Electrical Machines I

Week 5-6: Singly Excited Systems

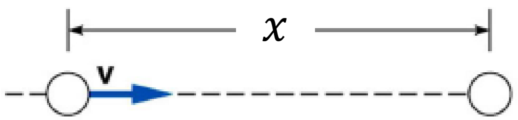
RECALL… REMEMBER…. !!

- Energy stored in linear and non linear systems
- Relationship between force and stored energy in magnetic systems
- Energy and Co-energy

Singly Excited Magnetic Field System: Force and torque

Linear system

involves an object moving from one point to another in a straight line



Force: $\mathcal{F}$

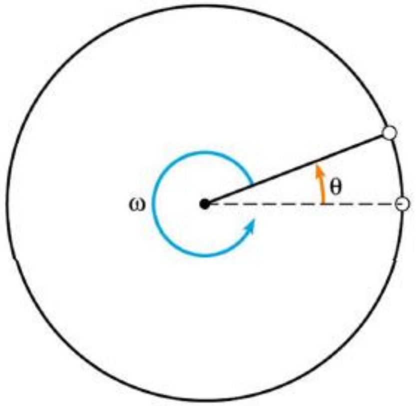
Distance (displacement): x

Velocity: $x' = v$

Mass: m

Rotational system

involves an object rotating about an axis



Torque: T

Distance (displacement): θ

Angular velocity: $\theta' = \omega$

Moment of inertia: J

Singly Excited Magnetic Field System: Force and torque

Linear system

Power: $\mathcal{F} \times v$

Newton's law of motion:

$$\sum \mathcal{F} = mass \times x''$$

$$\mathcal{F}_{aiding} - \mathcal{F}_{opposing} = m \times v'$$

Opposing force due to spring and friction for example

Rotational system

Power: $T \times \omega$

Newton's law of motion:

$$\sum T = moment\ of\ inertia \times \theta''$$

$$T_{aiding} - T_{opposing} = J \times \omega'$$

Opposing force due to load torque and friction for example

Singly Excited Magnetic Field System:

Previously we have calculated the force acting on a "plunger" as a function of the system variables as follows:

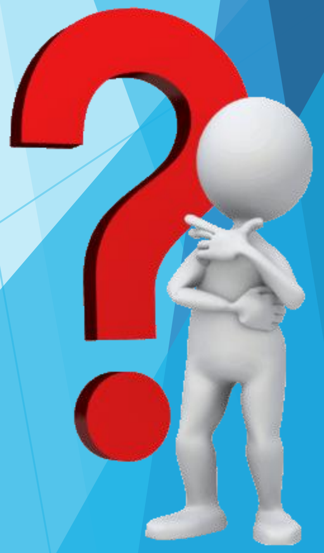
$$F_e = \frac{1}{2} i^2 \frac{dL}{dx}$$

$$F_e = \frac{\partial W_m}{\partial x}$$

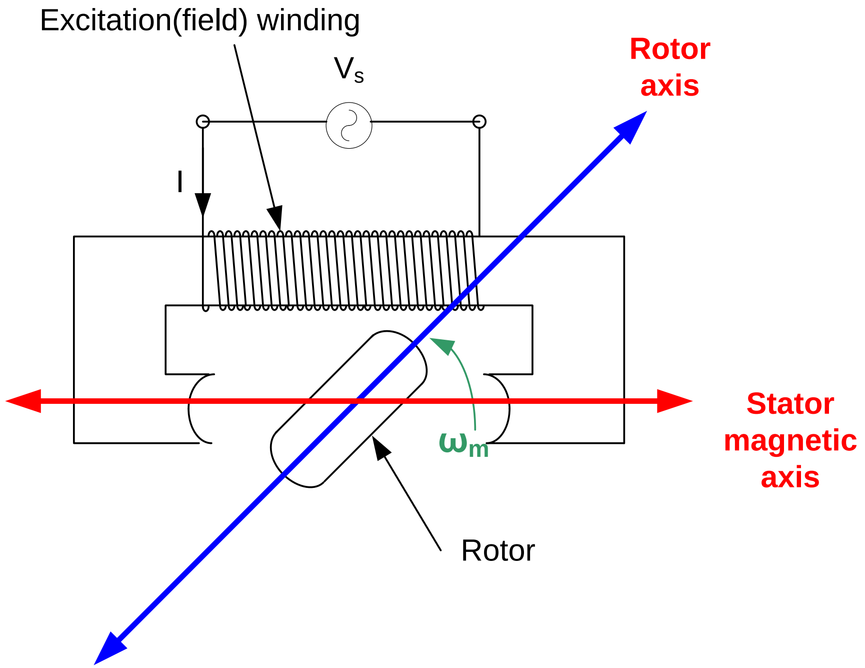
It is our objective today is to calculate the force or torque acting on a "single excited reluctance machine" in terms of system variables

