

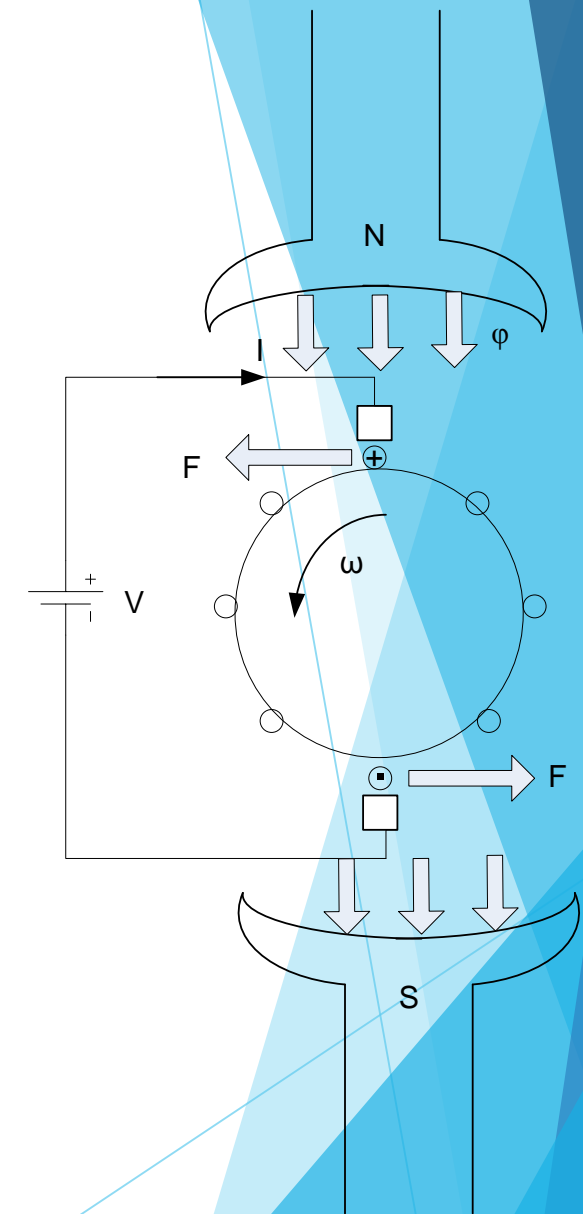
# Electrical Machines I

Week 11-12: DC Motor operation, characteristics and speed control

# DC Motor: 1- Theory of Operation:

## How dc motor works

- Stator field produces flux  $\phi$  from N pole to S poles.
- Brushes touch the terminal of the rotor coil under the pole.
- When brushes are connected to an external dc source of potential  $V$ , current  $I$  enters the rotor coil under the N pole and exits from the terminal that is under the S pole.
- **Rotor current + stator flux = force  $F$**  on coil (Lorentz force). This force will produce **torque  $T$**  that rotates the armature counterclockwise.
- Then the coil carrying current moves away from the brush and is disconnected from the external source and the next coil moves under the brush and the theory repeats itself.
- The force  $F$  is continuously produce and the motor keeps rotating.
- Commutator and brush “switch” the coils mechanically.



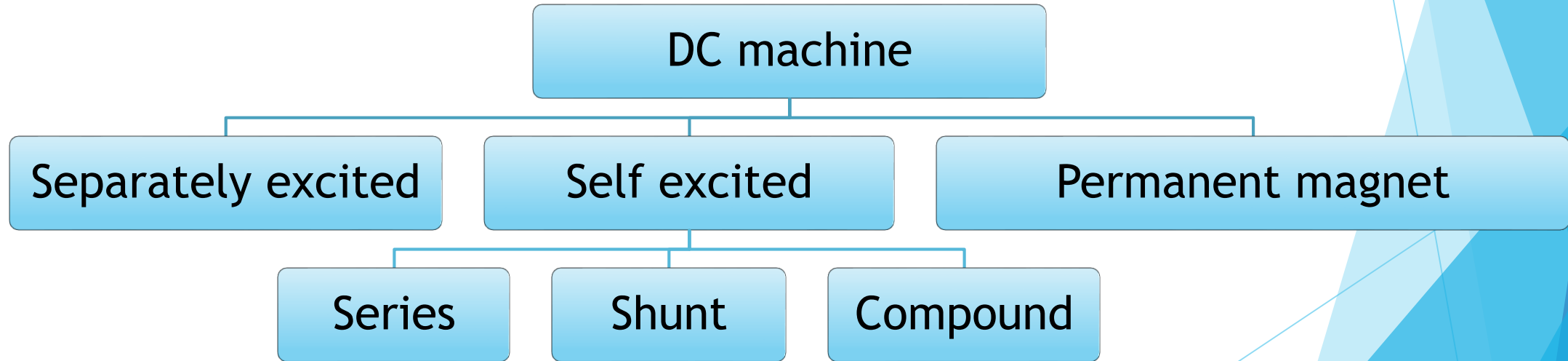
Operation of typical  
DC machine

## DC Motor: 3- Important rules:

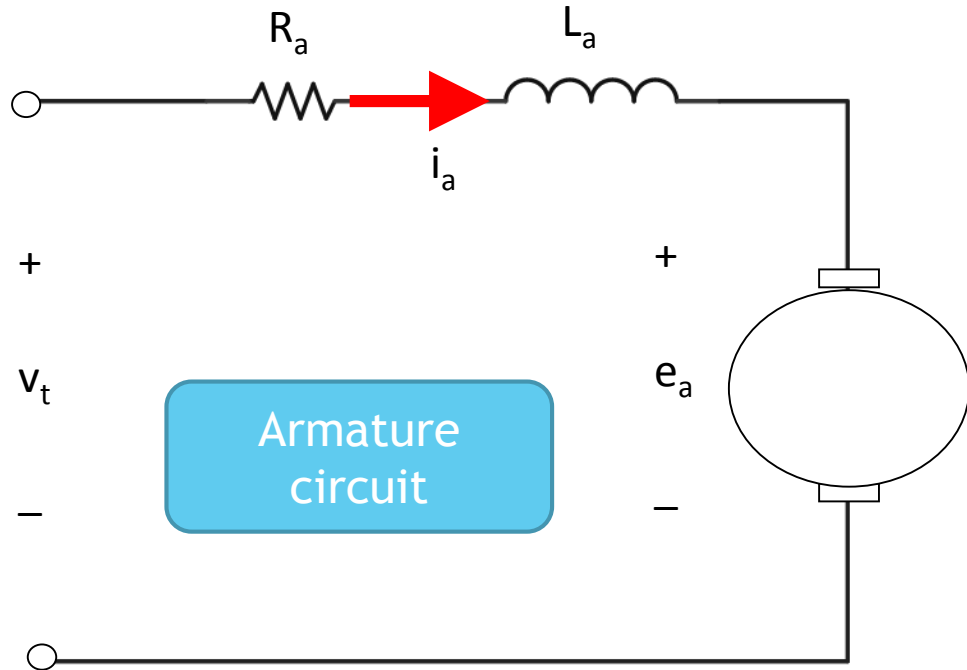
Some limitations:

- High maintenance (commutators & brushes)
- Expensive
- Speed limitations
- Sparking

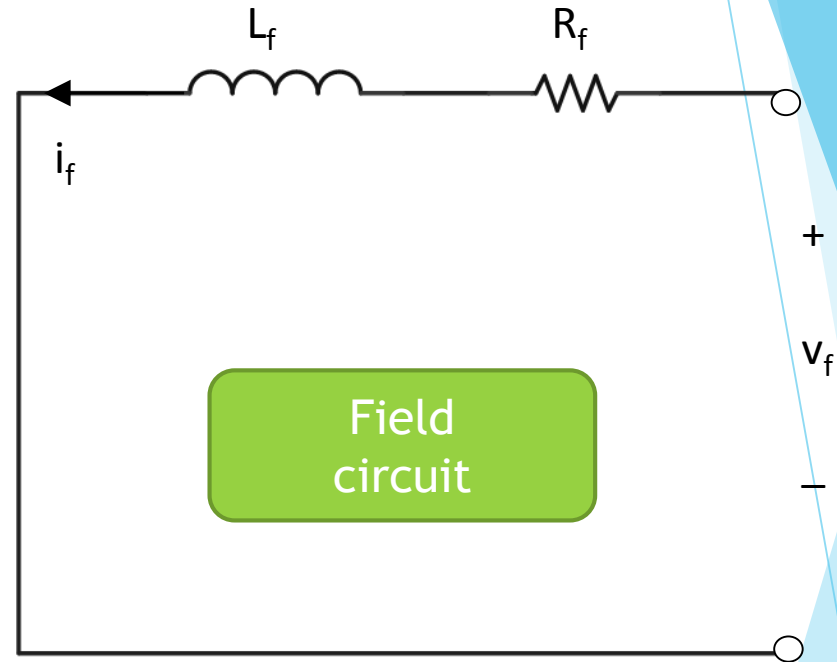
## DC Motor: 4- Connections:



