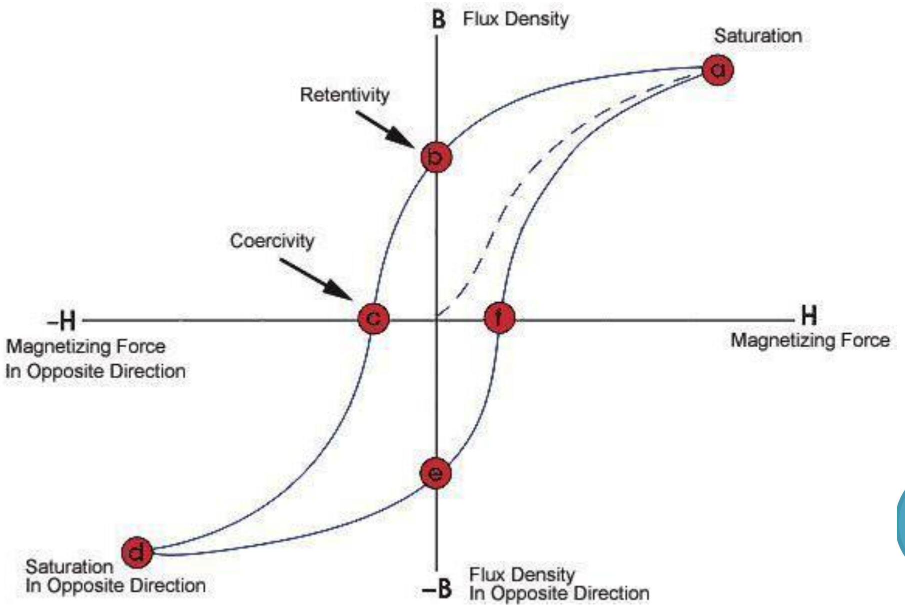
Electrical Machines I

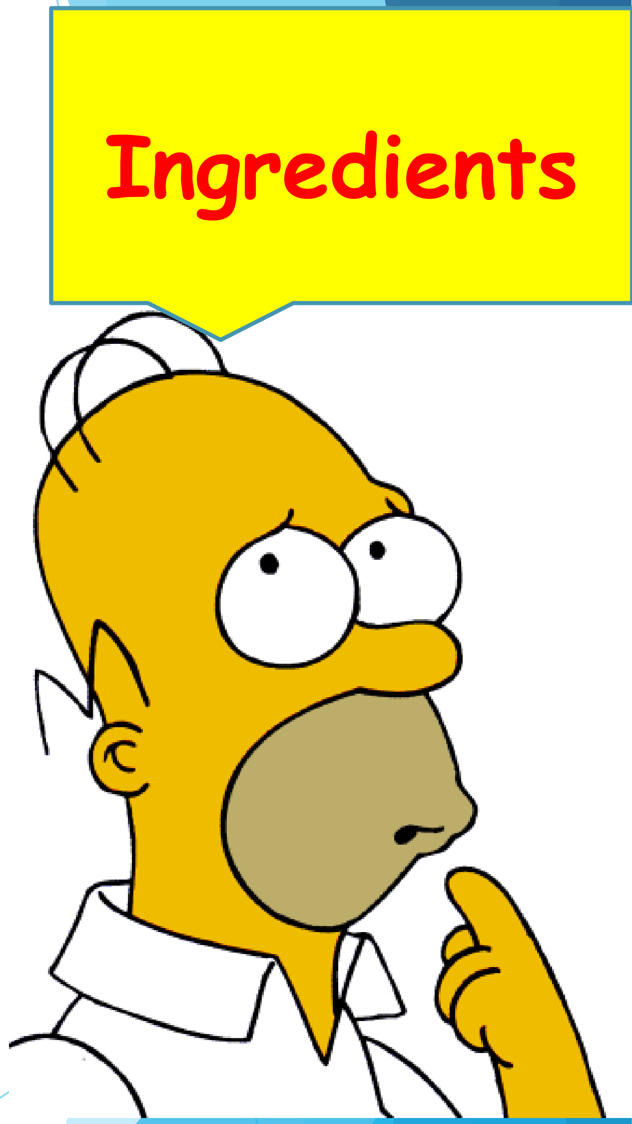
Week 3: Energy Storage

RECALL… REMEMBER…. !!

- Magnetic circuits and electrical circuits are co-related
- What is hystresis
- Magnetic Losses??



WHY DO WE NEED ALL OF THIS ANYWAY!!!!