A reluctance motor is an ac synchronous motor whose reluctance changes as a function of angular displacement θ. Owing to its constant speed operation, it is commonly used in electric clocks, record players, and other precise timing devices

The word synchronous actually mean "existing or occurring at the same time" and in electric machines mean that the motor rotates with a speed that is related to the supply frequency



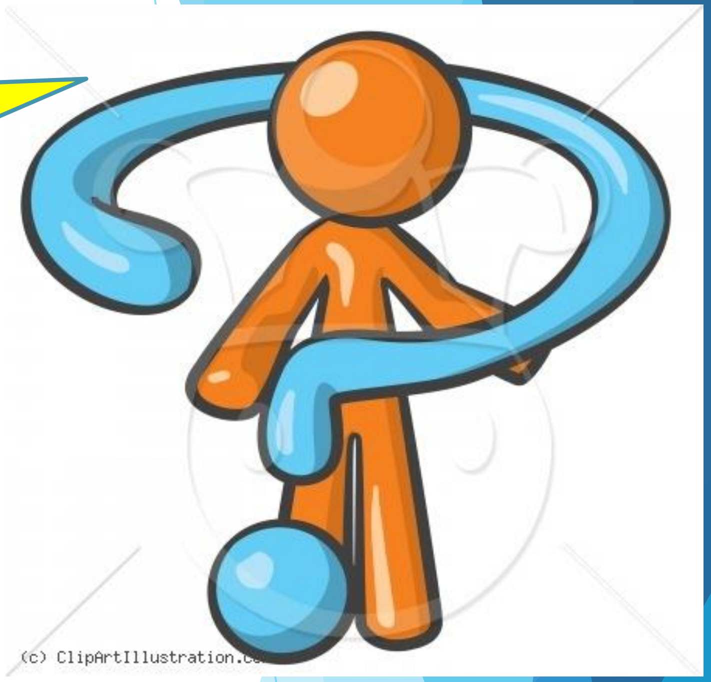
Singly Excited Magnetic Field System: Reluctance motor

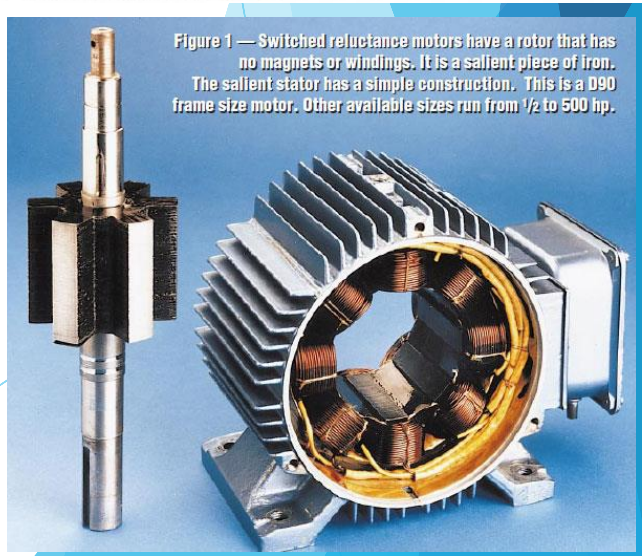


Single phase, 2 pole reluctance motor

ω_m: Mechanical speed and is NOT EQUAL to $\omega_s = 2\pi f$

What is the required torque to be able to make this motor rotate in terms of system variables?