# a- Separately Excited DC motor:



$$v_t = R_a i_a + e_a$$



$$v_f = R_f i_f$$

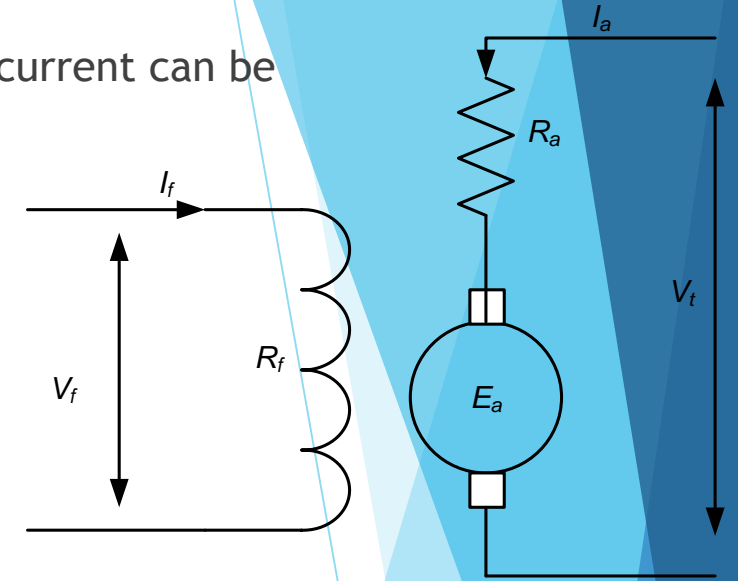
# a- Separately Excited DC motor: Field and armature

- ▶ Field is excited from separate DC source,  $V_f$ . Field resistance is high. The field current can be calculated as:

$$I_f = \frac{V_f}{R_f}$$

- ▶ External source is connected to armature  $V_t$  to provide the electric energy needed to drive the load.
- ▶ Relative to the field, the armature carries a much higher current than that of the field. The armature resistance  $R_a$  is smaller than  $R_f$ .
- ▶ Field current is usually between 1%-10% of rated armature current. The field and armature voltages are usually the same magnitude.
- ▶ The emf  $E_a$  and current  $I_a$  are related as:

$$I_a = \frac{V_t - E_a}{R_a}$$



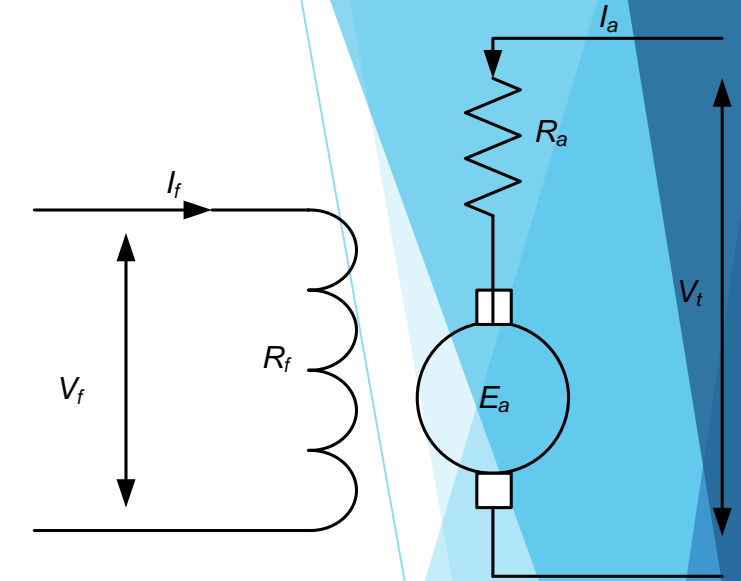
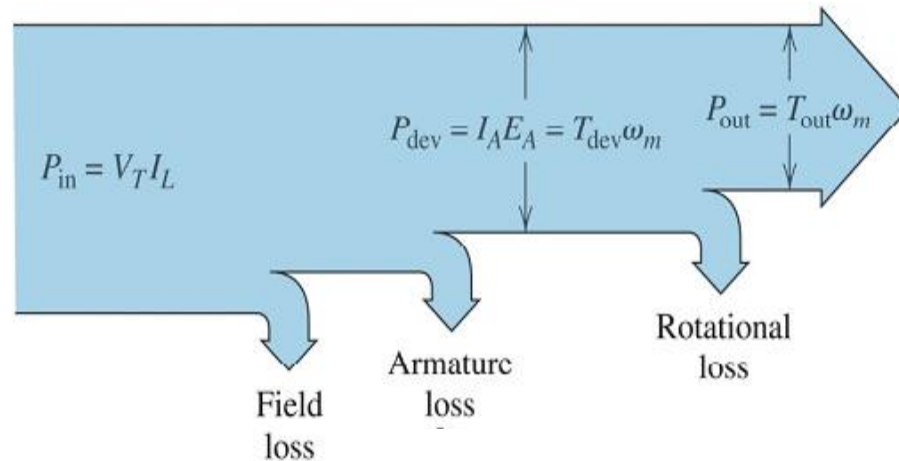
**Separately excited DC machine**

# a- Separately Excited DC motor: Developed Power

- ▶ The developed power,  $P_d$  is given by:

$$P_d = E_a I_a = T_d \omega$$

- ▶ The developed power  $P_d$  is also equal to the output power consumed by the load plus the rotational losses (friction and windage).
- ▶ Similarly, the developed torque,  $T_d$  is equal to the load torque plus the rotational torque.



Separately excited DC machine

# i- Separately Excited DC motor:

- ▶ Using the torque expression instead of force, and using angular speed instead of  $v$ ,  $E_a$  and  $T_d$  can be written as:

$$E_a = \Phi P \frac{n}{60} \times \frac{Z}{a}$$

$$T_d = \Phi P \frac{I_a}{2\pi} \times \frac{Z}{a}$$

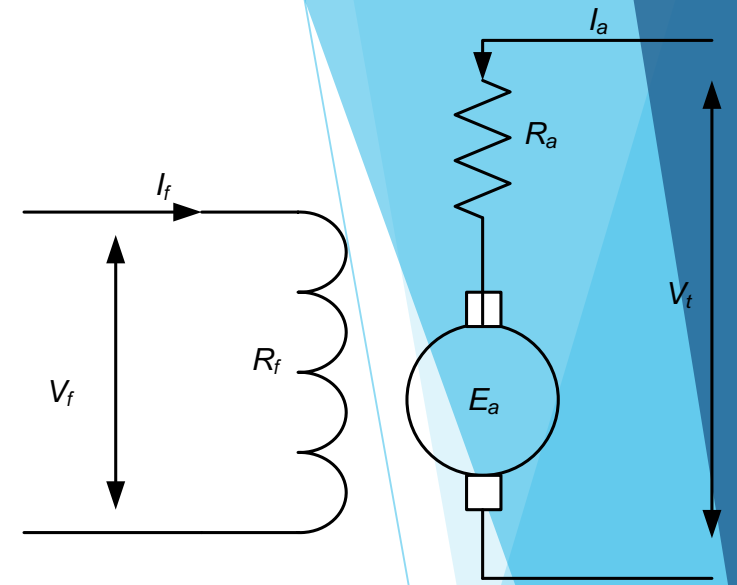
$$E_a = k\phi\omega$$
$$T_d = k\phi I_a$$

$$I_a = \frac{V_t - E_a}{R_a}$$

Speed - torque equation is thus:

$$T_d = k\phi \frac{V_t - E_a}{R_a}$$

$I_a$



Separately excited DC machine

$i_a$  = armature conductor current  
 $E$  = induced emf in conductor  
 $\Phi$  = flux (proportional to field current)  
 $K$  = constant dependent on machine (poles, parallel paths, number of conductors)

# a- Separately Excited DC motor:

$$T_d = k\phi \frac{V_t - E_a}{R_a} \rightarrow I_a$$

► By substituting  $E_a$  and re-writing:

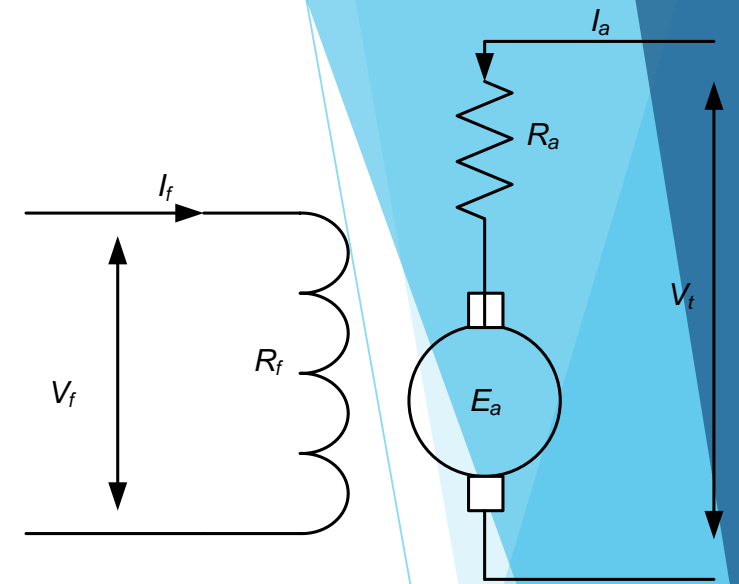
$$T_d = k\phi \frac{V_t - k\phi\omega}{R_a} \rightarrow E_a$$