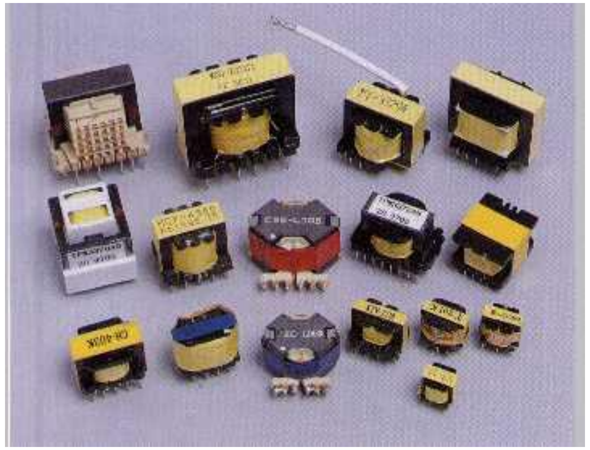
Energy Storage in Magnets

- Solonoid Video

WE NEED TO STUDY HOW TO CALCULATE THIS ENERGY? WHAT AFFECTS IT?

Power Transformers

Motors and generators



Coils, Inductances, low power Transformers

relays

Circuit breakers

Flux Linkage, Flux Leakage, Self Inductance

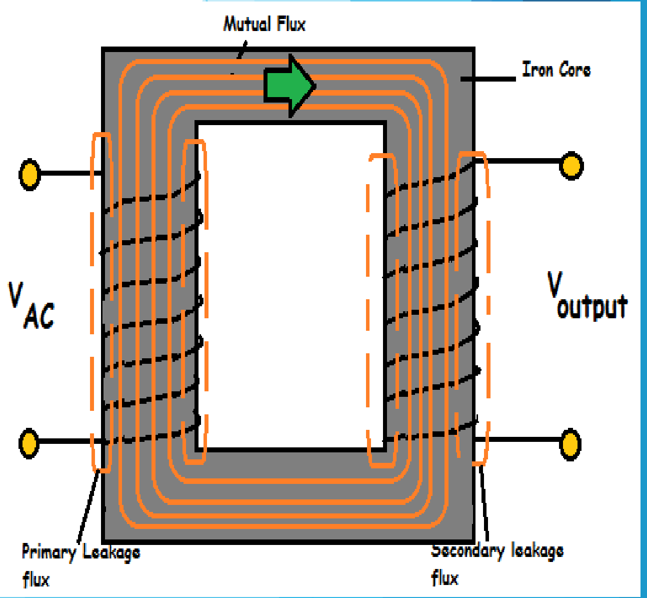
- The total flux produced due to the presence of an alternating source of emf exciting a coil wrapped around a core is called : **Total Flux φ_T**

- A new term is being introduced here which is called “FLUX LINKAGE”, **λ** **and can be calculated as**

$$\lambda_T = \varphi_T N$$

Flux linkage: flux lines that make a complete loop either cutting the other coil or remain circulating in the core

- If you look at the diagram you will see that NOT all of φ_T (which travels in circular paths and produced by the coil on the left) will reach the right coil and is divided into two types:
  - ***Leakage flux*** **: those flux lines which will “leak” to the air ….. φ_L**
  - ***Mutual flux*** **travelling and cutting another coil or will remain circulating in the core……. φ_M (WE CAN CALL IT LINKAGE FLUX AS IT LINKS TWO COILS TOGETHER)**



$$\varphi_T = \varphi_M + \varphi_L$$

$$\lambda_M = \varphi_M N \qquad \lambda_L = \varphi_L N$$

Flux Linkage, Flux Leakage, Self Inductance

$\lambda_M = \varphi_M N$……… lets take it for short $\lambda = \varphi N$

According to Faraday's law, if we have flux lines cutting a wire, an induced emf will generate proportional to the rate of change of flux with respect to time.
SO THE **MUTUAL FLUX** (OR LINKAGE FLUX) is the component of the total flux which will produce this emf!!!

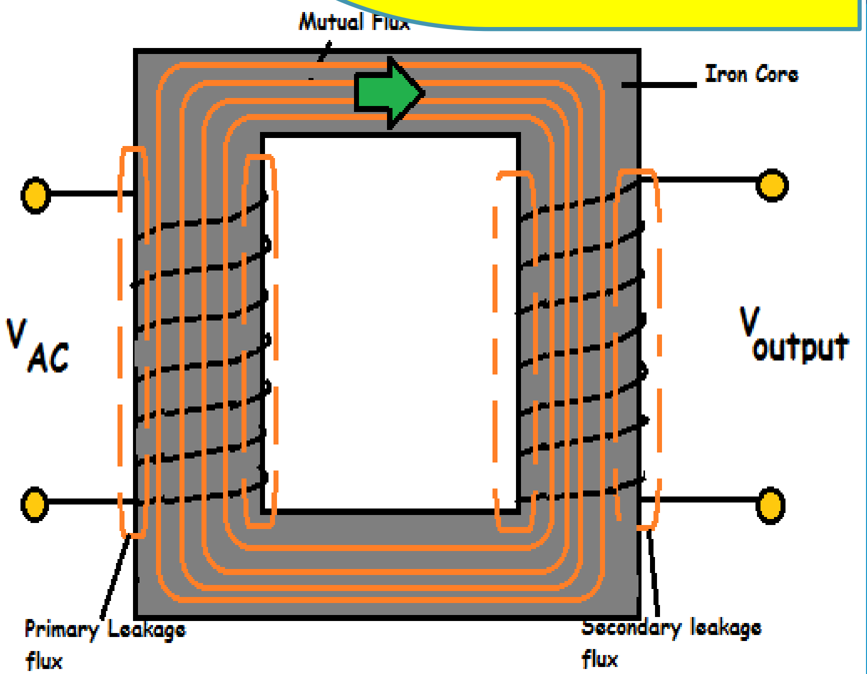
So generally speaking, emf is the rate of change of flux *LINKAGE* with respect to time