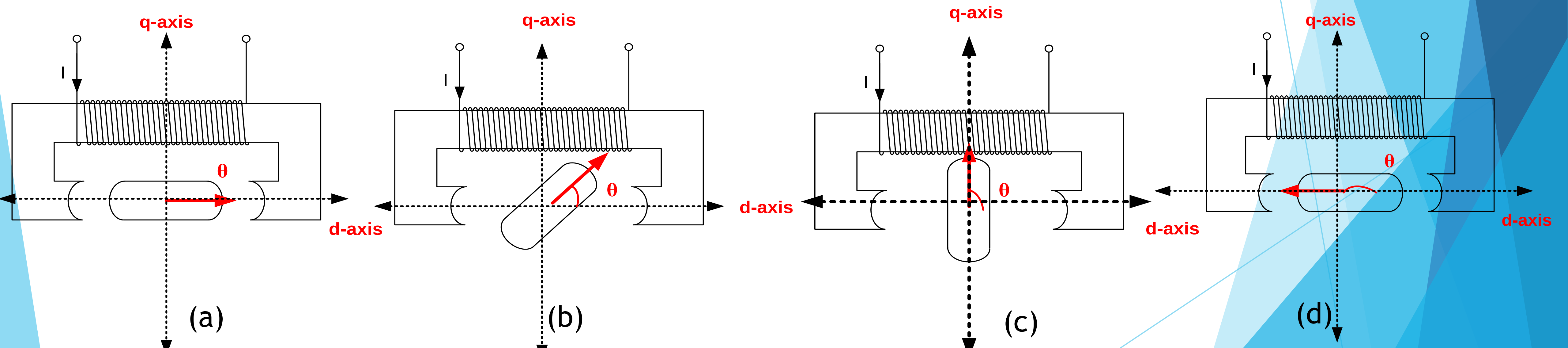
Singly Excited Magnetic Field System: Reluctance motor

$$\because F_e = \frac{\partial W_m}{\partial x}$$

$$\therefore T_e = \frac{\partial W_m}{\partial \theta}$$

$$\because W_m = \frac{1}{2} L i^2$$

We will study the rotor movement from 0 to 360˚ and started to plot the inductance and reluctance variation with time



