# CHAOTIC BEHAVIOUR OF IMPATT DIODE OSCILLATORS

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Abstract

A complete non-linear circuit has been developed for IMPATT diode oscillators. The circuit has been used to design microwave oscillators and to simulate their performance using a non-linear CAD procedure. It is shown that, under certain load conditions, period doubling occurs which leads ultimately to chaotic behaviour. The behaviour has also been verified experimentally and results suggest that IMPATT noise oscillators may be in fact chaotic oscillators.

## I. Method of Analysis

The method used for analysing a general non-linear microwave network is based on the state-variable approach [1-3]. The combination of Kirchoff's laws and the voltage-current relations of each element produces a system of differential, difference and non-linear equations which are solved simultaneously. The CAD package developed is known as ANAMIC.

When Kirchoff's laws and the voltage-current relations of the elements ar combined the following equations are derived.

$$\dot{x}_1(t) = A_1x(t) + B_1u(t) + C_1\dot{u}(t)$$

$$x_2(t) = A_2x(t-T_i) + B_2u(t-T_i) \qquad (1)$$

$$0 = A_{31}x_1(t) + A_{32}x_2(t) + B_3u(t) + f(x(t),u(t),t)$$

where

- $x_1$ is a vector of the lumped state variables (independent voltages on capacitors and independent currents in inductors).
- $x_2$ is a vector of all the distributed state variables (reflected voltages at all the transmission line ports).
- $x_3$ is a vector of the variables associated with non-linear resistors (voltages on RV elements and currents in RC elements).
- $x = [x_1:x_2:x_3]$ is the global state vector incorporating $x_1$, $x_2$ and $x_3$.
- u is the source vector containing independent sources.
- A, B, C, D are matrices which are functions of the values of the circuit elements.
- f(x(t), u(t), t) is a vector of the non-linear functions describing the non-linear, parametric and time dependent resistors,

and

- $T_i$ are the delays on the individual transmission lines or the modes on coupled lines.

The general output equation for the network can also be derived, which expresses all the circuit voltages and currents (the vector y) in terms of the global state vector x and the sources vector u

$$y(t) = A_4x(t) + B_4u(t) + C_4u(t) \qquad (2)$$

By using appropriate non-linear models [3] the matrices A, B and C in equations (1) and (2) become constant. As the solution of the state equations involves complicated matrix operations including several iterations and integrations, the fact that A, B and C are constant results in considerable computational efficiency. In this case the state equations are derived only once instead of being modified at every time step due to the non-linearity of the elements.

## II. Impatt Diode Model

J. W. Gannett and L. O. Chua [4] have presented a non-linear model for IMPATT diodes. Their model is reported by B. D. Bates and P. J. Khan [5] to be accurate in predicting the performance of IMPATT oscillators.

Based on this model a new non-linear model, shown in Fig. 1, is developed in which all the non-linear RLC elements are replaced with linear RLC elements and non-linear controlled sources.

Moreover, the model was expanded to predict the behaviour of the diode both in avalanche and non-avalanche regions. In this model the avalanche region is modelled by $C_a$, $L_{a1}$, $I_{an}$, $V_r$ and $V_n$. The drift region is modelled by $C_d$, $V_{nc}$, $I_e$, $C_m$ and $I_1$. The current source $I_o$ represents the current through the device under any bias condition. When the diode is biased in the non-avalanche region, the inductor $L_{a1}$ and the current source $I_{an}$ will behave like an open circuit, and the model reduces to the simple model of $P^+$-N junction diodes. The generality of the model is very attractive for characterisation and simulation on CAD programs.

An IMPATT diode was characterised by measurement and then simulated on ANAMIC [1-3]. In fig. 2 the results of this simulation are compared with measured values of the conductance and susceptance of the diode for an avalanche current of 60mA, and frequencies from 2 to 7GHz. Measurements at different diode currents give similar results. Fig. 3 shows the predicted and measured C-V characteristic of the diode in the non-avalanche reverse region. The close agreement

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between the predicted and measured values confirm the validity of the non-linear model.

## III. Oscillator Simulation

The next step was to design a complete IMPATT diode oscillator and analyse the circuit using the CAD procedure developed and compare the results with measurements.

The oscillator circuit is shown in Fig. 4 and the predicted and measured output power are shown in Fig. 5. The results of the diode characteristics and the oscillator output gives confidence in both the diode model and the CAD procedure.