Proof:

$$\text{emf} = -\frac{d\lambda}{dt} = -\frac{dN\varphi}{dt} = -N\frac{d\varphi}{dt} - \varphi\frac{dN}{dt}$$

Of course since $\varphi\frac{dN}{dt} = 0$, the equation is summarized as:

$$e = \frac{d\lambda}{dt} = \frac{d(N\Phi)}{dt} = N\frac{d\Phi}{dt}$$



Flux Linkage, Flux Leakage, Self Inductance

Then what happens to the leakage flux??

Is this what only happens when flux cuts a coil surrounding a core?

There is inductance in the title, where did this come from?

Flux Linkage, Flux Leakage, Self Inductance

BACK TO THE BASICS AGAIN....

When an alternating current travels through a loop of coil, it produces a changing magnetic field. By Faraday's law, a changing magnetic flux will induce an emf within a coil of wire. By Lenz's law, this induced emf will oppose the change of flux:

1. if the current in the loop is increasing, then the induced emf will create and induce current that points in the opposite direction.
2. if the current in the loop decreasing, then the induced emf will create and induce current that points in the same direction.

A changing current will create a changing flux.

$$I \,\alpha\, \varphi$$

Faraday's law states that the changing flux will create an emf. Direction from Lenz's law

$$e = \frac{d\lambda}{dt}$$

The emf acts to oppose the ***change*** in flux.

$$e = -N\frac{d\Phi}{dt}$$

$$\Phi\Re = NI$$

$$\Phi = \frac{NI}{\Re}$$

$$\therefore e = -N\frac{d\Phi}{dt} = -N\frac{d\frac{NI}{\Re}}{dt} = -\frac{N^2}{\Re}\frac{dI}{dt}$$

$$\therefore e = L\frac{dI}{dt}, \therefore L = \frac{N^2}{\Re}$$

Flux Linkage, Flux Leakage, Self Inductance

$$E = -L\frac{dI}{dt} \Rightarrow L = \frac{N^2}{\Re} \Rightarrow L = \frac{\mu N^2 A}{l} \Rightarrow L = \frac{N\varphi}{I} \Rightarrow LI = N\varphi$$

$$\Re = \frac{l}{\mu A}$$

$$B = \mu H = \frac{\mu NI}{l}, B = \frac{\varphi}{A}$$

$$\therefore \varphi = BA = \frac{\mu NIA}{l}$$

$$I = \frac{N}{L}\varphi$$

$I \; \alpha \; \varphi$

Inductance

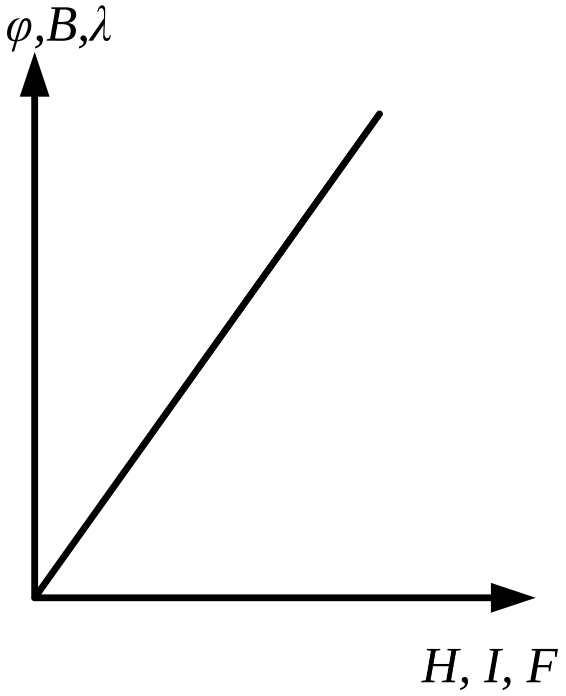
- This means that each flux component will naturally cause an inductance to appear to “resist” the flow of current in the coil
- Since we have two flux components, then we have the following inductances:
  - Leakage inductance: due to the leakage flux
  - Mutual inductance: due to the mutual flux