(a) $\varphi_{max}, L_{max}, \Re_{min}$

(b) $\varphi \Downarrow, L \Downarrow, \Re \Uparrow$

(c) $\varphi_{min}, L_{min}, \Re_{max}$

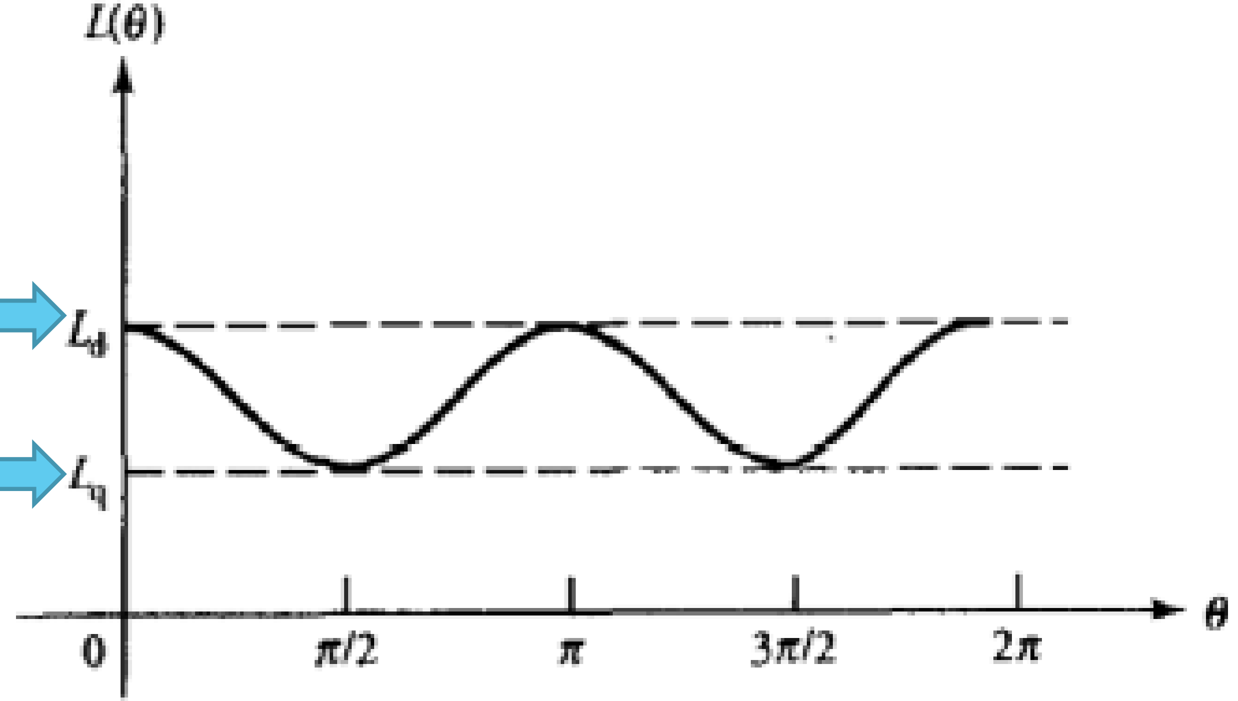
(d) $\varphi_{max}, L_{max}, \Re_{min}$

Singly Excited Magnetic Field System: Reluctance motor

When the magnetic axes of the rotor and the stator are at right angles to each other (the quadrature or q-axis position), the reluctance is maximum, leading to a minimum inductance. As the rotor rotates with a uniform mechanical speed ω_m the inductance goes through maxima and minima