► Thus  $\omega$  can be re written as:

$$\omega = \frac{V_t}{k\phi} - \frac{R_a}{(k\phi)^2} T_d$$

This is a straight line relation ship

Due to loading, speed decreases as load increases



Separately excited DC machine

$i_a$  = armature conductor current  
 $E$  = induced emf in conductor  
 $\Phi$  = flux (proportional to field current)  
 $K$  = constant dependent on machine (poles, parallel paths, number of conductors)



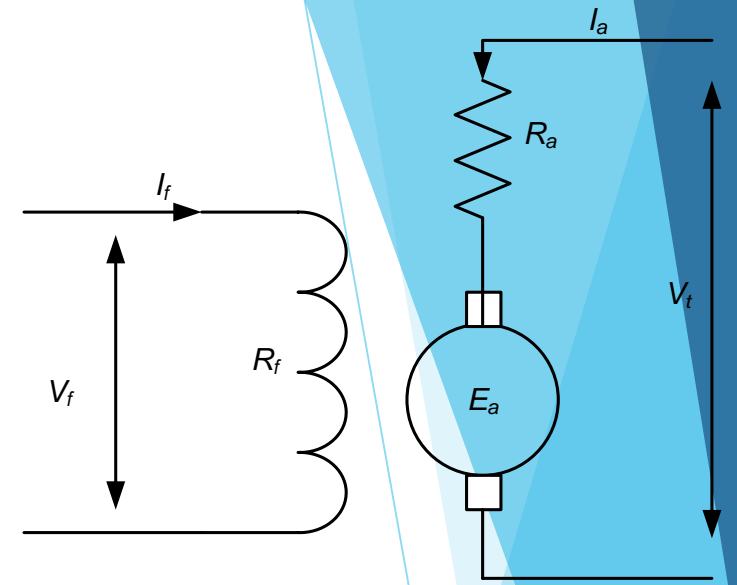
# a- Separately Excited DC motor:

$$\omega = \frac{V_t}{k\phi} - \frac{R_a}{(k\phi)^2} T_d$$

- ▶ The speed- current equation can be obtained if  $\frac{T_d}{k\phi}$  is replaced by  $I_a$ :


$$\omega = \frac{V_t}{k\phi} - \frac{R_a I_a}{k\phi}$$


Due to loading



Separately excited DC machine

- ▶ At no-load, armature current is equal to zero. Hence the no-load speed can be calculated using any of the above  $\omega$  equations by setting the no-load current and load torque equal to zero:

Speed due to load 

No -load speed 

$$\omega_0 = \frac{V_t}{k\phi}$$

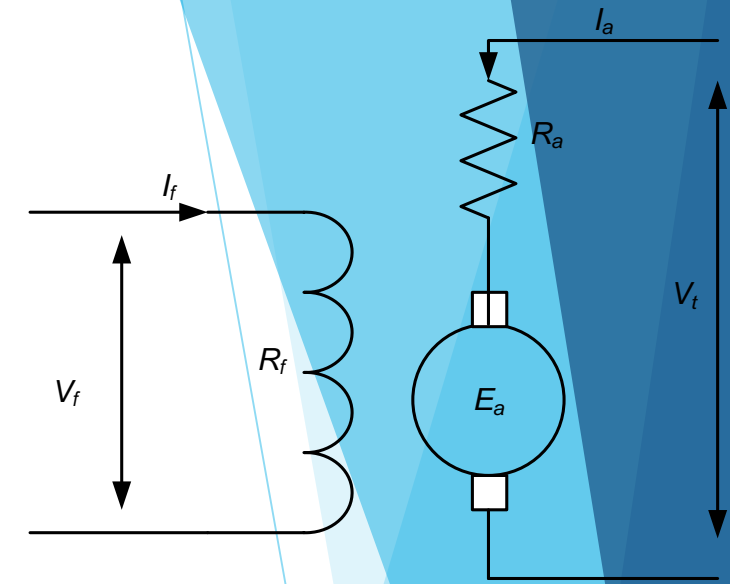
$$\therefore \Delta\omega = \frac{R_a}{(k\phi)^2} T_d$$

# a- Separately Excited DC motor:

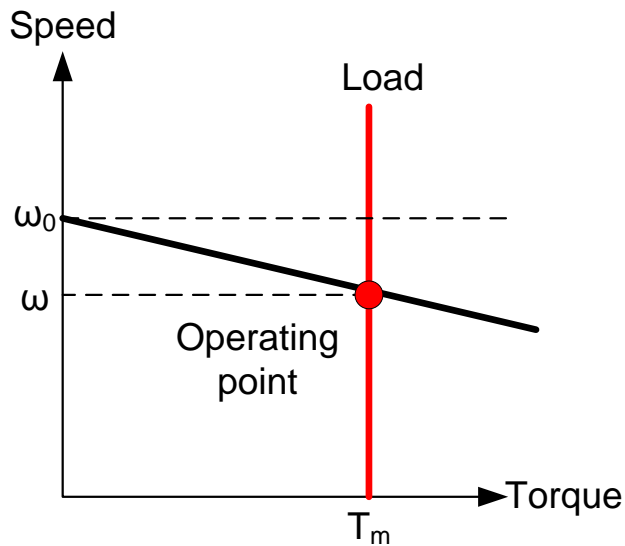
- ▶ For large motors,  $R_a$  is very small because the armature carries higher current and the cross section of the wire must be large. For these motors,  $\Delta\omega$  is very small. The motors are considered as a constant speed machine

$$\therefore \omega = \frac{V_t}{k\phi} - \frac{R_a}{(k\phi)^2} T_d$$

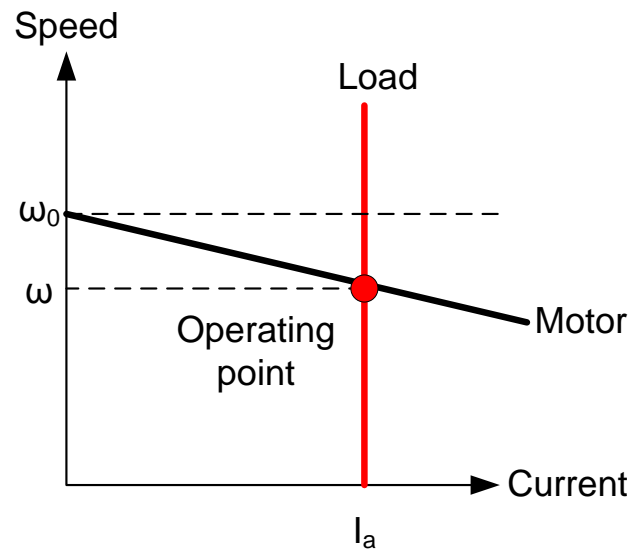
$$\omega = \frac{V_t}{k\phi} - \frac{R_a I_a}{k\phi}$$



Separately excited DC machine



Torques - speed characteristics  
Separately excited



Speed- current characteristics  
Separately excited

$$\omega = \omega_0 - \Delta\omega$$

## b- Shunt DC motor:

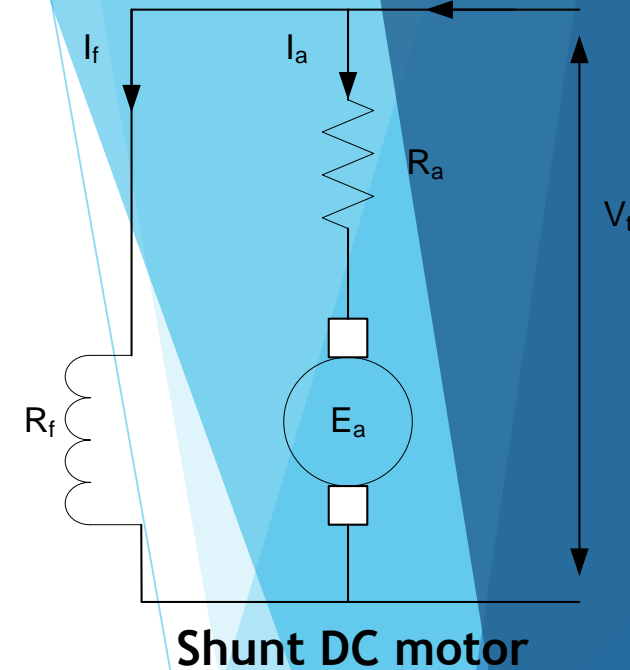
- ▶ Series and shunt field resistances are connected in shunt (parallel)
- ▶ Exhibits identical characteristics as that of the separately excited motor
- ▶ The field current is constant regardless of the loading of the machine.