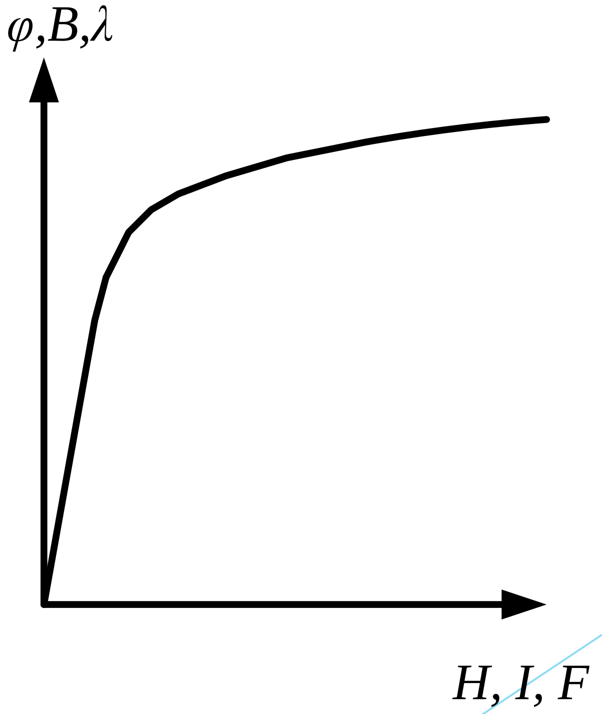
$$LI = N\varphi = \lambda$$

Any length of wire has inductance. Inductance is a measure of a coil's ability to store energy in the form of a magnetic field. It is defined as the rate of change of flux with current -

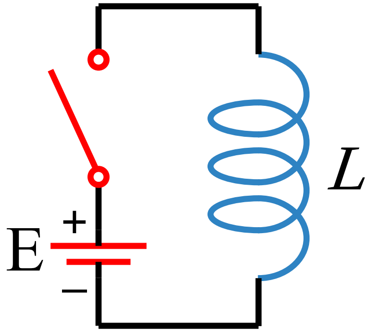
Energy Storage In Magnetic Systems:

Linear System	Non- Linear System





Energy Storage in Inductance (1): assuming linear system



e= voltage (v)

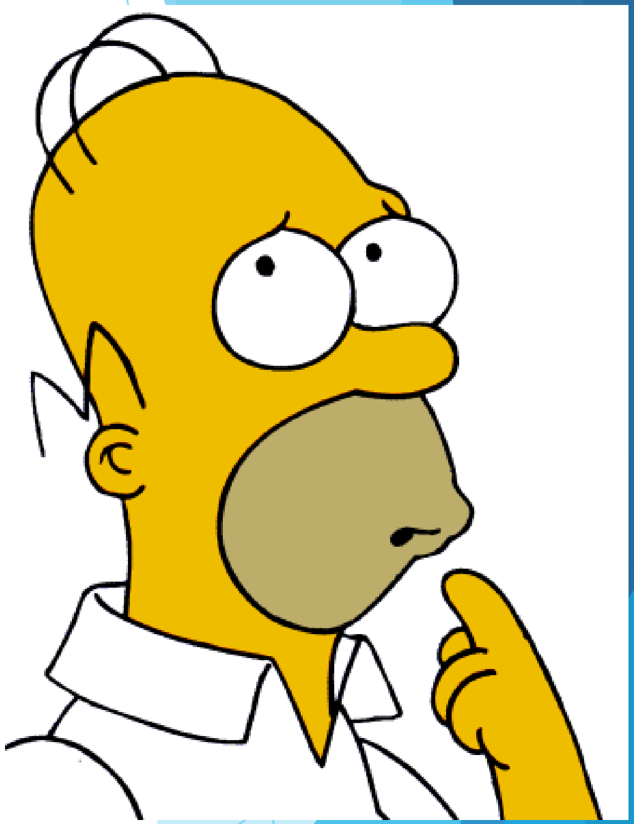
i= current (A)

P= source power (W)

W = work (energy) done (Joules)

- **Suppose that an inductor of inductance L is connected to a variable DC voltage supply.**
- **The supply is adjusted so as to increase the current i flowing through the inductor from zero to some final value of i.**
- **As the current through the inductor is ramped up, an emf is generated, which acts to oppose the increase in the current. Clearly, work must be done against this emf by the voltage source in order to establish the current in the inductor.**

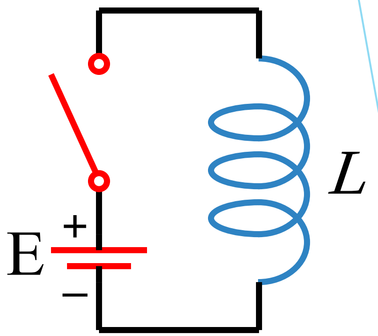
We need to calculate that work done by the voltage source during a time interval (dt)



Will this mean anything?