## IV. Chaotic Behaviour

To illustrate the chaotic behaviour of the oscillator, the circuit shown in Fig. 6 was both tested and simulated. The output match of the oscillator was varied until period doubling occurred and ultimately chaotic behaviour was observed. Fig. 7 shows the simulated output starting from a harmonic signal through period doubling and ultimately to chaos. Fig. 8 shows the measured spectrum for harmonic and chaotic behaviour and Fig. 9 shows the resulting phase plane (attractor) when the voltage across the avalanche capacitor is plotted versus the current in the series inductor.

The Lyapunov Exponent has proven to be the most useful dynamical diagnostic for chaotic systems. It is defined as the average exponential rates of divergence or convergence of nearby orbits in phase space.

It was shown in [6] and [7] that any system possessing at least one positive Lyapunov Exponent is defined to be chaotic. Since the positive exponent reflects a direction in which the system experiences repeated stretching and folding that decorrelates nearby states on the attractor and the magnitude of the exponent reflects the timescale on which the system dynamics become unpredictable.

The calculation of the Lyapunov Exponent can be carried out by means of the differential equations or by the time series solution of the simulated circuit. In our case the second method is more efficient. Following a program developed by Wolf et al. [7] we calculated the Lyapunov Exponents for all the state variables in the circuit. Two exponents were found to be positive, namely, the avalanche inductor current and the drift capacitor voltage, the first of which is shown in Fig. 10.

The calculation of the Lyapunov Exponent is based upon the following relation [7]

$$\lambda_1 = \frac{1}{t_M - t_o} \sum_{k=1}^{M} \log_2 \frac{L'(t_k)}{L(t_{k-1})}$$

where

M is the total number of replacement steps in the fixed evolution program.

$L(t_{k-1})$ is the distance between the nearest neighbour at $t_{k-1}$.

$L'(t_k)$ is the evolved distance at later time $t_k$.

Since the location of the nearest neighbour is limited by the accuracy of the computer, the results were further verified to be chaotic by calculating both the auto-correlation and cross-correlation of the foregoing state variables, the quick decay of the auto-correlation manifests the loss of memory of the signal to itself after a finite time. The latter is shown in Fig. 11.

## V. Conclusions

We have shown that chaotic oscillations occur in IMPATT diode oscillators. The load impedance is used as the bifurcation parameter and both the theoretical and practical results agree to a high degree of accuracy. The results also give confidence in the non-linear model the CAD package used.

## References


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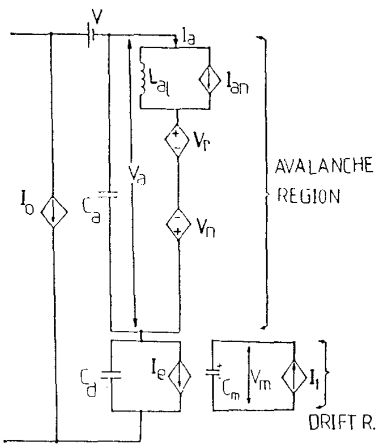


AVALANCHE REGION

$$V_a = \frac{K_e \tau_e}{\alpha'(E_{dc})} \left[ L_n(I_a + I_o) \right] + \frac{I_s}{\alpha'(E_{dc})} \left[ \frac{1}{I_o} - \frac{1}{I_a + I_o} \right] - \frac{\alpha''(E_{dc})}{2 x_a \alpha'(E_{dc})} V_a^2$$

DRIFT REGION

$$I(t) = I_o + I_e + C_d \frac{dV_d}{dt}$$

FIG. 1 - EQUIVALENT CIRCUIT OF IMPATT DIODE

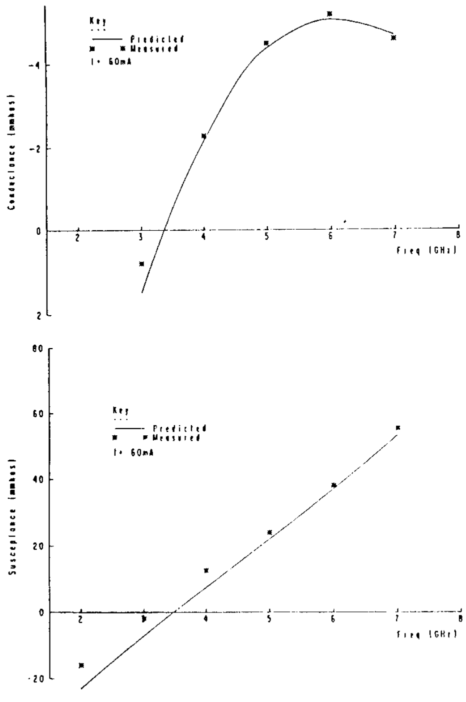


Fig. 2 - Diode Conductance (Top) and Susceptance (Bottom) at 60 m.A.