### Shunt DC motor Speed Control:

1. Field resistance control
2. Terminal voltage control
3. Series resistance insertion in armature (less common practice due to the added losses accompanied by the insertion of new resistance)

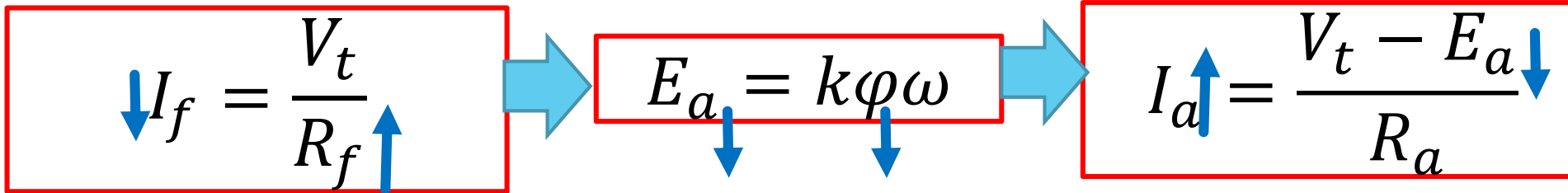
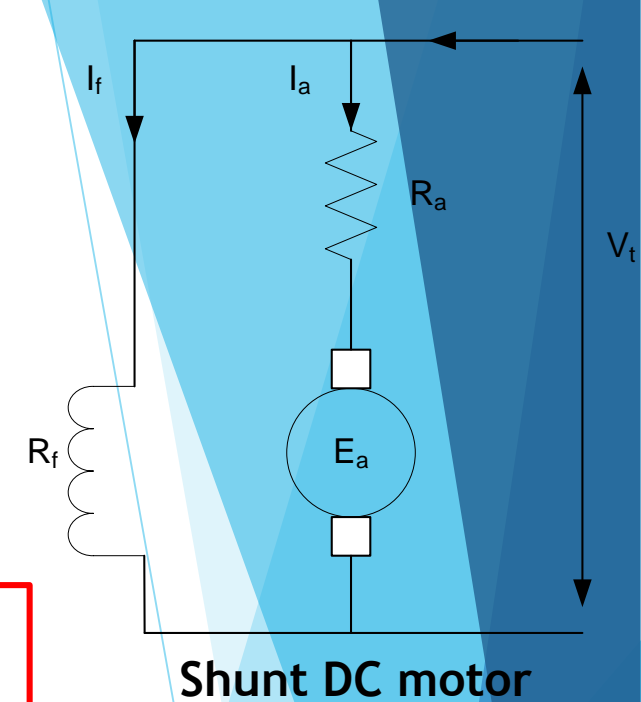
$$\omega = \frac{V_t - R_a I_a}{k\phi}$$

$$\omega = \frac{V_t}{K\phi} - \frac{R_a}{(K\phi)^2} T_d$$

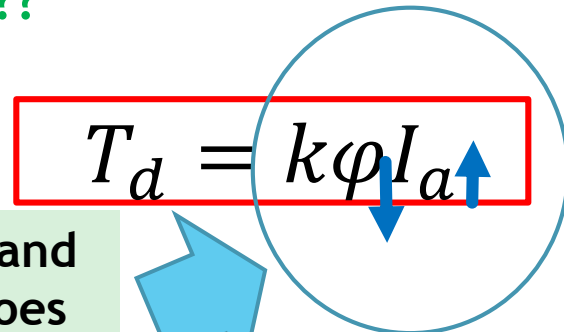


## b- Shunt DC motor: 1- Field resistance Control

- ▶ When  $R_f$  field resistance increases, field current  $I_f$  decreases and so the flux  $\phi_f$  decreases. Since flux decreases, induced emf  $E_a$  decreases and thus the armature current  $I_a$  increases



Since flux  $\phi_f$  decreased while the armature current  $I_a$  increased, how will this affect the developed torque  $T_d$ ???



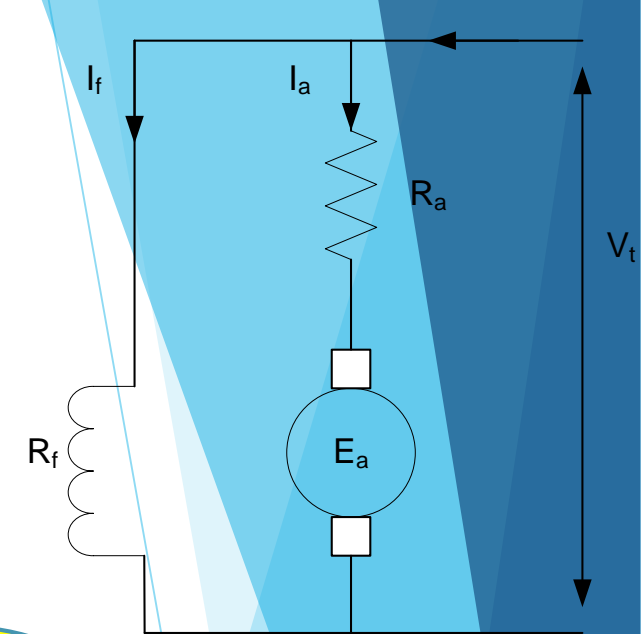
One variable increases and the other decreases, does this mean that the torque will remain constant???



It is also called "field weakening"

# b- Shunt DC motor: 1- Field resistance Control

▶ Example: Assume  $R_a = 0.25\Omega$ , terminal voltage = 250V, induced emf = 245V.



Shunt DC motor

$$I_a = \frac{250 - 245}{0.25} = 20 \text{ A}$$

$\phi$  decreased by 1%

$$E_a \text{ decreased by } 1\% = 0.99 * 245 = 242.55 \text{ V}$$

So what happens if the flux decreased by 1%?

$\uparrow T_d = k\phi I_a \uparrow$   
The torque will increase.  
Since  $T_d > T_L$  motor speeds up

$$\uparrow I_a = \frac{250 - 242.55}{0.25} = 29.8 \text{ A}$$

Armature current increased by 49% for a decrease in flux by 1%

$\omega$  increases,  $E_a$  increases,  $I_a$  decreases,  $T_d$  decreases till  $T_d = T_L$  **at a higher speed**



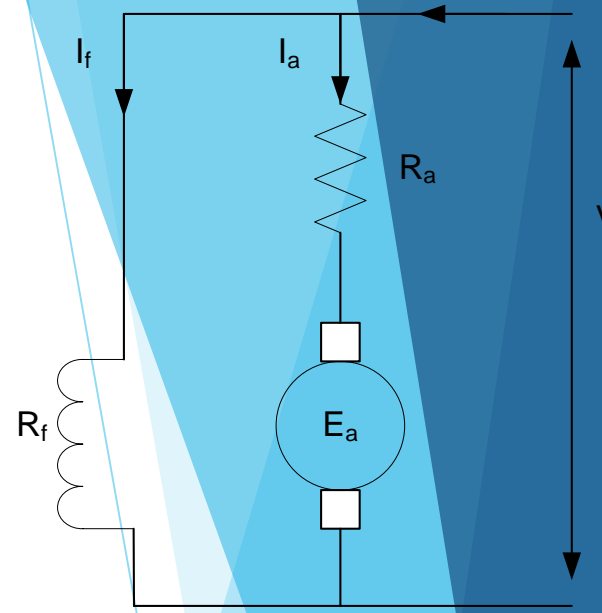
# b- Shunt DC motor: 1- Field resistance Control