Energy Storage in Inductance (2): assuming linear system

What is the amount of power supplied to the inductor to oppose the emf of the inductor?



e= voltage (v)

i= current (A)

P= source power (W)

W = work (energy) done (Joules)

$$P = EI$$

$$E = \frac{d\lambda}{dt} \Rightarrow \lambda = LI = N\varphi \Rightarrow E = \frac{d(LI)}{dt} = L\frac{dI}{dt} + I\frac{dL}{dt}$$

$$\because I\frac{dL}{dt} = 0$$

$$\Rightarrow E = L\frac{dI}{dt}$$

In this example inductance doesn't vary with position and thus doesn't vary with time

$$\therefore P = EI = L\frac{di}{dt}I = LI\frac{di}{dt}$$

the amount of power supplied to the inductor to oppose the emf of the inductor

Energy Storage in Inductance (3): assuming linear system

Now for our mystery question, What is the work done to increase the current I from 0 to a value I in in time dt?

$$P = EI = LI\frac{di}{dt}$$

Energy Storage in “the loops of wire” in terms of (L,I)

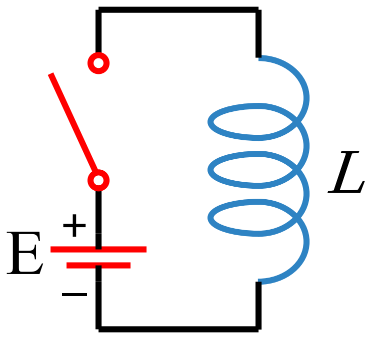
The change in the work done on the inductor:

$$dW = Pdt = EI\,dt$$

$$dW = \underbrace{LI\frac{dI}{dt}}_{P}dt = L.IdI$$

$$W = \int_0^I dW = \int_0^I I\,L\,dI = \frac{1}{2}LI^2$$

Energy Storage in Inductance (4): assuming linear system



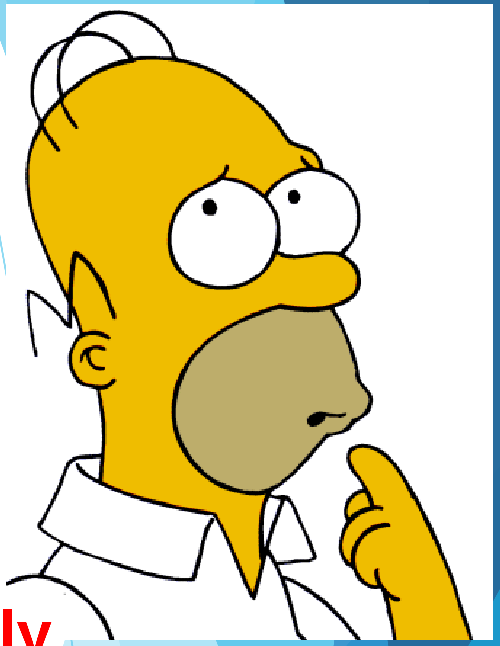
e= voltage (v)

i= current (A)

P= source power (W)

W = work (energy) done (Joules)

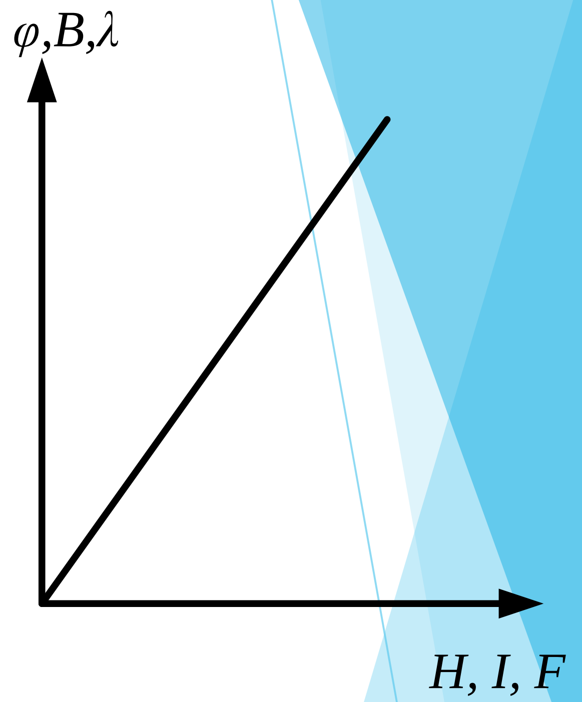
So let me get this straight



This energy which we just calculated in previous slide is actually the stored in the magnetic field generated by the current flowing through the inductor. In a pure inductor, the energy is stored without loss, and is returned to the rest of the circuit when the current through the inductor is ramped down, and its associated magnetic field collapses.

