Arab Academy for Science and Technology and Maritime Transport

Maintenance Planning-ME542

Linear regression

By: Eng. Ahmad Elruby

Introduction

linear regression is an approach to modeling the relationship between a scalar dependent variable *Y* and one or more explanatory variables denoted *X*.

Linear regression formulas

$$\overline{\boldsymbol{Y}} = a + b\,\boldsymbol{X}$$ → Straight line equation

$$\boldsymbol{b} = \frac{n(\sum XY) - (\sum X)(\sum Y)}{n(\sum X^2) - (\sum X)^2}$$ → Slope

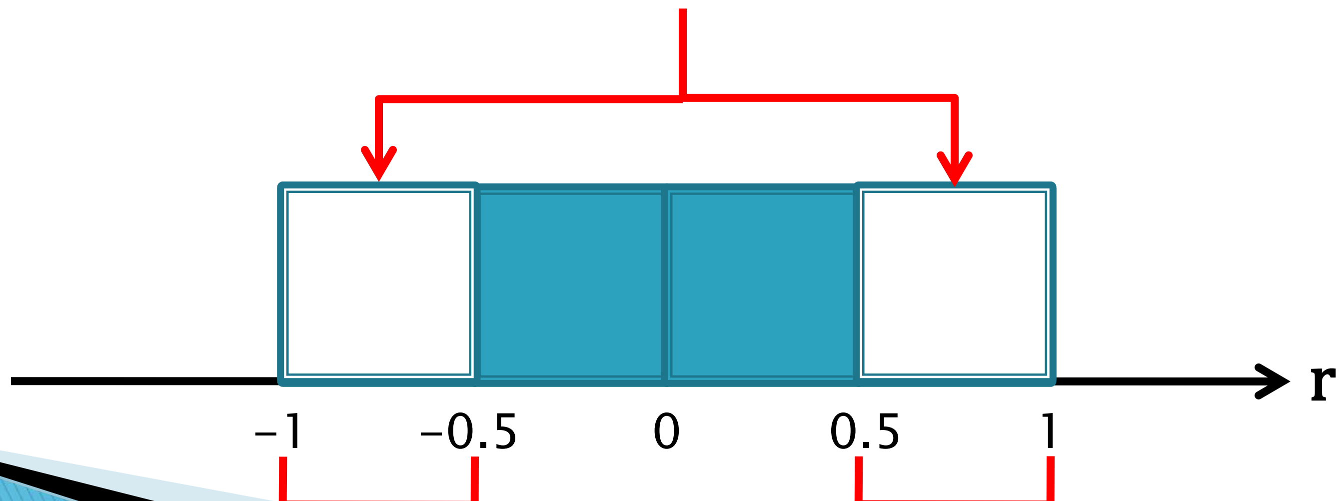
$$\boldsymbol{a} = \frac{\sum Y}{n} - \boldsymbol{b}\,\frac{\sum X}{n}$$ → Y intercept

Coeff. of Linear Regression "r"

It is used to check the validity of linear regression

$$r = \frac{n(\sum XY) - (\sum X)(\sum Y)}{\sqrt{[n(\sum X^2) - (\sum X)^2][n(\sum Y^2) - (\sum Y)^2]}}$$

Acceptable range of linear regression



Root Mean Square error

$$R.M.S._{error} = \sqrt{(Y_1 - \overline{Y}_1)^2 + (Y_2 - \overline{Y}_2)^2 + (Y_3 - \overline{Y}_3)^2 + \ldots + (Y_n - \overline{Y}_n)^2}$$

Example 1

Year	1999	2000	2001	2002	2003
Unit Price ($)	1450	1300	1200	1000	?

Required:

1) estimate unit price of year 2003
2) Check the validity of linear regression
3) Calculate the R.M.S. error

Example 1

Year	1999	2000	2001	2002	2003
Unit Price ($)	1450	1300	1200	1000	?

Solution:

	X	Y	XY	X^2	Y^2
n= 4	1	1450	1450	1	2,102,500
	2	1300	2600	4	1,690,000
	3	1200	3600	9	1,440,000
	4	1000	4000	16	1,000,000
Total					

Example 1

Year	1999	2000	2001	2002	2003
Unit Price ($)	1450	1300	1200	1000	?

Solution:

$$\boldsymbol{b} = \frac{n(\sum XY) - (\sum X)(\sum Y)}{n(\sum X^2) - (\sum X)^2} = -145$$

$$\boldsymbol{a} = \frac{\sum Y}{n} - \boldsymbol{b}\,\frac{\sum X}{n} = 1600$$

Example 1

Year	1999	2000	2001	2002	2003
Unit Price ($)	1450	1300	1200	1000	?

Solution:

1) Estimated unit price of year 2003

$$\bar{Y} = \underbrace{1600}_{a} - \underbrace{145}_{b} X \quad = 875 \ \$$$

Example 1

Year	1999	2000	2001	2002	2003
Unit Price ($)	1450	1300	1200	1000	?

Solution:

2) Checking validity of linear regression

$$\boldsymbol{r} = \frac{n(\sum XY) - (\sum X)(\sum Y)}{\sqrt{[n(\sum X^2) - (\sum X)^2][n(\sum Y^2) - (\sum Y)^2]}}$$

$$\boldsymbol{r} = \frac{4 * 1600 - 10 * 4950}{\sqrt{[4 * (30) - (10)^2][4 * (6{,}232{,}500) - (4950)^2]}} = -0.99$$

Example 1

Year	1999	2000	2001	2002	2003
Unit Price ($)	1450	1300	1200	1000	?