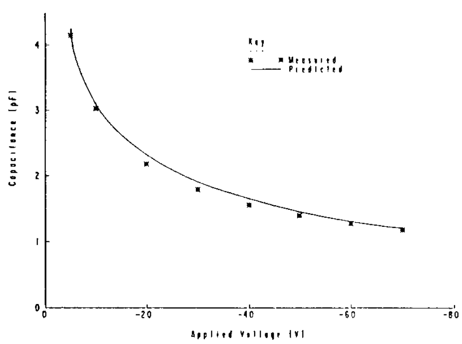


Fig. 3 - C-V Characteristics in the Non-Avalanche Region

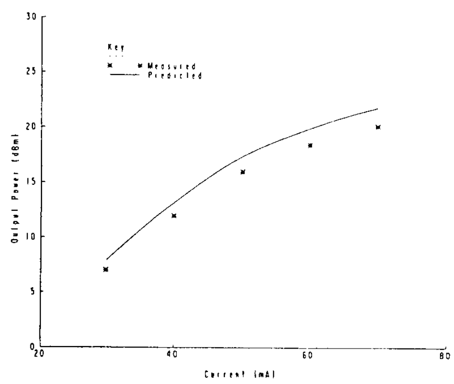


Fig. 5 - Measured and Predicted Output Power for Different Avalanche Current

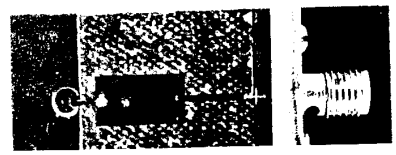

Fig. 4 - The IMPATT Oscillator

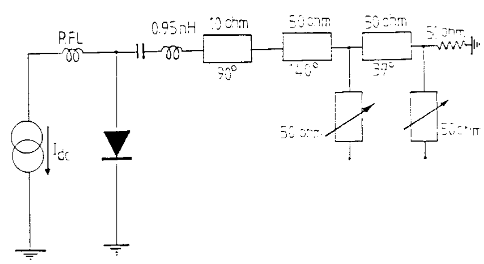


FIG. 6 - IMPATT DIODE OSCILLATOR

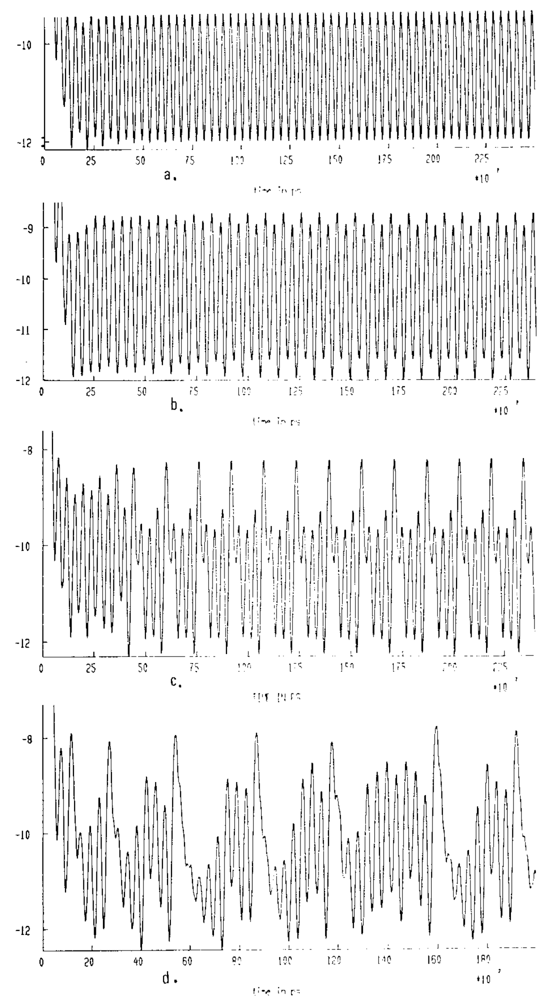


Fig. 7 - Calculated output of IMPATT oscillator with varying load impedance
(a) Harmonic (b) Period Doubling (c) Period 4 (d) Chaotic