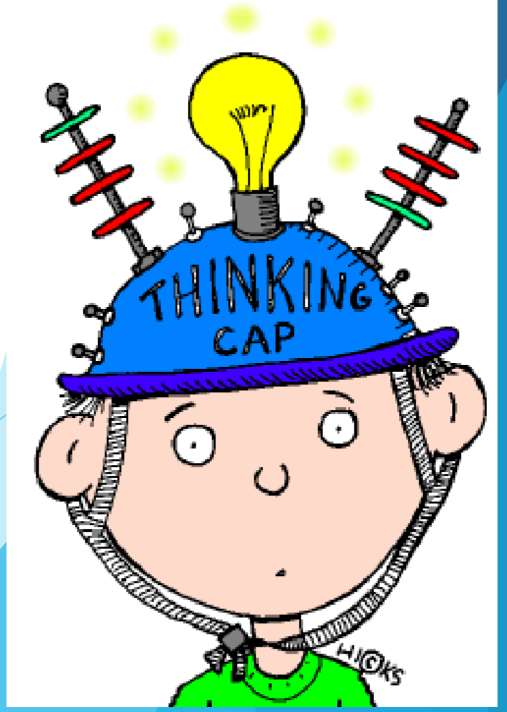
Maximum inductance = Minimum reluctance

Minimum inductance = Maximum reluctance



One electrical cycle of $\omega_s \equiv$ half one mechanical cycle of ω_m

$\omega_s \equiv \frac{\omega_m}{2}$ since this is a two pole machine



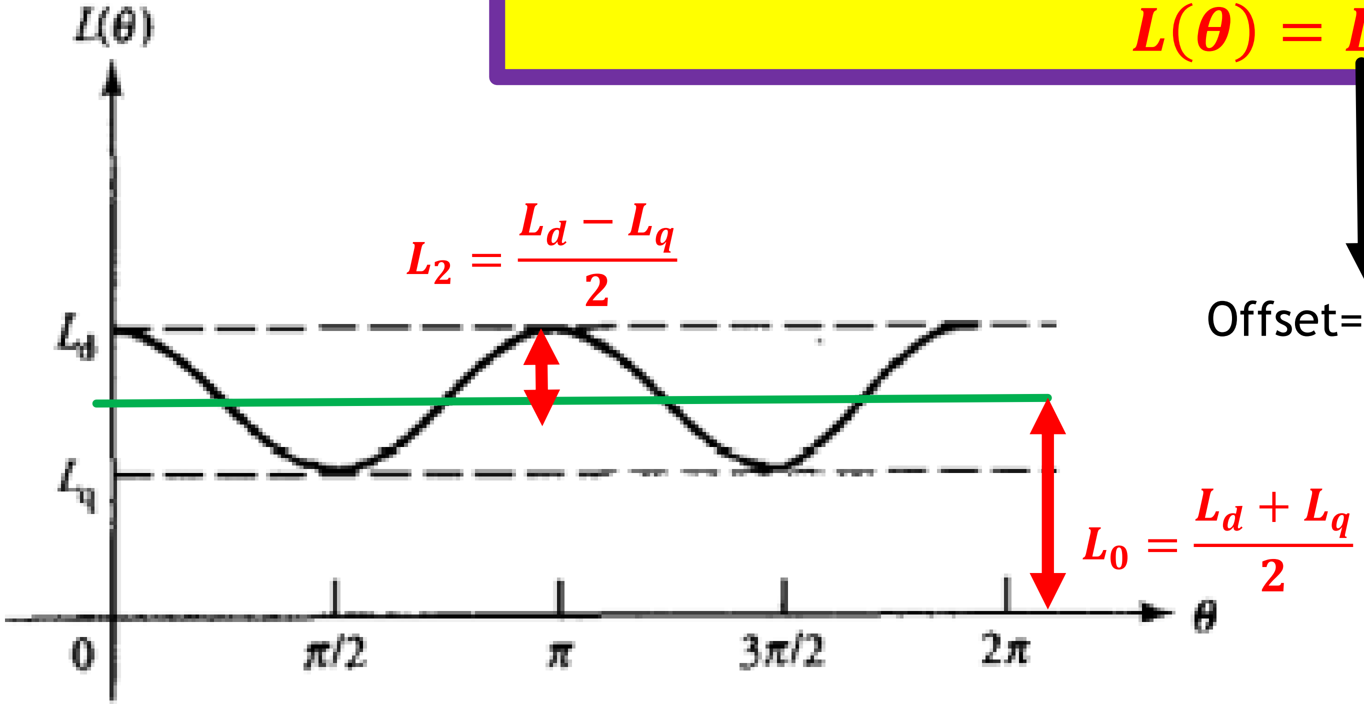
Singly Excited Magnetic Field System: Reluctance motor

If we try to write the general equation of this inductance as a function of theta it will be in the form of:

$$L(\theta) = L_0 + L_2 cos(2\theta)$$

L_0: Offset=shift

L_2: Peak inductance value



2θ is because 2 complete rotations are made in the 360˚

$$L(\theta) = 0.5(L_d + L_q) + 0.5(L_d - L_q)cos(2\theta)$$

Singly Excited Magnetic Field System: Reluctance motor

$$L(\theta) = 0.5(L_d + L_q) + 0.5(L_d - L_q)cos(2\theta)$$

$$\because T_e = \frac{\partial W_m}{\partial \theta}$$

$$\because W_m = \frac{1}{2} L(\theta) i^2$$

$$\therefore T_e = \frac{1}{2} i^2 \frac{\partial L}{\partial \theta}$$

$$\therefore T_e = -\frac{1}{2} i^2 \left[\frac{1}{2} (L_d - L_q) * 2(\sin 2\theta) \right]$$

This is a "mechanical angle"

Replace the variable θ

$$\theta = \omega_m t + \delta$$

Where δ is the initial position of the rotor's magnetic axis with respect to the stator's magnetic axis.

Singly Excited Magnetic Field System: Reluctance motor

$$\therefore T_e = -\frac{1}{2} i^2 \left(L_d - L_q\right) \sin 2\left(\omega_m t + \delta\right)$$

