Electrical Machines II

Week 7: Induction Motor efficiency, power flow torque speed characteristics

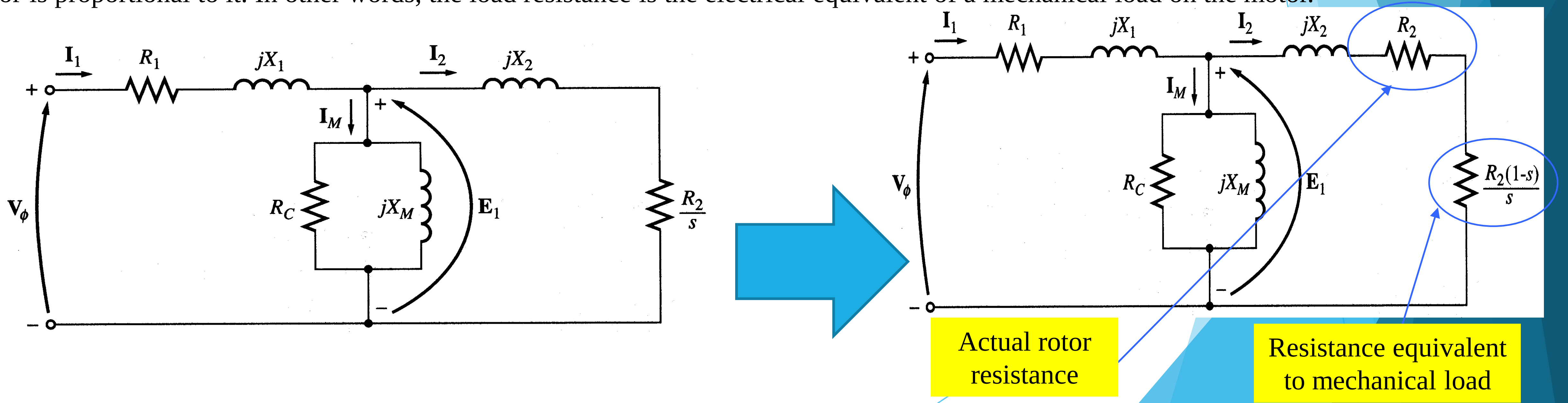
Equivalent Circuit and Losses: Power Relations

- Since the load resistance varies with the slip and the slip adjusts itself to the mechanical load on the motor, the power delivered to the load resistance is equivalent to the power developed by the motor. Thus, the performance of the motor at any slip can be determined from its equivalent circuit as modified below

$$\frac{R_2}{s} = R_2 + R_2\left(\frac{1-s}{s}\right)$$

$$\frac{R_r}{s} = R_r + R_r\left(\frac{1-s}{s}\right)$$

The above equation establishes the fact that the hypothetical resistance $R_2/_s$ can be divided into two components: **the actual resistance** of the rotor R_2, and an additional resistance $R_2\left(\frac{1-s}{s}\right)$. The additional resistance is called **the load resistance** or the dynamic resistance. The load resistance depends upon the speed of the motor and is said to represent the load on the motor because the mechanical power developed by the motor is proportional to it. In other words, the load resistance is the electrical equivalent of a mechanical load on the motor.



Equivalent Circuit and Losses

- Now as we managed to solve the induced voltage and different frequency problems, we can combine the stator and rotor circuits in one equivalent circuit

