

# A Data Association Approach for Multitarget Tracking Based on a Hidden Markov Model

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**Abstract**— When tracking multiple targets, the task of determining which measurement belongs to each target is a challenging one. There are many data association techniques to solve this challenging task in multitarget tracking systems. In most previous studies, there is a severe tradeoff between computational complexity and tracking performance. In this paper, a new data association approach, based on a Hidden Markov Model (HMM), is proposed. The proposed association approach utilizes the HMM to model the state space and capture the transition probabilities, through training, among the states of the target. The proposed approach has the advantage of a balance between computational complexity and tracking performance, thus it achieves higher performance with a lower computational complexity compared to some association approaches reported in the literature. Tracking performance of the proposed association approach is evaluated in some examples of multitarget tracking systems. The results show that the proposed association approach outperforms the nearest neighbor standard filter association technique.

**Keywords;** multitarget tracking, HMM, data association

## I. INTRODUCTION

Nowadays, target tracking has become prominent with the advancement in robotics, computer visions, wireless sensor networks, and Radars [1-4]. The first stage in tracking multiple targets accurately is data association. Data association is the process of determining sensor data set that relates to a certain target when multiple received measurements exist for more than one target. In a noisy environment, this is quite a difficult task because a target could easily be mistaken for another especially in points where targets are in close contact.

The optimal solution of multitarget tracking, based upon use of the a posteriori probability is the multiple hypothesis tracker (MHT) [5-7]. This is recognized as the theoretically optimum approach in Bayesian sense under idealized modeling assumptions. Unfortunately, the computational complexity of the MHT limits its practical realization. Furthermore, a priori statistical knowledge of the noise environment is required. Thus suboptimal realizations of the MHT have many practical advantages. For this reason, many different multitarget tracking techniques have been developed which sacrifice optimal performance for the sake of computational feasibility.

The nearest neighbor association, in which one measurement is used to update each target trajectory, is

considered in many literatures [1,5-7]. The optimal nearest neighbor partitioning of measurements into tracks is based on the maximum likelihood technique. The goal of this technique is to define an expression for computing the relative likelihood that each of the correlation hypotheses is correct. This expression is defined to be the likelihood function. The main feature of this complicated nearest neighbor data association technique is the computation of the probability associated with the partitioning of measurements, received over a number of scans, into tracks and false alarms. This optimal and complicated solution is based on batch processing and is generally computationally infeasible for a real-time processing. Thus suboptimal, but computationally feasible, sequential nearest neighbor solutions are developed.

Among the suboptimal solutions that tackle the problem of data association are the nearest neighbor standard filter (NNSF), which is usually implemented in practice, and the strongest neighbor filter (SNF) [1,5]. In these techniques, only one measurement, is used to update a given target trajectory in such a way to optimize an overall association measure. The NNSF is characterized by its simplicity as data association is performed based on computation of distance measures between the received measurements and the targets under surveillance. However, these simple approaches result in degraded performance in case of closing and crossing targets.

Other categories of suboptimal solutions are based on neural network and fuzzy logic techniques [8-10]. The main disadvantage of the neural network implementations are that they require a large number of neurons and require training with a very large set of tracks [1,5]. Other techniques combine measurements rather than select a single measurement. Measurements are combined either based on association probabilities such as in the PDA and the JPDA [4], or on possibility distributions such as in the all-neighbour fuzzy association approach [5]. Since HMM has lower complexity than neural networks, as it requires a low number of states for representing the underlying model and uses a reasonable data set for training [11,12], it is utilized in this paper to perform data association in multitarget tracking systems.

This paper proposes a new data association approach for multiple targets tracking based on HMM. In the proposed association approach, one measurement is selected for each target, based on a new similarity metric. The similarity metric is based on a likelihood calculated by projecting a sequence of

observations on an HMM and then it is used for associating measurements to targets. This paper is organized as follows. Problem formulation is addressed in Section 2. The proposed data association approach is developed in Section 3. The performance of the proposed technique and comparison to the NNSF technique are presented in Section 4. Finally, Section 5 contains the conclusion.

