 \text{ Re}_L^{0.5} \text{ Pr}^{1/3} = 0.664 \times (4.13 \times 10^4)^{0.5} \times 2870^{1/3} = 1918 \]
FLOW ACROSS CYLINDERS AND SPHERES

\[ \text{Nu}_{\text{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62 \text{Re}^{1/2} \text{Pr}^{1/3}}{[1 + (0.4/\text{Pr})^{2/3}]^{1/4}} \left[ 1 + \left( \frac{\text{Re}}{282,000} \right)^{5/8 - 4/5} \right] \]

\[ \text{Re} \quad \text{Pr} > 0.2 \]

\[ \text{Nu}_{\text{sph}} = \frac{hD}{k} = 2 + [0.4 \text{Re}^{1/2} + 0.06 \text{Re}^{2/3}] \text{Pr}^{0.4} \left( \frac{\mu_{\infty}}{\mu_s} \right)^{1/4} \]

\[ 3.5 \leq \text{Re} \leq 80,000 \quad \text{and} \quad 0.7 \leq \text{Pr} \leq 380 \]
## FLOW ACROSS CYLINDERS AND SPHERES

<table>
<thead>
<tr>
<th>Cross-section of the cylinder</th>
<th>Fluid</th>
<th>Range of Re</th>
<th>Nusselt number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle</td>
<td>Gas or liquid</td>
<td>0.4–4, 4–40, 40–4000, 4000–40,000, 40,000–400,000</td>
<td>$\text{Nu} = 0.989 \text{Re}^{0.330} \text{Pr}^{1/3}$, $\text{Nu} = 0.911 \text{Re}^{0.385} \text{Pr}^{1/3}$, $\text{Nu} = 0.683 \text{Re}^{0.466} \text{Pr}^{1/3}$, $\text{Nu} = 0.193 \text{Re}^{0.618} \text{Pr}^{1/3}$, $\text{Nu} = 0.027 \text{Re}^{0.805} \text{Pr}^{1/3}$</td>
</tr>
<tr>
<td>Square</td>
<td>Gas</td>
<td>5000–100,000</td>
<td>$\text{Nu} = 0.102 \text{Re}^{0.675} \text{Pr}^{1/3}$</td>
</tr>
<tr>
<td>Square (tilted 45°)</td>
<td>Gas</td>
<td>5000–100,000</td>
<td>$\text{Nu} = 0.246 \text{Re}^{0.588} \text{Pr}^{1/3}$</td>
</tr>
</tbody>
</table>
### FLOW ACROSS CYLINDERS AND SPHERES

<table>
<thead>
<tr>
<th>Shape</th>
<th>Medium</th>
<th>Reynolds Number</th>
<th>Nusselt Number Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hexagon</td>
<td>Gas</td>
<td>5000–100,000</td>
<td>( N_u = 0.153R_e^{0.638}P_r^{1/3} )</td>
</tr>
<tr>
<td>Hexagon (tilted 45°)</td>
<td>Gas</td>
<td>5000–19,500</td>
<td>( N_u = 0.160R_e^{0.638}P_r^{1/3} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19,500–100,000</td>
<td>( N_u = 0.0385R_e^{0.782}P_r^{1/3} )</td>
</tr>
<tr>
<td>Vertical plate</td>
<td>Gas</td>
<td>4000–15,000</td>
<td>( N_u = 0.228R_e^{0.731}P_r^{1/3} )</td>
</tr>
<tr>
<td>Ellipse</td>
<td>Gas</td>
<td>2500–15,000</td>
<td>( N_u = 0.248R_e^{0.612}P_r^{1/3} )</td>
</tr>
</tbody>
</table>
A long 10-cm-diameter steam pipe whose external surface temperature is 110°C passes through some open area that is not protected against the winds. Determine the rate of heat loss from the pipe per unit of its length when the air is at 1 atm pressure and 10°C and the wind is blowing across the pipe at a velocity of 8 m/s.