$$y = C + mx$$

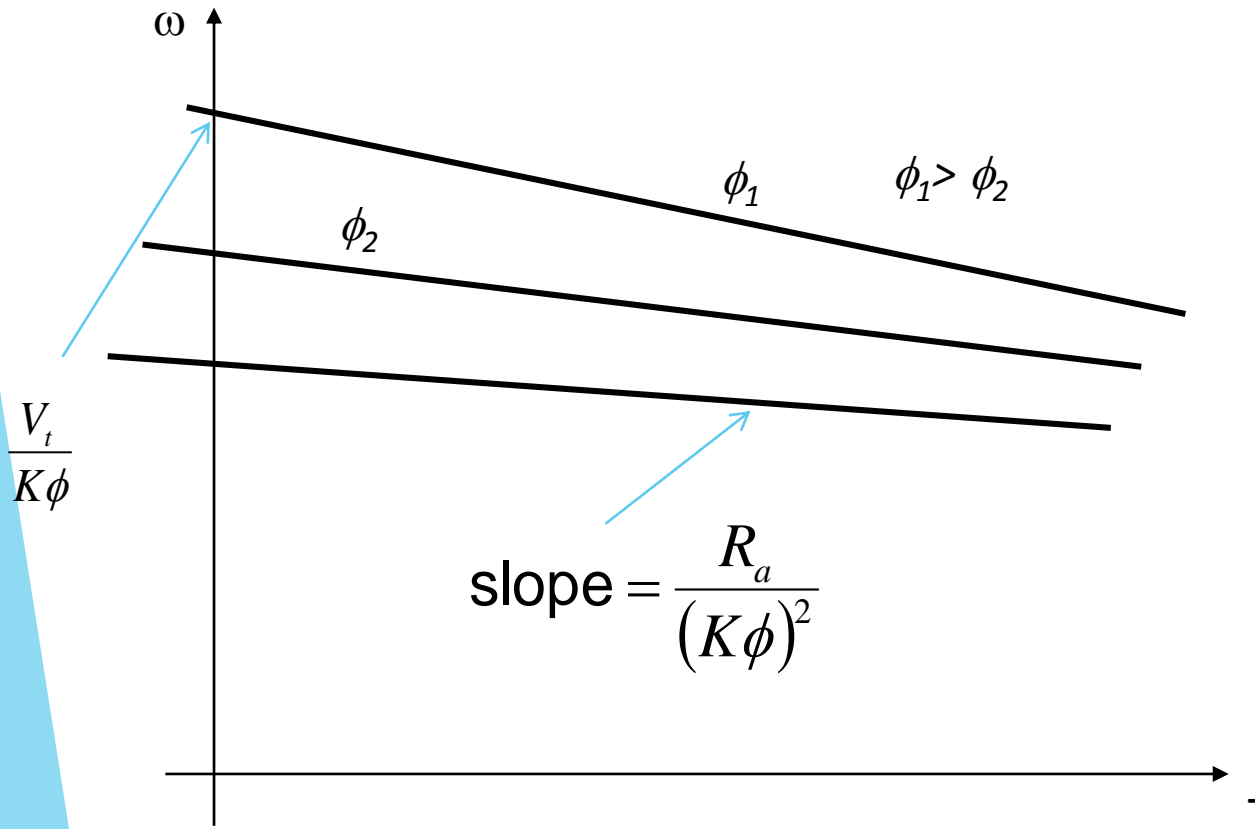
$$\omega = \frac{V_t}{k\phi} - \frac{R_a}{(k\phi)^2} T_d$$

No load speed= C intercept

Slope of a st. line= m



Shunt DC motor

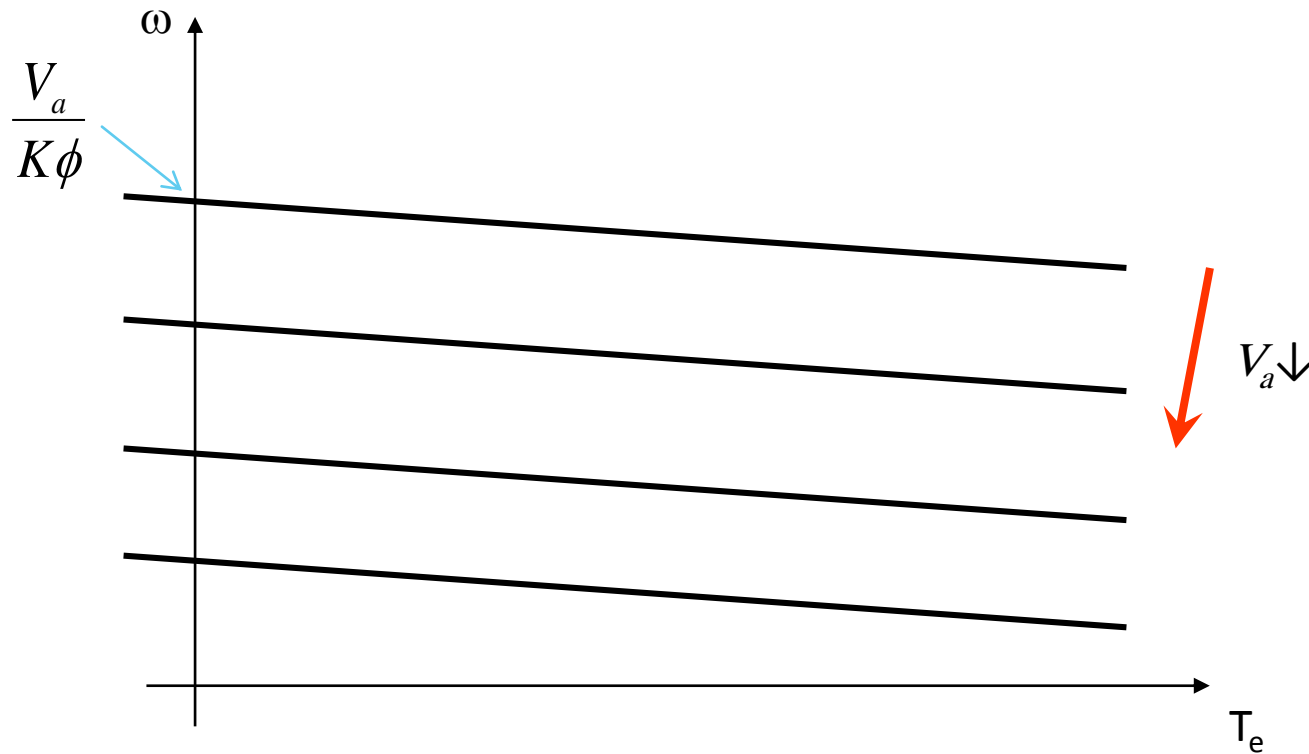
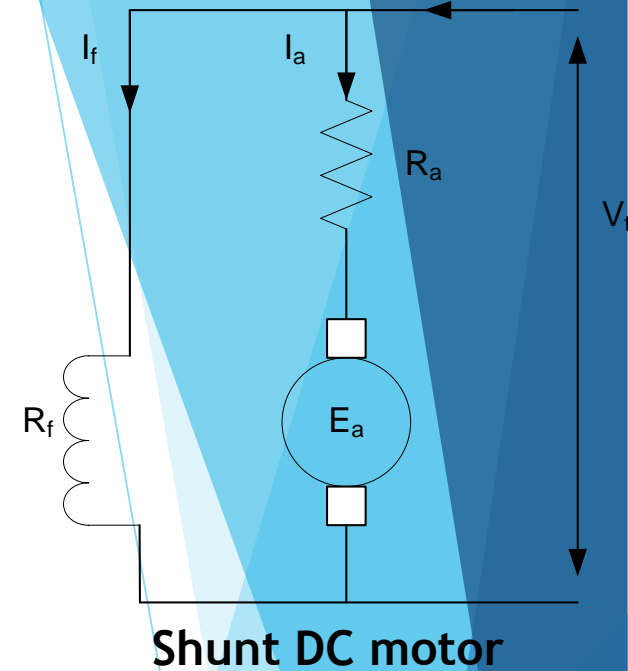


As the field resistance increase, the flux decreases, and the no-load speed of the motor increases, while the slope of the torque speed curve becomes steeper

# b- Shunt DC motor: 2- Terminal voltage control

$$\omega = \frac{V_t}{k\phi} - \frac{R_a}{(k\phi)^2} T_d$$

No load speed= **CHANGES**    Slope = **UNCHANGES**



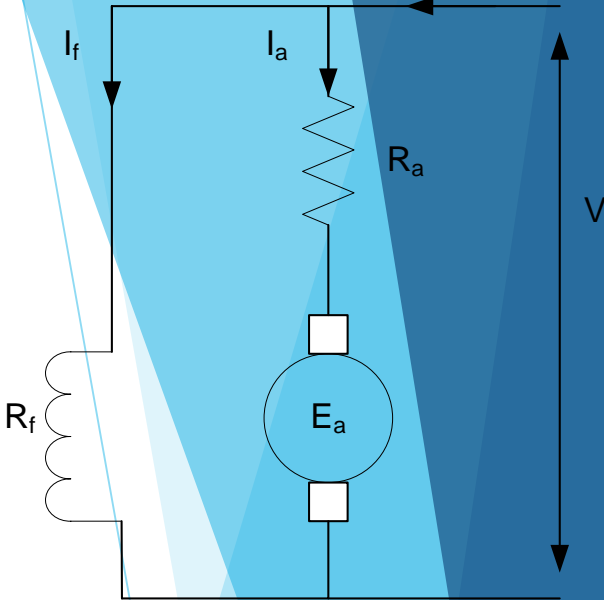
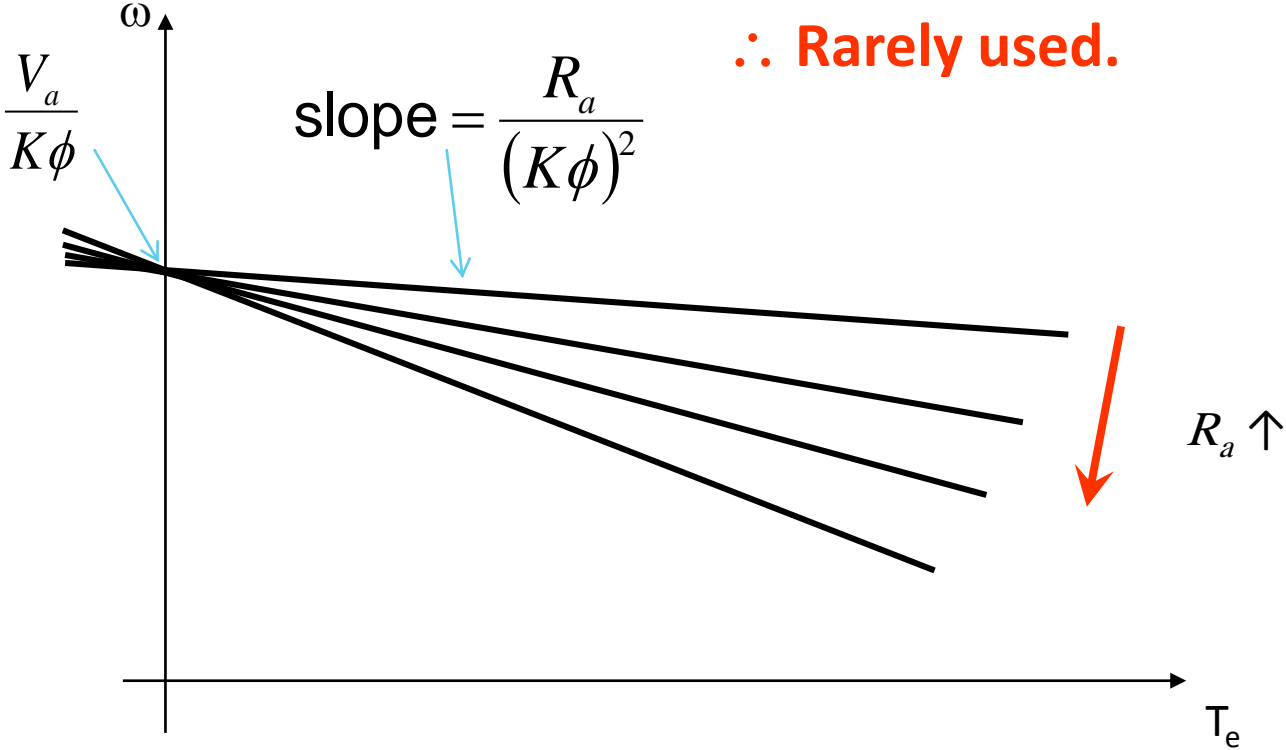
# b- Shunt DC motor: 3- Armature resistance control