Energy Storage in Inductance (5): assuming linear system

$$W = \int_0^I I L dI = \frac{1}{2} L I^2$$



Another way of calculating the work (energy) is by getting the area under the (λ, I) curve.

$$W = area under\ curve = \frac{1}{2} base * height = \frac{1}{2} I\lambda$$

$$\because \lambda = LI$$

$$\therefore W = \frac{1}{2} I * LI = \frac{1}{2} LI^2$$

Energy Storage in Inductance (6): assuming linear system

Energy Storage in Inductance (λ, I)

$$W = \int_0^I I\,L\,dI = \frac{1}{2}LI^2 = \frac{1}{2}LI\,I = \frac{1}{2}\lambda I$$

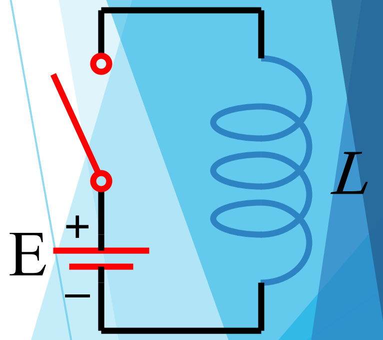
$\lambda = LI$

e= voltage (v)

i= current (A)

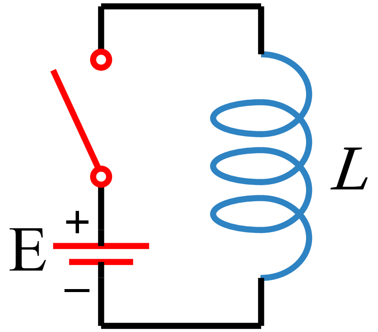
P= source power (W)

W = work (energy) done (Joules)



Energy Storage in Inductance (7): assuming linear system

Energy Storage in Inductance (ϕ, R)



e= voltage (v)

i= current (A)

P= source power (W)

W = work (energy) done (Joules)

$$W = \int_0^I I L dI = \frac{1}{2} L I^2 = \frac{1}{2}\frac{N^2}{\Re} I^2 = \frac{1}{2}\frac{NI}{\Re} NI = \frac{1}{2}\varphi^2 \Re$$

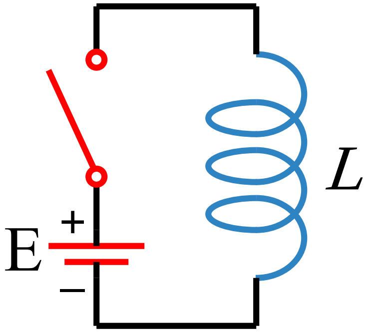
$$\frac{N^2}{\Re}$$

$$\varphi \Re = NI$$

$$\therefore \varphi = \frac{NI}{\Re}$$

Energy Storage in Inductance (8): assuming linear system

Energy Storage in Inductance (B, H)



e= voltage (v)

i= current (A)

P= source power (W)

W = work (energy) done (Joules)

$$W = \int_0^I I\,L\,dI = \frac{1}{2}\varphi^2\Re = \frac{1}{2}\frac{\Re}{A}\varphi\varphi A = \frac{1}{2}BHlA = \frac{1}{2}BH\,vol$$

$$*\frac{A}{A}$$

$$B = \frac{\varphi}{A}$$

$$\varphi\Re = Hl$$

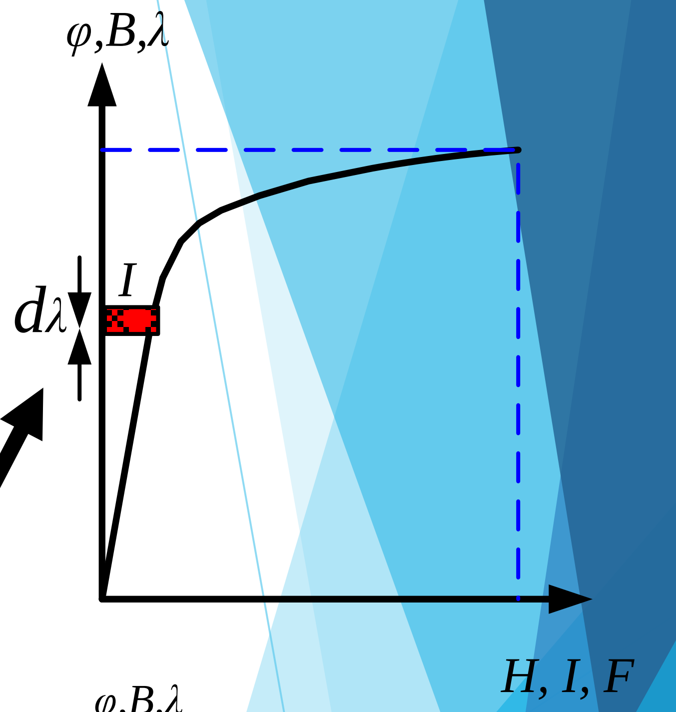
$$vol = l * A$$

Energy Storage in mag. elements (1): **assuming NON- LINEAR system**

- **In this case we need to look at the whole graph and in this case to get the area under the curve we must use *integration*.**