## II. PROBLEM FORMULATION

The problem of multiple targets tracking in a noisy environment, has two main stages; data association and state estimation. Data association is responsible for determining which measurements belong to each target at each time instant  $t$ . The target state dynamic and measurement models are assumed to be;

$$x_i(t+1) = F x_i(t) + v_i(t), \quad (1)$$

$$z_i(t) = H x_i(t) + w_i(t), \quad (2)$$

where  $x_i(t)$  is the state vector of target  $i$  at an instant  $t$ ,  $F$  is the state transition matrix,  $z_i(t)$  is the measurement vector for target  $i$  at instant  $t$ ,  $H$  is the measurement matrix,  $v_i(t)$  is the process noise and  $w_i(t)$  is the measurement noise. The covariance matrices of the process and measurement noise are:

$$Q_i(t) = \text{Cov}(v_i(t)), \quad (3)$$

$$R_i(t) = \text{Cov}(w_i(t)). \quad (4)$$

The standard Kalman filter equations are used to construct the state estimate  $\hat{x}_i$  of target  $i$ , from the received noisy measurement vector given by (2). This is achieved through three main stages; prediction, correction, and update stages [5,13]. In the prediction stage we determine;

$$\hat{x}_i(t+1/t) = F \hat{x}_i(t/t), \quad (5)$$

$$P_i(t+1/t) = F P_i(t/t) F^T + Q_i(t), \quad (6)$$

where  $P_i(t)$  is the covariance matrix of the estimation error of target  $i$  at time instant  $t$ . In the correction stage, the Kalman filter gain  $K_i(t+1)$  and the innovation  $\tilde{z}_i(t+1)$  are determined according to:

$$K_i(t+1) = P_i(t+1/t) H^T [H P_i(t+1/t) H^T + R_i(t+1)]^{-1}, \quad (7)$$

$$\tilde{z}_i(t+1) = z_i(t+1) - H \hat{x}_i(t+1/t). \quad (8)$$

In the update stage, the state estimate  $\hat{x}_i(t+1/t+1)$  and the error covariance matrix  $P_i(t+1/t+1)$  are determined as [6,13]:

$$\hat{x}_i(t+1/t+1) = \hat{x}_i(t+1/t) + K_i(t+1) \tilde{z}_i(t+1), \quad (9)$$

$$P_i(t+1/t+1) = [I - K_i(t+1) H] P_i(t+1/t). \quad (10)$$

In case of NNSF, the measurement at a given time instant;  $z_i(t+1)$ , is selected according to the minimum distance between all the received measurements and the state estimate  $\hat{x}_i(t+1/t)$ . First, the innovations are calculated using (8) and then the distance metric is calculated according to:

$$d_{ij} = \sqrt{(\tilde{z}_{ij}(t+1))^T \tilde{z}_{ij}(t+1)}, \quad (11)$$

where  $d_{ij}$  is the distance between measurement  $j$  and target  $i$ . According to (11), the measurement that is closest to the state estimate, i.e. with minimum distance, is selected to update the target state estimate of target.

The purpose of this work is to provide a better metric than the metric distance given in (11), upon which a single measurement is selected for each target, yet maintaining lower complexity than other approaches that combine weighted measurements.

## III. PROPOSED DATA ASSOCIATION TECHNIQUE

A Markov model is a system which may be described as being in one of a set  $N$  of discrete states  $S_1, S_2, \dots, S_N$  at any time instant  $t$  [12]. At each time step the system undergoes a change of its state, either looping back to the same state or transitioning to a different state. The transition is governed by a set of probabilities called the state transition probabilities. For any given state the state transition probabilities obey the following standard stochastic constraints:

$$a_{ij} = P[q_t = S_j | q_{t-1} = S_i] \quad (12)$$

$$a_{ij} \leq 1 \quad (13)$$

$$\sum_{j=1}^N a_{ij} = 1 \quad (14)$$

where  $a_{ij}$  denotes the probability of transition from state  $i$  to state  $j$ , and  $q_t$  denotes the state at time  $t$ .