\[ k = 0.02808 \text{ W/m} \cdot \text{°C} \quad \text{Pr} = 0.7202 \]
\[ \nu = 1.896 \times 10^{-5} \text{ m}^2/\text{s} \]

\[ \text{Re} = \frac{\nu D}{\nu} = \frac{(8 \text{ m/s})(0.1 \text{ m})}{1.896 \times 10^{-5} \text{ m}^2/\text{s}} = 4.219 \times 10^4 \]

\[ \frac{hD}{k} = 0.3 + \frac{0.62 \text{ Re}^{1/2} \text{ Pr}^{1/3}}{[1 + (0.4/\text{Pr})^{2/3}]^{1/4}} \left[ 1 + \left( \frac{\text{Re}}{282,000} \right)^{5/8} \right]^{4/5} \]
\[ = 0.3 + \frac{0.62(4.219 \times 10^4)^{1/2} (0.7202)^{1/3}}{[1 + (0.4/0.7202)^{2/3}]^{1/4}} \left[ 1 + \left( \frac{4.219 \times 10^4}{282,000} \right)^{5/8} \right]^{4/5} \]
\[ = 124 \]
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INTERNAL FORCED CONVECTION

Re = \frac{\rho \nu_m D}{\mu} = \frac{\nu_m D}{v}

D_h = \frac{4A_e}{P}

\begin{align*}
\text{Re} &< 2300 \quad \text{laminar flow} \\
2300 &\leq \text{Re} \leq 10,000 \quad \text{transitional flow} \\
\text{Re} &> 10,000 \quad \text{turbulent flow}
\end{align*}
Chapter 1: Introduction

**Heat Transfer**

The entrance region is characterized by the transition from a boundary layer to a fully developed region. The hydrodynamic entrance region is typically defined as the length where the flow becomes fully developed, and it is given by:

\[ L_{h, \text{turbulent}} \approx L_{i, \text{turbulent}} \approx 10D \]

The thermal entrance region is similarly defined, with the thermal entrance length given by:

\[ L_{h, \text{laminar}} \approx 0.05 \, \text{Re} \, D \]

\[ L_{i, \text{laminar}} \approx 0.05 \, \text{Re} \, \text{Pr} \, D = \text{Pr} \, L_{h, \text{laminar}} \]
INTERNAL FORCED CONVECTION

GENERAL THERMAL ANALYSIS

\[ \dot{Q} = \dot{m} C_p (T_e - T_i) \]

\[ q_s = h_x (T_s - T_m) \quad \text{(W/m}^2\text{)} \]

Constant Surface Heat Flux (\( q_s = \text{constant} \))

\[ \dot{Q} = q_s A_s = \dot{m} C_p (T_e - T_i) \]

\[ T_e = T_i + \frac{q_s A_s}{\dot{m} C_p} \]
INTERNAL FORCED CONVECTION

Constant Surface Temperature ($T_s = \text{constant}$)

\[
\dot{Q} = hA_s \Delta T_{ave} = hA_s (T_s - T_m)_{ave}
\]

\[
\Delta T_{ave} \approx \Delta T_{am} = \frac{\Delta T_i + \Delta T_e}{2} = \frac{(T_s - T_i) + (T_s - T_e)}{2} = T_s - \frac{T_i + T_e}{2} = T_s - T_b
\]

where $T_b = (T_i + T_e)/2$ is the bulk mean fluid temperature.
INTERNAL FORCED CONVECTION

Constant Surface Temperature \( (T_s = \text{constant}) \)

\[
\dot{m} C_p \, dT_m = h(T_s - T_m) \, dA_s
\]

\[
\frac{d(T_s - T_m)}{T_s - T_m} = -\frac{h \rho}{\dot{m} C_p} \, dx
\]

\[
\ln \frac{T_s - T_e}{T_s - T_i} = -\frac{h A_s}{\dot{m} C_p}
\]

\[
T_e = T_s - (T_s - T_i) \exp(-h A_s / \dot{m} C_p)
\]
INTERNAL FORCED CONVECTION

Constant Surface Temperature ($T_s = \text{constant}$)

$$\dot{Q} = hA_s \Delta T_{\ln}$$

$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$$

is the logarithmic mean temperature difference.
Example

Water enters a 2.5-cm-internal-diameter thin copper tube of a heat exchanger at 15°C at a rate of 0.3 kg/s, and is heated by steam condensing outside at 120°C. If the average heat transfer coefficient is 800 W/m² °C, determine the length of the tube required in order to heat the water to 115°C.
Laminar Flow in Pipes