$$dW = EI\,dt \qquad E = \frac{d\lambda}{dt}$$

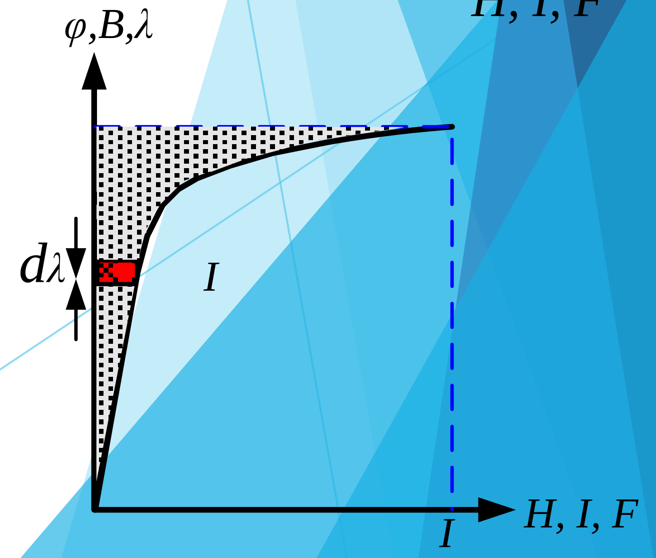
$$dW = \frac{d\lambda}{dt}.I\,dt = I\,\frac{d\lambda}{dt}.dt = I d\lambda$$



$$W = \int_{0}^{\lambda} I d\lambda$$

Assume a horizontal strip of width $d\lambda$ and length of i and this strip moves from 0 to λ

Represents the area above the curve it represents the energy stored and is calculated from the basic principles. IT IS THE SHADED PART OF THE GRAPH



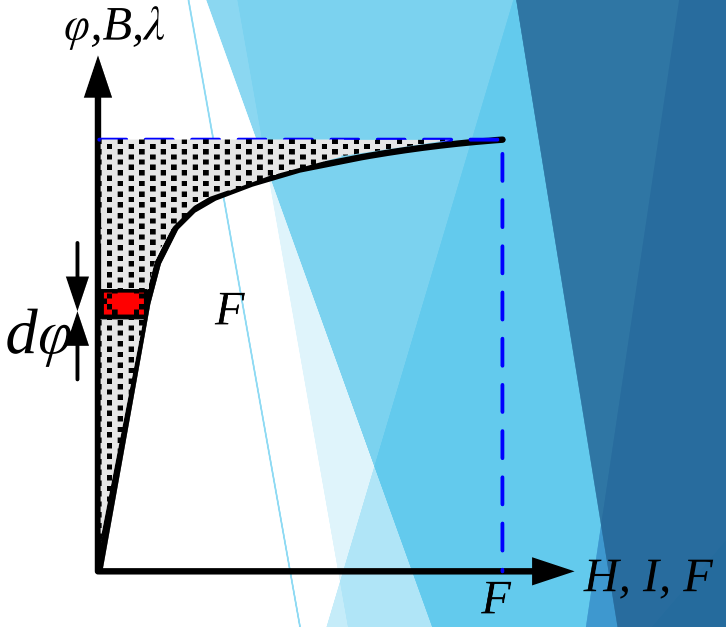
Energy Storage in mag. elements (1): assuming NON- LINEAR system

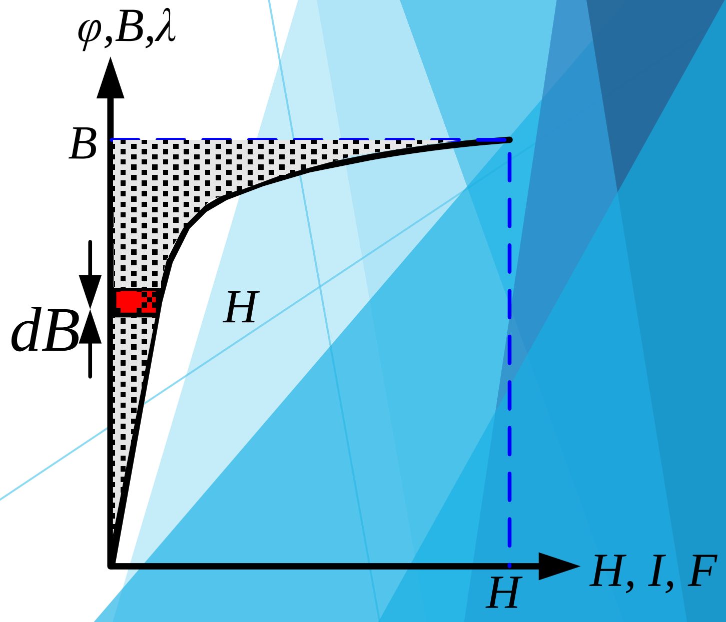
$$W = \int_0^{\lambda} I d\lambda = \int_0^{\varphi} N I d\varphi = \int_0^{\varphi} F d\varphi$$