The above model is an observable model [12,14]. Unlike the Markov model, where it has complete certainty that the system is in a specific state at time  $t$ , the Hidden Markov Model (HMM) lacks such certainty. In this case, each state of the model has a probability distribution over possible observations. So a path through the model between two states would have a probability  $P_{ij}$  given by;

$$P_{ij} = P[q_t | S_j] a_{ij} P[q_{t-1} | S_i] \quad (15)$$

Thus an HMM is a doubly-embedded stochastic process, the first stochastic process is a finite set of states, the transitions between the states are statistically defined by a set of transition

probabilities. The second stochastic process is the distribution of observations over a particular state (since observations hold no certainty to which state they belong), usually the distribution of observable events over a state is a multidimensional probability distribution; typically a Gaussian Mixture Model (GMM) [15,16]. In this case, each state is a GMM that is composed of a weighted sum of Gaussian density functions and the probability density function of the outputs over a particular state is given by [15, 17]:

$$f_x(x) = \sum_{i=1}^m w_i g(x|\mu_i, \sigma_i) \quad (15)$$

where  $w_i$  is the weight given to density function  $i$ ,  $m$  is the number of mixtures in the model and  $g(x|\mu_i, \sigma_i)$  is a Gaussian distribution with a mean  $\mu_i$  and a standard deviation  $\sigma_i$ .

An HMM is characterized by the following parameters: (1) the number of states in the model  $N$ . This number is of great importance, as a model with a small  $N$  might not capture all the statistical details of the problem, and a large value of  $N$  would result in a model that is practically impossible to train. From simulation results, it is found that  $N=5$  is adequate, (2) the state transition probability distribution  $A=\{a_{ij}\}$ ,  $a_{ij} > 0$  for all  $i$  and  $j$ . We consider an ergodic model, i.e., any state can transition to any other state, and (3) the observation probability distribution in state  $S_j$  of the model. The probability distribution for each state is a GMM, where the number of mixtures is  $m=5$  of initial weights, means and variances given as:

$$w_i=1/m = 0.2, \mu_i=0, \sigma_i=1. \quad (16)$$

First a large number of tracks is used to train an HMM. Through training the state transition probabilities as well as the weights, means and variances of the GMM's of each state are calculated. The training is performed by the Baum-Welch algorithm [15,18]. The training phase is not part of the data association algorithm, rather an initial step to construct a model to capture the dynamics of the targets. After the model is constructed it is used to calculate data association metrics as follows.

When a number of measurements  $h$  exist for a target, each measurement  $z_j$  is tested for  $1 < j < h$ . First a sequence is created from the previously determined target states and a measurement as:

$$\begin{aligned} O &= x(t-N+2), x(t-N+3), \dots, x(t-1), x(t), z_j \\ &= O_1, O_2, \dots, O_N. \end{aligned} \quad (17)$$

This forms an observation sequence  $O$  of length  $N$ . The probability that this sequence originated from the constructed model  $\lambda$  is calculated using the Viterbi decoding algorithm [12,19]. The path through the model that maximizes likelihood is chosen as the state sequence and the likelihood of this path is calculated as:

$$l(O|\lambda) = \prod_{k=1}^N \mathbf{P}(O_k|S_j) a_{ij}. \quad (18)$$

For each measurement, a sequence is constructed and the likelihood of this sequence is calculated. The sequence with the highest likelihood contains the most probable measurement  $z_j$  and this measurement is associated to the target. Fig. 1 shows a simple example for selection and Fig. 2 shows an overview of the proposed data association and tracking algorithm.

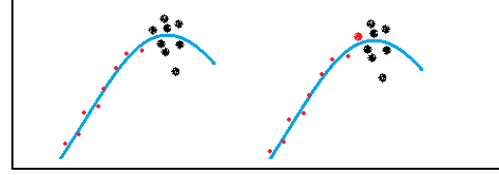


Figure 1. (LHS) Red dots mark previous states, black dots mark measurements, (RHS) The measurement with highest likelihood is selected

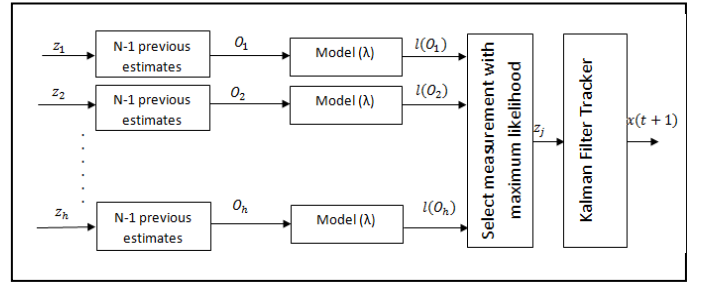


Figure 2. An overall view of the system

#### IV. PERFORMANCE AND COMPARISON

The model and test sequences are composed of  $N=5$  states. In our simulation, the measurements are considered to be affected by white Gaussian noise. In simulated examples the number of measurements under consideration is  $h=2$ . The state vector consists of the  $x$ - and  $y$ - position and the  $x$ - and  $y$ -velocity as;

$$\mathbf{X}(k) = \begin{bmatrix} x(k) \\ v_x(k) \\ y(k) \\ v_y(k) \end{bmatrix} \quad (19)$$