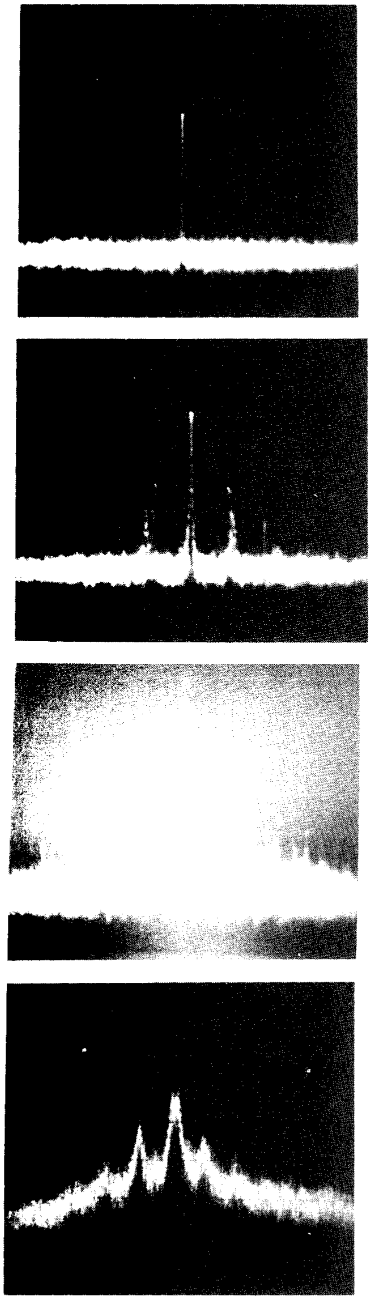

Fig. 8 - Transition from Harmonic to Chaotic Behaviour of IMPATT Diode Oscillator.

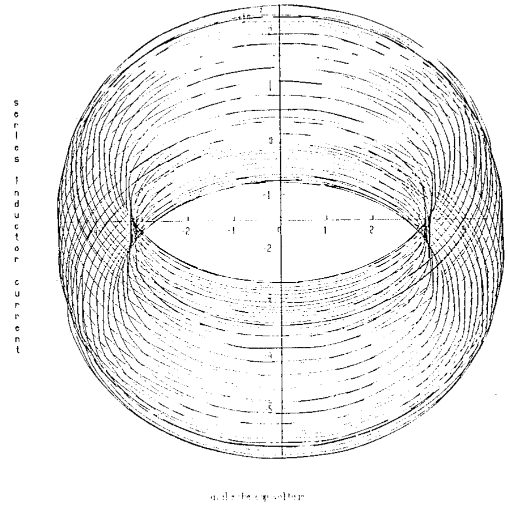


Fig. 9 - Attractor of IMPATT Oscillator Circuit

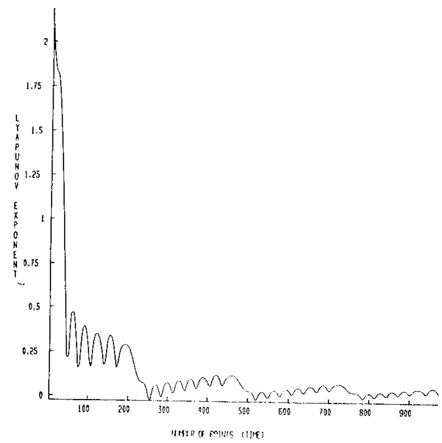


Fig. 10 - Lyapunov exponent of attractor

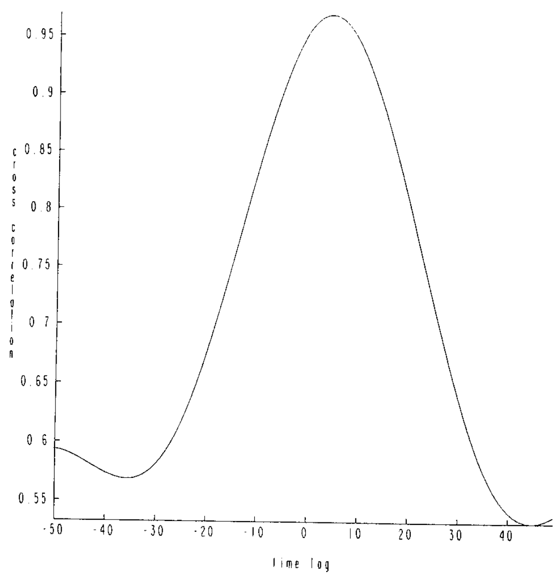


Fig. 11 - Cross-correlation Function