Since the current is sinusoidal and can be expressed as:

$$i = I_{max} \cos \omega_s t$$

If you assume it as a "sin" function its totally correct as well. It is preferred that we assume that the supply voltage is "sinusoidal" and since the coil is inductive, current waveform will lag the sinusoidal voltage by 90˚. This means that the current will be a "cosine" function

$$\therefore T_e = -\frac{1}{2}\left(L_d - L_q\right)\left[I_{max}{}^2 \cos^2\left(\omega_s t\right)\right] \sin 2\left(\omega_m t + \delta\right)$$

$$\because \cos^2 a = \frac{1}{2}\left(1 + \cos 2a\right)$$

$$\therefore T_e = -\frac{1}{4} I_{max}{}^2 \left(L_d - L_q\right)\left[\sin 2\left(\omega_m t + \delta\right) + \sin 2\left(\omega_m t + \delta\right) \cos 2\left(\omega_s t\right)\right]$$

$$\because 2 \sin a \cos b = \sin\left(a + b\right) + \sin\left(a - b\right)$$

Singly Excited Magnetic Field System: Reluctance motor

$$\therefore T_e = -\frac{1}{4} I_{\max}{}^2 \left(L_d - L_q\right) \begin{bmatrix} \sin 2\left(\omega_m t + \delta\right) + 0.5\sin\left(2\left(\omega_s t - \omega_m t\right) - 2\delta\right) \\ + 0.5\sin\left(2\left(\omega_s t + \omega_m t\right) + 2\delta\right) \end{bmatrix}$$

This “big” equation implies that for this reluctance machine to be able to rotate the average value of the torque terms of this equation **MUST NOT** equal to zero

1. First term: average is zero as it is periodical so no torque is produced by this term
2. Second term: if we are able to make $\omega_m = \omega_s$, this term will be re-written as

$$0.5\sin\left(-2\delta\right)$$

This condition will produce an average torque

Singly Excited Magnetic Field System: Reluctance motor

$$\therefore T_e = \frac{1}{4} I_{\max}{}^2 \left(L_d - L_q\right) \begin{bmatrix} \sin 2\left(\omega_m t + \delta\right) + 0.5\sin\left(2\left(\omega_s t + \omega_m t\right) - 2\delta\right) \\ + 0.5\sin\left(2\left(\omega_s t + \omega_m t\right) + 2\delta\right) \end{bmatrix}$$

3. Third term: if we are able to make $\omega_m = -\omega_s$, this term is discard (same as the second term but with a negative sign which does not imply)

Thus the condition which will successfully drive this motor will be represented as

$$\therefore T_e = -\frac{1}{8} I_{\max}{}^2 \left(L_d - L_q\right)\left[\sin\left(-2\delta\right)\right]$$

Given that the machine $\omega_m = \omega_s$

Maximum torque

Singly Excited Magnetic Field System: Reluctance motor

$$\therefore T_e = -\frac{1}{8} I_{\max}^2 \left(L_d - L_q\right)\left[\sin(-2\delta)\right]$$

Given that the machine $\omega_m = \omega_s$

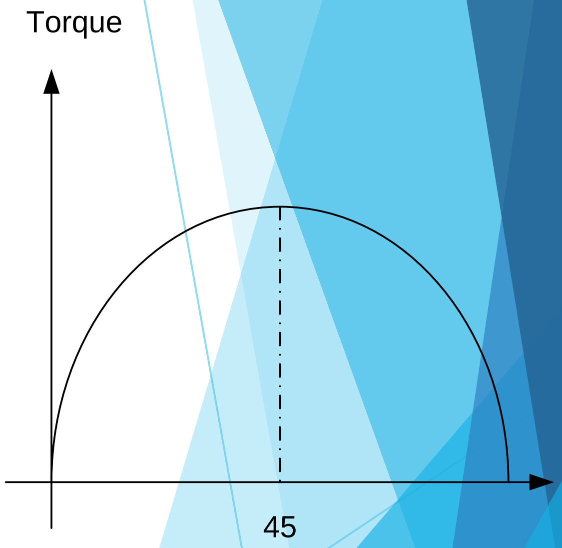
The torque sign depends on the value of δ,

- **if δ is (+ve), T will be (-ve) in this case, the torque is opposite to the direction of rotation. i.e. the machine acts as generator.**
- **if δ is (-ve), T will be (+ve) in this case, the torque is same direction of rotation. i.e. the machine acts as motor.**

Average torque will be equal to zero if

$$L_d = L_q$$

This condition happens if the rotor is cylindrical (not salient)



Maximum torque occurs when $sin(2\delta) = 1, \delta = 45^{\circ}$