We consider two examples in 2-D space. The first example considers a case of two crossing targets moving in a straight line with constant velocity (zero acceleration). White Gaussian noise is added to the true trajectory to simulate the received target measurements. In this case, the state transition matrix  $F$  in (1) will be [20]:

$$\mathbf{F} = \begin{bmatrix} 1 & \delta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \delta \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (20)$$

where  $\delta$  is the sampling interval.

The measurements are the x- and y- positions of targets, i.e. the measurement matrix  $H$  is given by:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (21)$$

We assume that the noise covariance matrix is given by:

$$R = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix} \quad (22)$$

where  $\sigma_x$  and  $\sigma_y$  are the standard deviation of the noise in the x- and y- direction, respectively. The standard deviation of both target measurements are taken as  $\sigma_{x1} = \sigma_{y1} = 100$ ,  $\sigma_{x2} = \sigma_{y2} = 150$ . The measurement error is defined as;

$$e = \sqrt{e_x^2 + e_y^2} = \sqrt{(x_{\text{true}} - x_{\text{estimate}})^2 + (y_{\text{true}} - y_{\text{estimate}})^2} \quad (23)$$

The true target trajectories and the measured trajectories are shown in Fig. 3. The estimated target trajectories using NNSF are shown in Fig. 4, and the estimated target trajectories using the proposed association approach are shown in Fig. 5. and Fig. 6 compares the measurement error of the NNSF and the proposed association approach. From Fig. 4-6, it is clear that the proposed association approach has much better performance than the NNSF.

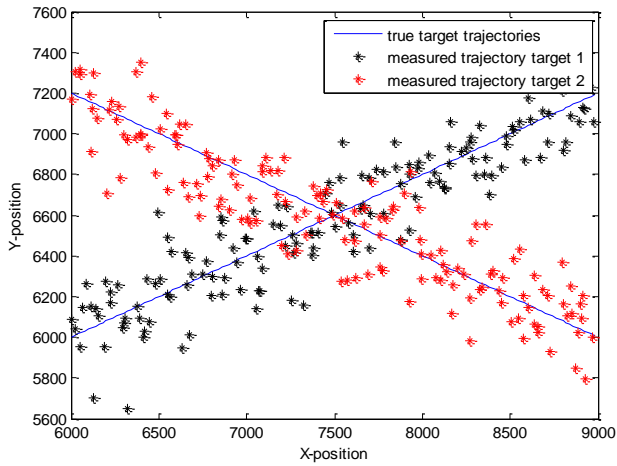


Figure 3. True and measured target tracks for non-maneuvering target.

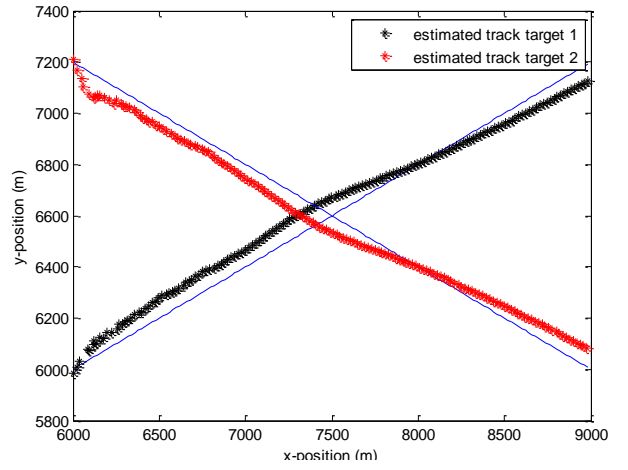


Figure 4. Estimated target tracks with NNSF.