$$\omega = \frac{V_t}{k\phi} - \frac{R_a}{(k\phi)^2} T_d$$

Simple control

Losses in external resistor

∴ Rarely used.



Shunt DC motor



## b- Shunt DC motor:



Is the choice REALLY that simple?  
Can I either choose field control or  
armature control or is there a  
hidden secret

Field control: The lower the field, the faster the machine turns and vice versa. Therefore at minimum value for speed, the field current is maximum. For very very low values of speed, the field winding might be damaged due to the high field current

Armature control: The lower the terminal voltage, the slower the machine is and vice versa. The maximum achievable speed is related to the maximum terminal voltage the machine can withstand. For very very high values of speed, the armature winding might be damaged due to the high terminal voltage.

## b- Shunt DC motor:

**Field control (field weakening control):** used for operation above the base speed

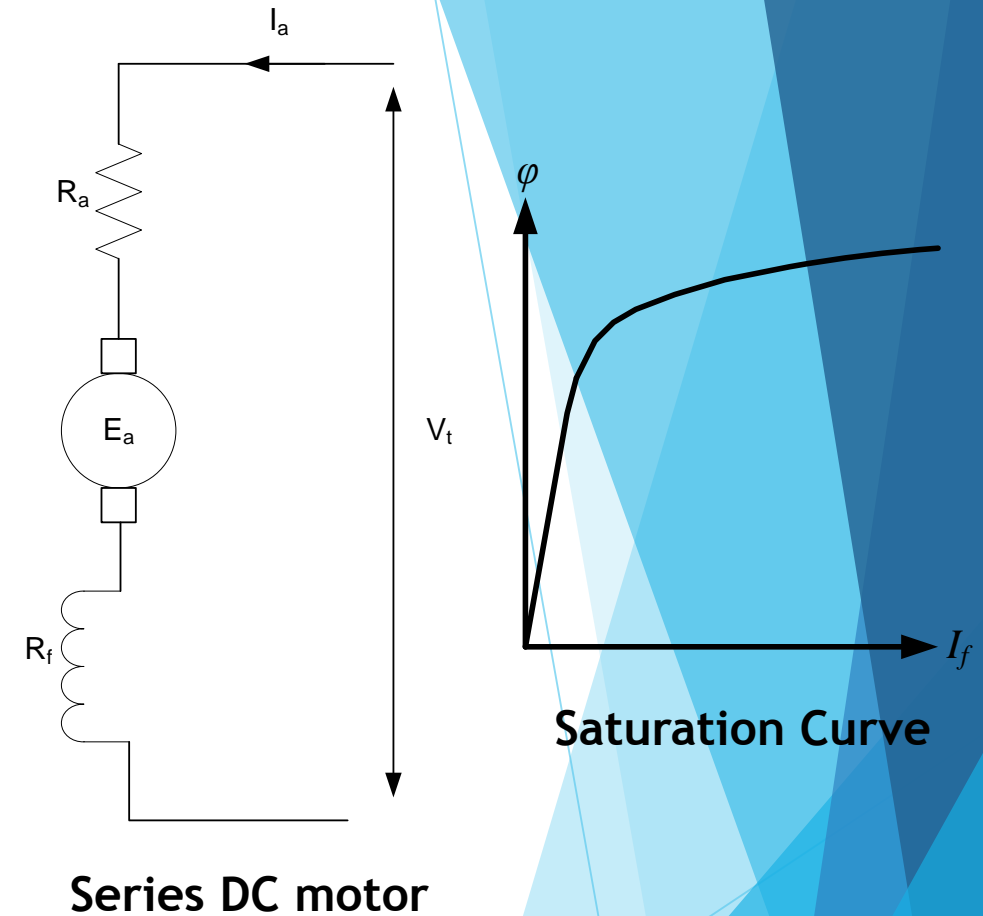


**Armature control (terminal voltage control):** used for operation below the base speed

Combining two together it is possible to control the machine in a good operating range. Separately excited and shunt DC motor have an excellent speed control range

## c- Series DC motor:

- ▶ Series and shunt field resistances are connected in series
- ▶ Series field resistance is composed of a small number of turns as compared to that of the shunt field resistance
- ▶ Current in the series winding is equal to that of the armature current. Hence the series field resistance carries much larger current than that of the shunt field.
- ▶ The series machine has field current varying with the loading of the motor- the heavier the load, the stronger the field. At light or no load conditions, the field of the series motor is very small.
- ▶ The effect of high field current must be taken into consideration when operating with series motor as to avoid saturation due to high field current



# c- Series DC motor:

$$E_a = k\phi\omega$$

$$T_d = k\phi I_a$$

$$I_a = \frac{V_t - E_a}{R_a + R_f}$$

$$T_d = k\phi \frac{V_t - k\phi\omega}{R_a + R_f}$$

$$\omega = \frac{V_t}{k\phi} - \frac{R_a + R_f}{(k\phi)^2} T_d$$

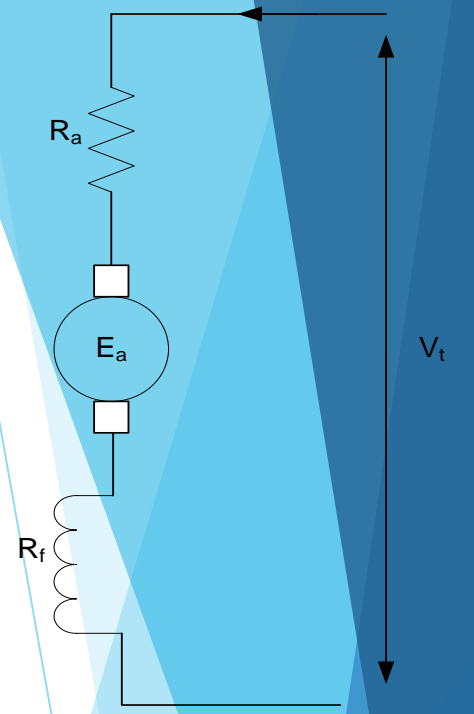
Modify the separately

➤ If we assume that the motor operates in the linear region of the saturation curve.