Solution:

3) Root Mean Square error

$$R.M.S._{error} = \sqrt{(Y_1 - \bar{Y}_1)^2 + (Y_2 - \bar{Y}_2)^2 + (Y_3 - \bar{Y}_3)^2 + \ldots + (Y_n - \bar{Y}_n)^2}$$

$$R.M.S._{error} = \sqrt{(1450 - 1455)^2 + (1300 - 1310)^2 + (1200 - 1165)^2 + (1000 - 1020)^2}$$

$$R.M.S._{error} = \pm 41.8 \,\$$$

X	Y	$\bar{Y}$
1	1450	1455
2	1300	1310
3	1200	1165
4	1000	1020

Example 1

Year	1999	2000	2001	2002	2003
Unit Price ($)	1450	1300	1200	1000	?

Solution:

Estimated price unit of year 2003

$$\bar{Y} = 875 \pm 41.8 \ \$$$

Example 2

hours	100	200	300	400	500	600	700	800	900
mm/s	0.58	1.08	1.58	2.08	2.58	3.08	3.58	4.08	4.58

The vibration readings of a 2000 rev/min fan is given in the previous table

Required:

1) Construct the vibration trend
2) Predict vibration level at 110 running hours
3) Predict the time to "call for service"
4) Predict the time to "Immediate repair"

Example 2

t	100	200	300	400	500	600	700	800	900	4500
v	0.58	1.08	1.58	2.08	2.58	3.08	3.58	4.08	4.58	23.22
t^2	10000	40000	90000	160000	250000	360000	490000	640000	810000	2850000
vt	58	216	474	832	1290	1848	2506	3264	4122	14610

1) Vibration trend:

$$\sum \mathbf{V} = \left(\sum \mathbf{t}\right)\mathbf{a} + \mathbf{n} * \mathbf{b} \longrightarrow \boldsymbol{b} = \frac{\mathbf{23.22} - \mathbf{4500}\boldsymbol{a}}{\mathbf{9}} \longrightarrow 1$$

$$\sum \mathbf{Vt} = \left(\sum \mathbf{t}\right)\mathbf{b} + \left(\sum \mathbf{t}^2\right)\mathbf{a} \longrightarrow \mathbf{14610} = \mathbf{4500}\boldsymbol{b} + \mathbf{2850000}\boldsymbol{a} \longrightarrow 2$$

From 1 and 2: a = 0.005 , b = 0.08

$$\boxed{\mathbf{V = 0.005\,t + 0.08}}$$

Example 2

t	100	200	300	400	500	600	700	800	900	4500
v	0.58	1.08	1.58	2.08	2.58	3.08	3.58	4.08	4.58	23.22
t^2	10000	40000	90000	160000	250000	360000	490000	640000	810000	2850000
vt	58	216	474	832	1290	1848	2506	3264	4122	14610

2) Vibration level at 110 running hours:

$$\boxed{\mathbf{V = 0.005\,t + 0.08}}$$

$$\boxed{\mathbf{V_{110} = 0.005\,(110) + 0.08} = 0.63 \text{ mm/s}}$$

3) Time to "call for service":

From tables : vibration level of "call for service" is 5.6 mm/s

$$\boxed{\mathrm{t} = \frac{V - 0.008}{0.005} = 1104 \text{ hours}}$$

Example 2

t	100	200	300	400	500	600	700	800	900	4500
v	0.58	1.08	1.58	2.08	2.58	3.08	3.58	4.08	4.58	23.22
t^2	10000	40000	90000	160000	250000	360000	490000	640000	810000	2850000
vt	58	216	474	832	1290	1848	2506	3264	4122	14610

4) Time to "immediate repair":

From tables : vibration level of "immediate repair" is 10 mm/s

$$\boxed{t = \frac{V - 0.008}{0.005} = 1984 \text{ hours}}$$

Example 3

hours	100	200	300	400	500	600	700	800	900
mm/s	0.58	1.08	1.58	2.08	2.58	3.08	3.58	4.08	4.58

During a predictive maintenance program of 2000 R.P.M. air blowing unit, the following vibration levels were obtained, the stud fixed vibrometer used has 0.7 damping ratio and a natural frequency of 10 Hz

Required:

Find the percentage error in the measured vibration levels, the highest level corresponds to 25 Hz

Then construct the vibration trend and predict the vibration level after 110 running hours.

Example 3

hours	100	200	300	400	500	600	700	800	900
mm/s	0.58	1.08	1.58	2.08	2.58	3.08	3.58	4.08	4.58

Solution:

$f_{measured=25\ Hz}$ $f_{natural=10\ Hz}$ $\xi\ 0.7$ $\left(\omega/\omega_n\right) = f/f_n$

$$\boldsymbol{Error = (1 - M) * 100\%}$$

$$\boldsymbol{Error = \left(1 - \frac{\left(\omega/\omega_n\right)^2}{\sqrt{\left(1 - \left(\omega/\omega_n\right)^2\right)^2 + \left(2\xi\left(\omega/\omega_n\right)\right)^2}}\right) * 100\%}$$

Thank you ☺