$$W = \int_0^{\varphi} F d\varphi = \int_0^{\varphi} H l d\varphi = \int_0^{\varphi} \frac{H l d\varphi A}{A}$$

$$= \int_0^{B} H dB.(lA) = vol \int_0^{B} H dB$$

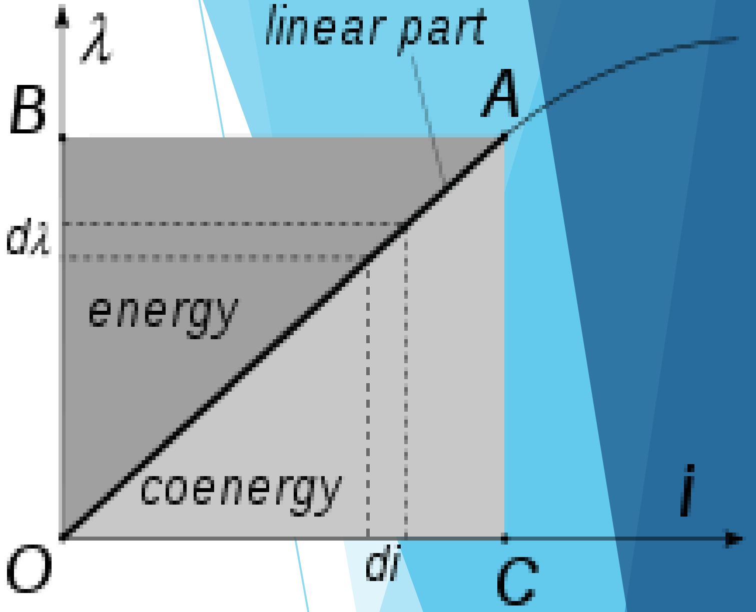
$$\frac{d\varphi}{A} = dB$$



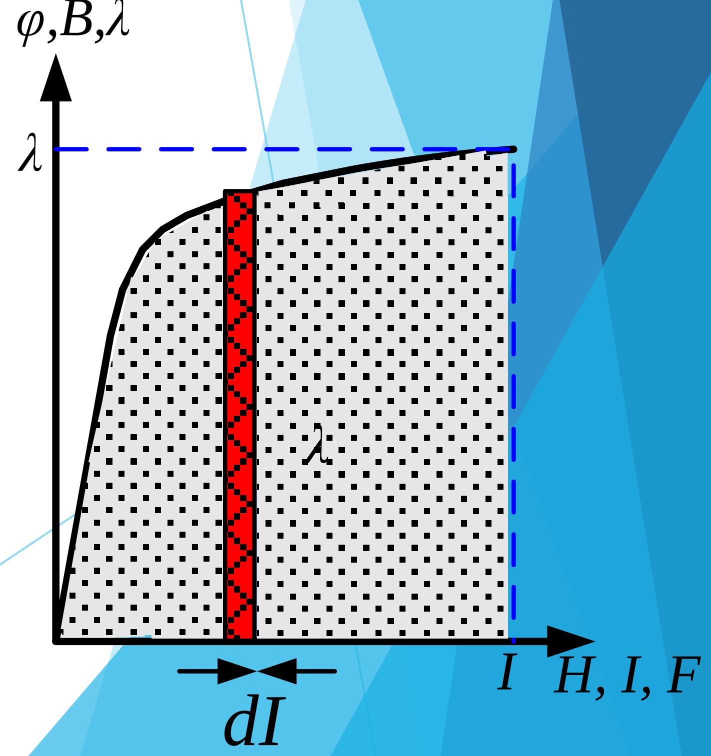


Co- Energy???

- In case of ***linear*** systems, the area under the y-axis (energy) is equal to the area under the x-axis so both energy and co-energy are numerically equal
- The area close to the x-axis is called "Co-energy". Physically, co-energy doesn't exist but instead it is only used for mathematical analysis purposes. Co-energy calculation is used with stepper motor and permanent magnet analysis.
- Have a "closer" look at the graph, CO-ENERGY better defines the system as it is more likely that we have systems equations in term of change in current rather than flux linkage. λdi is more reasonable than $id\lambda$
- Another important issue is in terms of calculating force, we might need an inversion mathematical process if we used the energy part, which might require additional mathematical manipulation and is avoided if we used co-energy.



$$W' = \int_0^I \lambda dI$$



Questions

- State applications of magnetic elements
- State and explain what is meant by flux linkage, flux leakage and total flux
- Derive expressions of energy stored in magnetic cores in terms of (flux linkage, current), (flux, reluctance), (flux density, field intensity) in both linear and non-linear regions
- Explain what is mean by energy and co-energy in magnetic circuits and express how to calculate the work done in each