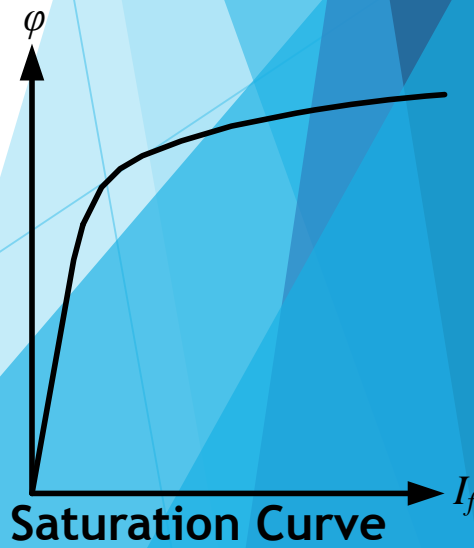
Flux proportional to armature current

$$\phi = C I_a$$

$$T_d = k\phi I_a = kC I_a^2$$

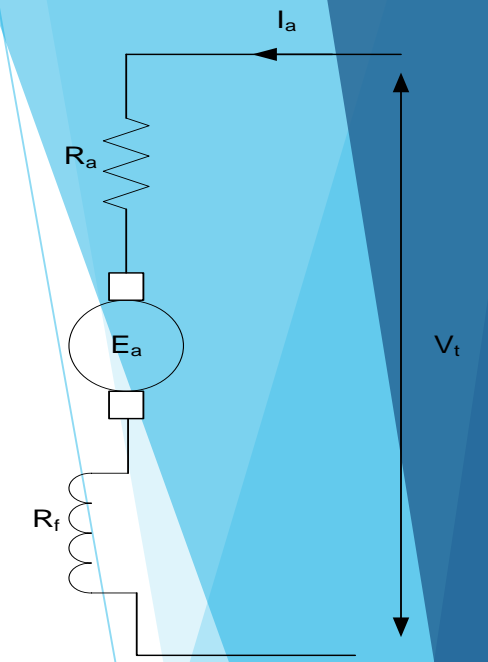


Series DC motor



Saturation Curve

# c- Series DC motor:



Series DC motor

$$T_d = k\phi I_a = kC I_a^2$$

$$k\phi = kC I_a$$

$$\phi = C I_a$$

$$\omega = \frac{V_t}{k\phi} - \frac{R_a + R_f}{(k\phi)^2} T_d$$

Speed at no load (or light load) is **excessively high**. For this reason, series motors must always be connected to a mechanical load.

$$\therefore \omega = \frac{V_t}{kC I_a} - \frac{R_a + R_f}{kC}$$

$$\therefore \omega = \frac{V_t}{\sqrt{kC T_d}} - \frac{R_a + R_f}{kC}$$


$$k\phi I_a = kC I_a^2$$

$$k\phi = kC I_a$$

$$1/I_a = kC / k\phi$$

$$T_d = k\phi \frac{k\phi}{kC} = \frac{(k\phi)^2}{kC}$$

$$k\phi = \sqrt{kC T_d}$$

$$\omega = \frac{V_t}{k\phi} - \frac{R_a}{(k\phi)^2} T_d$$


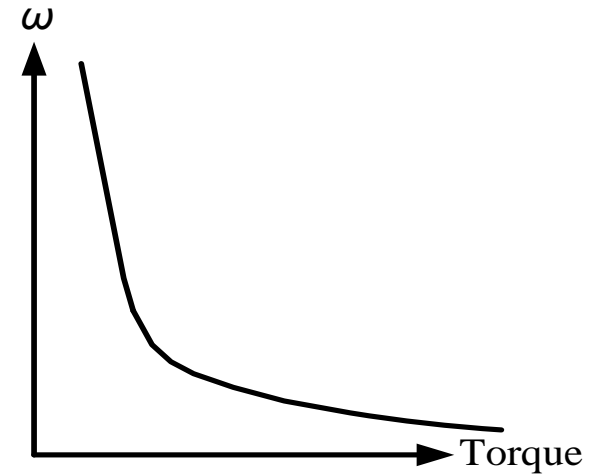
Separately excited and shunt

# c- Series DC motor: Methods of Speed control

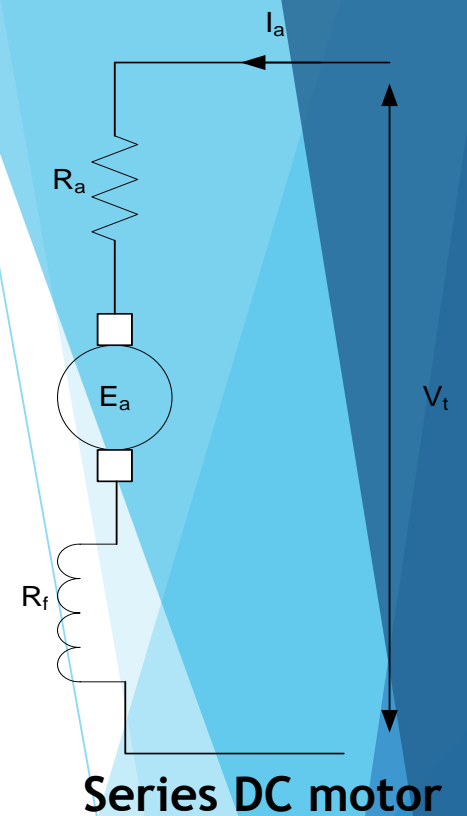
- ▶ Unlike the shunt machine, there is only one way which is changing the armature voltage (terminal voltage change).
- ▶ Increasing the terminal voltage  $V_t$ , will increase the first term of the equation and will result in higher speed for any given torque

$$\therefore \omega = \frac{V_t}{\sqrt{kCT_d}} - \frac{R_a + R_f}{kC}$$

Terminal voltage change



Torques - speed characteristics series motor



- ▶ Inserting another resistance can be used in speed control but will account for additional losses
- ▶ Until the last 40 years, there was no convenient way to change  $V_t$ , so the only method was through the resistance change. Everything changes today with the introduction of solid state control circuits

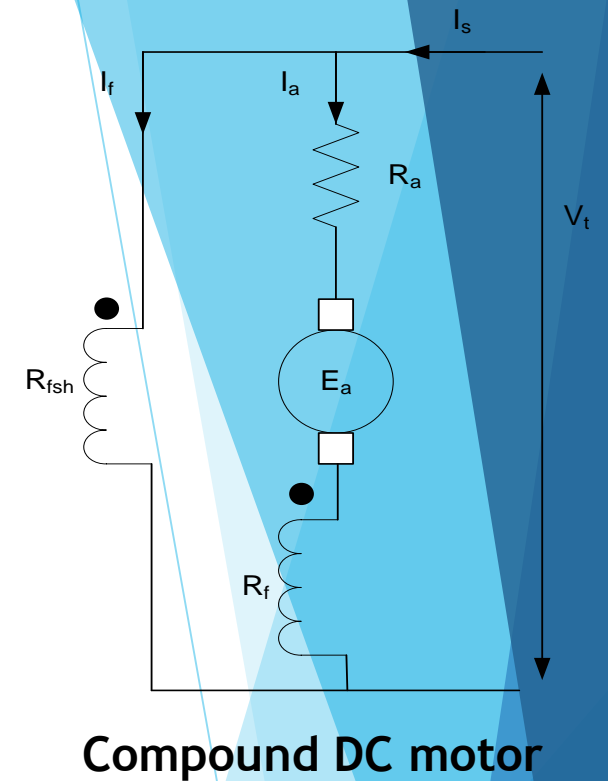
# d- Compound DC motor:

- The compound excitation characteristic in a dc motor can be obtained by combining the operational characteristic of both the shunt and series excited motor. The compound wound self excited dc motor or simply compound wound dc motor *essentially* contains the field winding connected both in series and in parallel to the armature winding.
- When the shunt field flux assists the main field flux, produced by the main field connected in series to the armature winding then its called cumulative compound dc motor.

$$\varphi_{total_{cum}} = \varphi_{series} + \varphi_{shunt}$$

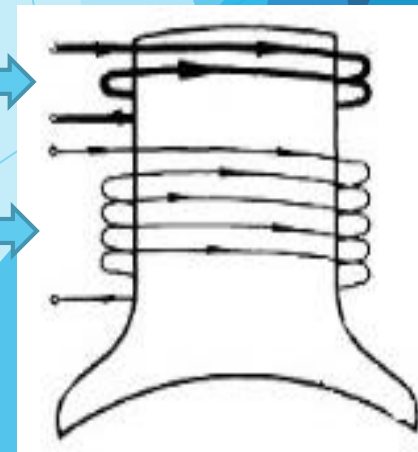
- In case of a differentially compounded self excited dc motor i.e. differential compound dc motor, the arrangement of shunt and series winding is such that the field flux produced by the shunt field winding diminishes the effect of flux by the main series field winding. The net flux produced in this case is lesser than the original flux and hence does not find much of a practical application.