Constant Surface Heat Flux

\[ \text{Nu} = \frac{hD}{k} = 4.36 \]

Constant Surface Temperature

\[ \text{Nu} = \frac{hD}{k} = 3.66 \]
## Chapter 1: Introduction

### Laminar Flow in Pipes

<table>
<thead>
<tr>
<th>Tube Geometry</th>
<th>$a/b$ or $\theta^\circ$</th>
<th>$Nusselt$ Number $T_s = \text{Const.}$</th>
<th>$\dot{q}_s = \text{Const.}$</th>
<th>Friction Factor $f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle</td>
<td></td>
<td>3.66</td>
<td>4.36</td>
<td>64.00/Re</td>
</tr>
<tr>
<td>Rectangle</td>
<td>$a/b$</td>
<td>2.98</td>
<td>3.61</td>
<td>56.92/Re</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3.39</td>
<td>4.12</td>
<td>62.20/Re</td>
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<tr>
<td></td>
<td>2</td>
<td>3.96</td>
<td>4.79</td>
<td>68.36/Re</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4.44</td>
<td>5.33</td>
<td>72.92/Re</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>5.14</td>
<td>6.05</td>
<td>78.80/Re</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>5.60</td>
<td>6.49</td>
<td>82.32/Re</td>
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<tr>
<td></td>
<td>$\infty$</td>
<td>7.54</td>
<td>8.24</td>
<td>96.00/Re</td>
</tr>
<tr>
<td>Ellipse</td>
<td>$a/b$</td>
<td>3.66</td>
<td>4.36</td>
<td>64.00/Re</td>
</tr>
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<td></td>
<td>1</td>
<td>3.74</td>
<td>4.56</td>
<td>67.28/Re</td>
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<tr>
<td></td>
<td>2</td>
<td>3.79</td>
<td>4.88</td>
<td>72.96/Re</td>
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<tr>
<td></td>
<td>4</td>
<td>3.72</td>
<td>5.09</td>
<td>76.60/Re</td>
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<td></td>
<td>8</td>
<td>3.65</td>
<td>5.18</td>
<td>78.16/Re</td>
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<tr>
<td>Triangle</td>
<td>$\theta$</td>
<td>1.61</td>
<td>2.45</td>
<td>50.80/Re</td>
</tr>
<tr>
<td></td>
<td>10$^\circ$</td>
<td>2.26</td>
<td>2.91</td>
<td>52.28/Re</td>
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<tr>
<td></td>
<td>30$^\circ$</td>
<td>2.47</td>
<td>3.11</td>
<td>53.32/Re</td>
</tr>
<tr>
<td></td>
<td>60$^\circ$</td>
<td>2.34</td>
<td>2.98</td>
<td>52.60/Re</td>
</tr>
<tr>
<td></td>
<td>90$^\circ$</td>
<td>2.00</td>
<td>2.68</td>
<td>50.96/Re</td>
</tr>
<tr>
<td></td>
<td>120$^\circ$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Turbulent Flow in Pipes

\[ \text{Nu} = 0.023 \text{Re}^{0.8} \text{Pr}^n \]

where \( n = 0.4 \) for heating and \( 0.3 \) for cooling of the fluid flowing through the tube.

The fluid properties are evaluated at the \textit{bulk mean fluid temperature}.

\[ \text{Nu} = \frac{(f/8) \text{Re} \text{Pr}}{1.07 + 12.7(f/8)^{0.5}(\text{Pr}^{2/3} - 1)} \]

\( 0.5 \leq \text{Pr} \leq 2000 \)

\( 10^4 < \text{Re} < 5 \times 10^6 \)
Turbulent Flow in Pipes

\[ \text{Nu} = \frac{(f/8)(\text{Re} - 1000) \Pr}{1 + 12.7(f/8)^{0.5} (\Pr^{2/3} - 1)} \begin{cases} 0.5 \leq \Pr \leq 2000 \\ 3 \times 10^3 < \text{Re} < 5 \times 10^6 \end{cases} \]

Smooth tubes: \[ f = (0.790 \ln \text{Re} - 1.64)^{-2} \quad 10^4 < \text{Re} < 10^6 \]
Thank you!