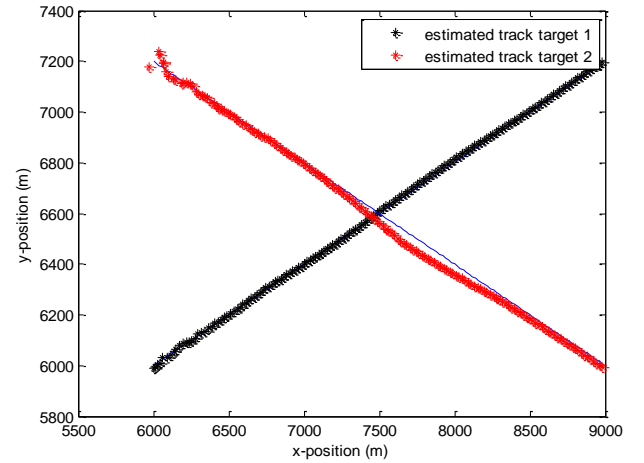


Figure 5. Estimated target tracks with the proposed approach.

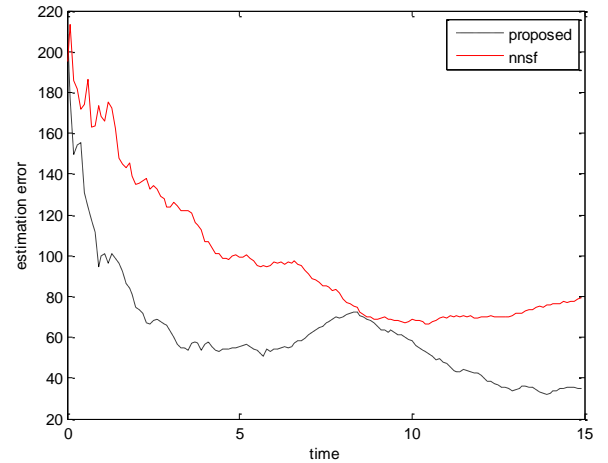


Figure 6. Estimation error for target tracking of non-maneuvering target with both methods

The second example considers the case of two moving targets with acceleration, i.e. maneuvering targets with turn. The state estimate in case of turning motion model is given by [5, 13, 20]:

$$\hat{\mathbf{x}}_i(t+1/t) = F \hat{\mathbf{x}}_i(t/t) + G a_i(t+1), \quad (24)$$

where  $G$ ; the gain matrix and  $a_i(t+1)$ ; the acceleration at time instant  $t+1$ , are given by:

$$G = \begin{bmatrix} \delta^2/2 & 0 \\ \delta & 0 \\ 0 & \delta^2/2 \\ 0 & \delta \end{bmatrix} \quad (25)$$

$$a_i = \begin{bmatrix} a_x \\ a_y \end{bmatrix} \quad (26)$$

The process noise covariance matrix is given by:

$$Q = q^2 \begin{bmatrix} \delta^3/3 & \delta^2/2 & 0 & 0 \\ \delta^2/2 & \delta & 0 & 0 \\ 0 & 0 & \delta^3/3 & \delta^2/2 \\ 0 & 0 & \delta^2/2 & \delta \end{bmatrix} \quad (27)$$

where  $q$  is a scalar equals to  $a\sqrt{\delta}$  [5,13,20] and  $a = a_x = a_y$  is the acceleration.

The standard deviation of the noise for both target measurements are taken as,  $\sigma_{x1} = \sigma_{y1} = 50$ ,  $\sigma_{x2} = \sigma_{y2} = 75$ . The true and measured target trajectories are shown in Fig. 7. The estimated trajectories in case of the NNSF and the proposed association approach are shown in Fig. 8 and 9, respectively. Fig. 10 compares the measurement error of the NNSF and the proposed association approach. The results of Fig.8-10 show the superiority of the proposed approach, in terms of the tracking performance, as compared to the NNSF approach.