$$\varphi_{total_{diff}} = \varphi_{shunt} - \varphi_{series}$$



Series field

Shunt field



# d- Compound DC motor: Cumulative Compound

Cumulative Compound DC motor

$$V_t = E_a + I_a(R_a + R_f)$$

$$I_a = I_s - I_f$$

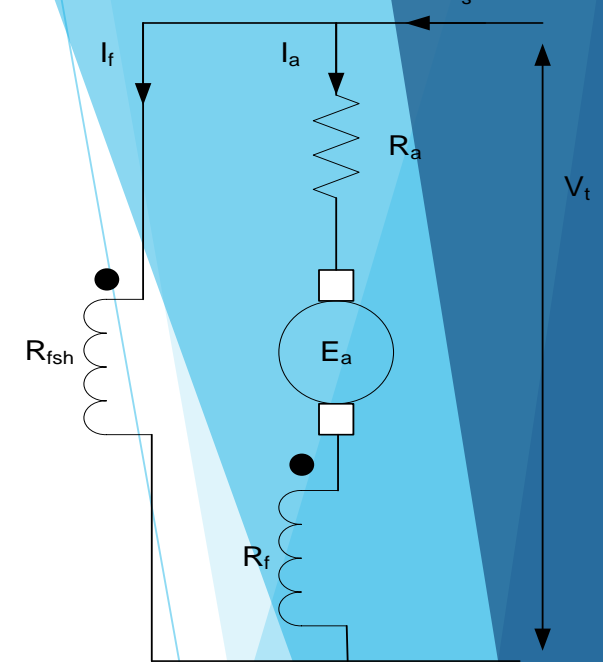
$$I_f = \frac{V_f}{R_{fsh}}$$

In cumulative compound DC motor there is a component of flux which is constant (not dependent on load) and another component which depends on load (armature current)

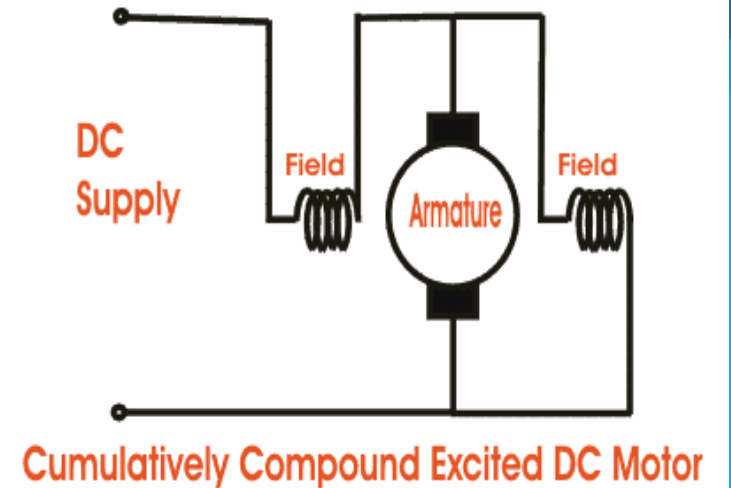
$$\varphi_{total\ cum} = \varphi_{series} + \varphi_{shunt}$$

Load dependent

Load independent



Cumulative Compound DC motor



Cumulatively Compound Excited DC Motor



# d- Compound DC motor: Cumulative Compound

Original separately excited equation  $\rightarrow$   $\omega = \frac{V_t - R_a I_a}{k\phi}$

$\downarrow$  Compound (cumulative)

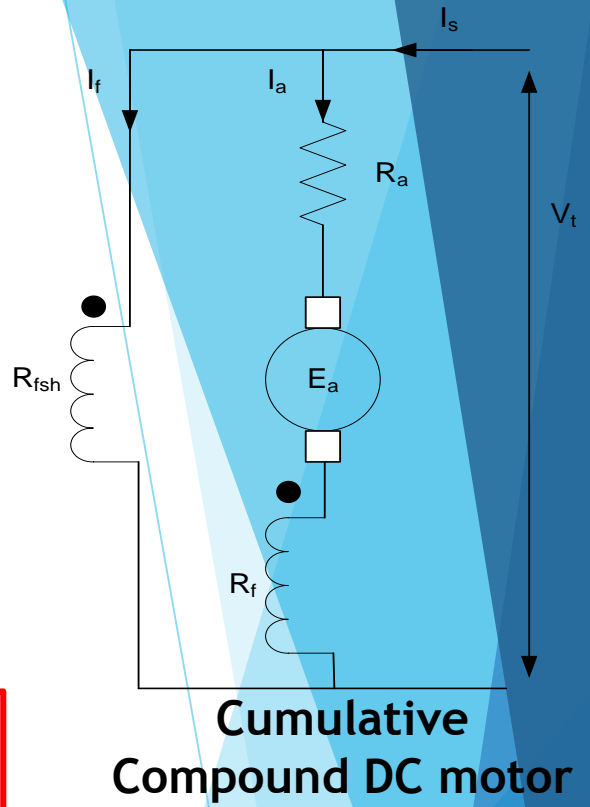
$$\omega = \frac{V_t - (R_a + R_f)I_a}{k(\phi_{total})}$$

$$\omega = \frac{V_t}{k(\phi_{series} + \phi_{shunt})} - \frac{(R_a + R_f)I_a}{k(\phi_{series} + \phi_{shunt})}$$

Assuming that the terminal voltage,  $V_t$  is constant as well as the  $\phi_{shunt}$  are constant:

$$\therefore \phi_{series} = C I_a$$

$$\therefore \omega = \frac{V_t}{\boxed{k C I_a} + k\phi_{shunt}} - \frac{(R_a + R_f)I_a}{\boxed{k C I_a} + k\phi_{shunt}}$$



# d- Compound DC motor: Cumulative Compound

$$\therefore \omega = \frac{V_t}{kCI_a + k\varphi_{shunt}} - \frac{(R_a + R_f)I_a}{kCI_a + k\varphi_{shunt}}$$

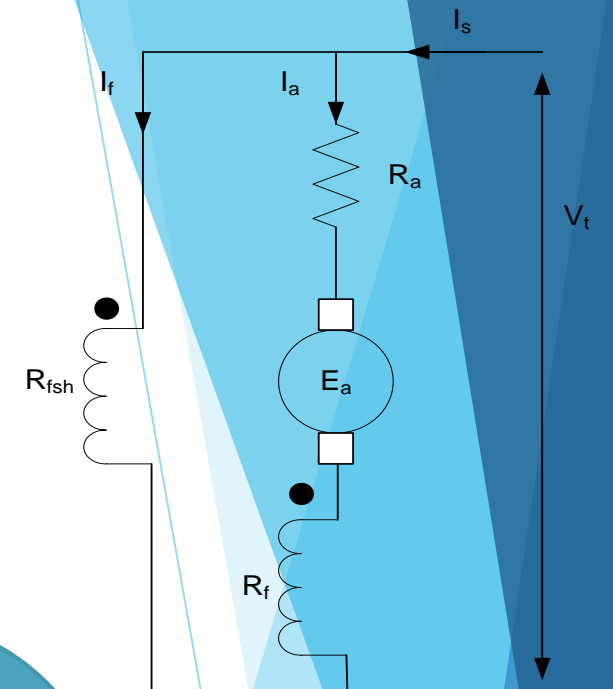
$$\therefore T_d = k(\varphi_{series} + \varphi_{shunt})I_a$$

$$\therefore I_a = \frac{T_d}{k(\varphi_{series} + \varphi_{shunt})}$$

Previously we concluded

$$\omega = \frac{V_t}{k(\varphi_{series} + \varphi_{shunt})} - \frac{(R_a + R_f)I_a}{k(\varphi_{series} + \varphi_{shunt})}$$

$$\therefore \omega = \frac{V_t}{k(\varphi_{series} + \varphi_{shunt})} - \frac{(R_a + R_f)T_d}{[k(\varphi_{series} + \varphi_{shunt})]^2}$$



Cumulative Compound DC motor

# d- Compound DC motor: Cumulative Compound

$$\omega = \frac{V_t}{k(\varphi_{series} + \varphi_{shunt})} - \frac{(R_a + R_f)T_d}{[k(\varphi_{series} + \varphi_{shunt})]^2}$$

At no-load when  $T_d=0$ , armature current is zero, and  $\varphi_{series}=0$

$$\varphi_{total} = \varphi_{series} + \varphi_{shunt}$$

$$\varphi_{total} = \varphi_{shunt}$$

=0

$$\therefore \omega_0 = \frac{V_t}{k(\varphi_{shunt})}$$

✓ **No load** speed using cumulative compound...  
"same as shunt"

✓ Also, the **speed drop** is avoided which occurs in **series** motor connection



$$\omega = \frac{V_t}{k\varphi} - \frac{R_a + R_f}{(k\varphi)^2} T_d$$

Series Motor

Bigger speed drop

shunt Motor

smaller speed drop

$$\omega = \frac{V_t}{k\varphi} - \frac{R_a}{(k\varphi)^2} T_d$$

# d- Compound DC motor: Cumulative versus Differential Compound

$$\omega = \frac{V_t - (R_a + R_f)I_a}{k(\varphi_{total})}$$

$$\varphi_{total} = \varphi_{series} + \varphi_{shunt}$$

Load dependent      Load independent

Cumulative compound

Load increase, armature current increase, series field increases, total flux increases, speed decreases

$$\varphi_{total} = \varphi_{shunt} - \varphi_{series}$$

Load independent      Load dependent

Differential compound

Load increase, armature current increase, series field increases, total flux decreases, speed increases

# Questions

Explain how DC motor works

Explain the methods of speed control for:

- Shunt and separate dc motor
- Series dc motor
- Compound dc motor

How does a compound dc motor combines the features of a series and shunt dc motor

Draw the torque speed characteristics of separately, shunt and series dc motor. Indicate how to plot these characteristics