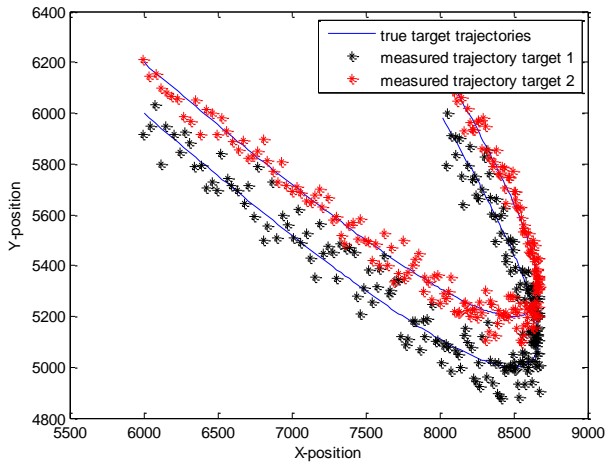


Figure 7. True and measured target tracks for maneuvering targets.

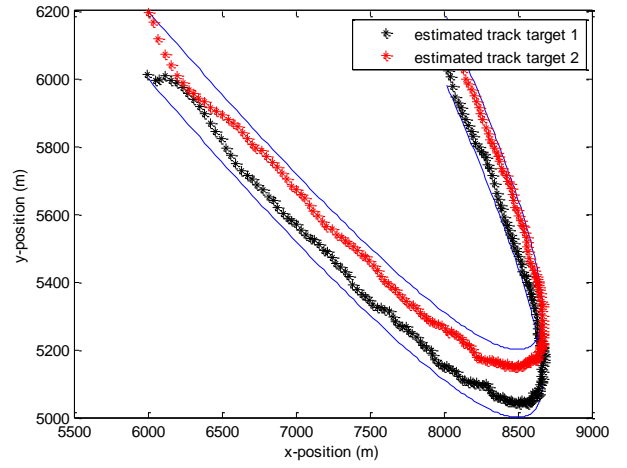


Figure 8. Estimated target tracks using NNSF.

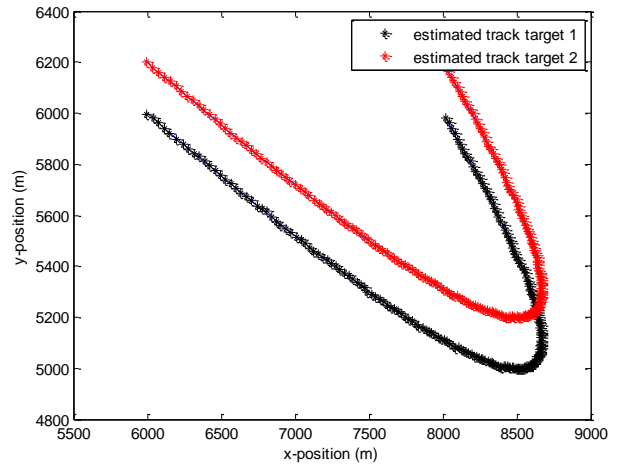


Figure 9. Estimated target tracks using the proposed approach.

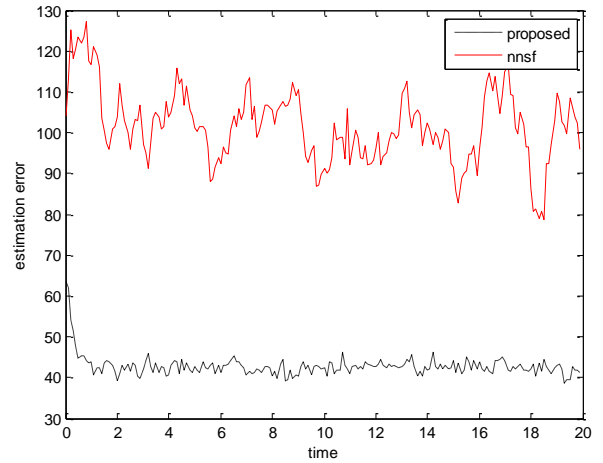


Figure 10. Estimation error for maneuvering target using NNSF and the proposed approach.

## V. CONCLUSION

In this paper, a new data association approach for multiple targets tracking, based on an HMM, has been proposed. The proposed approach utilizes HMM to provide a metric upon which measurements are associated to targets in multiple targets tracking in noisy environments. The performance of the proposed association approach is evaluated and compared to the performance of the NNSF approach, which is usually implemented in practice, in some simulated examples of multiple targets tracking systems. The results showed that the proposed association approach outperforms the NNSF approach in terms of tracking performance. It is worth noting that the proposed association approach provides a trainable model which is sometimes unfeasibly implemented using neural networks approaches.

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