  Where

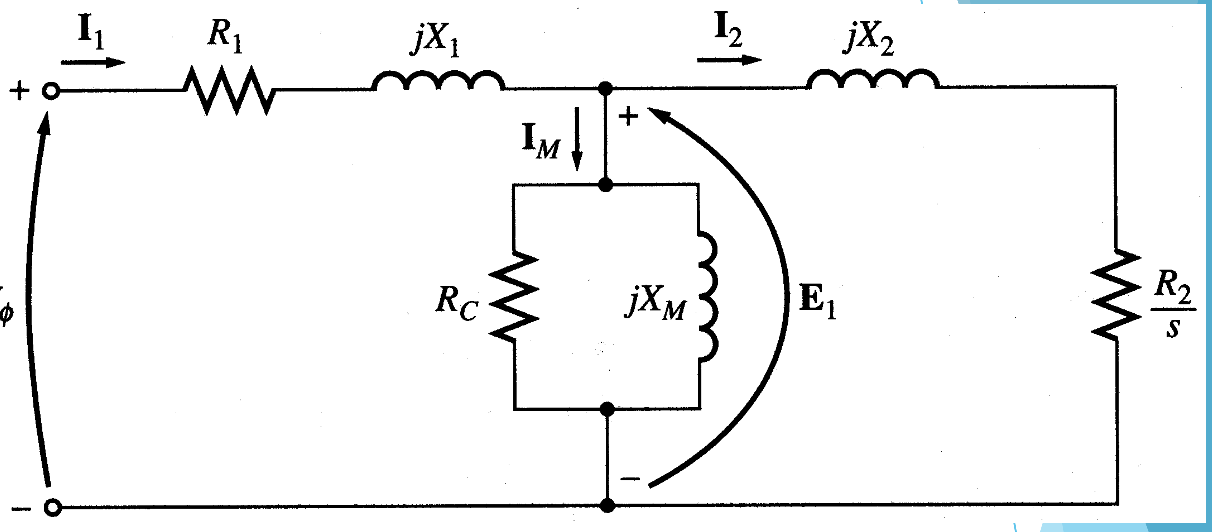
$$X_2 = a_{eff}^2 X_{R0}$$

$$R_2 = a_{eff}^2 R_R$$

$$I_2 = \frac{I_R}{a_{eff}}$$

$$E_1 = a_{eff} E_{R0}$$

$$a_{eff} = \frac{N_S}{N_R}$$



1. <u>Copper losses</u>
   - Copper loss in the stator (P_{SCL}) = $I_1^2R_1$
   - Copper loss in the rotor (P_{RCL}) = $I_2^2R_2$
2. <u>Core loss (P_{core})</u>
3. <u>Mechanical power loss due to friction and windage</u>

How this power flow in the motor?

Asynchronous (Induction) Motor: Power flow

Electrical Power

$$P_{in} = \sqrt{3} V_T I_L \cos\theta$$

Air gap Power P_{ag}

$$P_{gap} = 3 I_r'^2 \frac{R_r'}{s}$$

Developed Power P_{dev}

$$P_{dev} = 3 I_r'^2 R_r' \left(\frac{1-s}{s} \right)$$

Mechanical Power

$$P_{out} = T_L \omega_m$$

$$P_{gap} : P_{RCL} : P_{dev}$$

$$3 I_r'^2 \frac{R_r'}{s} : 3 I_r'^2 R_r' : 3 I_r'^2 R_r' \left(\frac{1-s}{s} \right)$$

$$\frac{1}{s} : 1 : \left(\frac{1-s}{s} \right)$$

$$1 : s : (1-s)$$

$$P_{dev} = (1-s) P_{gap}$$

$$P_{RCL} = s P_{gap}$$

Stator Copper Loss (SCL)

$$P_{SCL} = 3 I_s^2 R_s$$

Core losses (eddy current + hystresis)

Rotor Copper Loss (RCL)

$$P_{RCL} = 3 I_r'^2 R_r'$$

Rotational losses P_{rot} (Friction and windage and stray losses)

Equivalent Circuit

We can rearrange the equivalent circuit as follows

$$P_{in} = \sqrt{3}\, V_L I_L \cos\theta = 3 V_{ph} I_{ph} \cos\theta$$

$$P_{SCL} = 3 I_s^{'2} R_s$$

$$P_{gap} = P_{in} - (P_{SCL} + P_{core}) \quad = P_{dev} + P_{RCL} = \frac{P_{RCL}}{s}$$

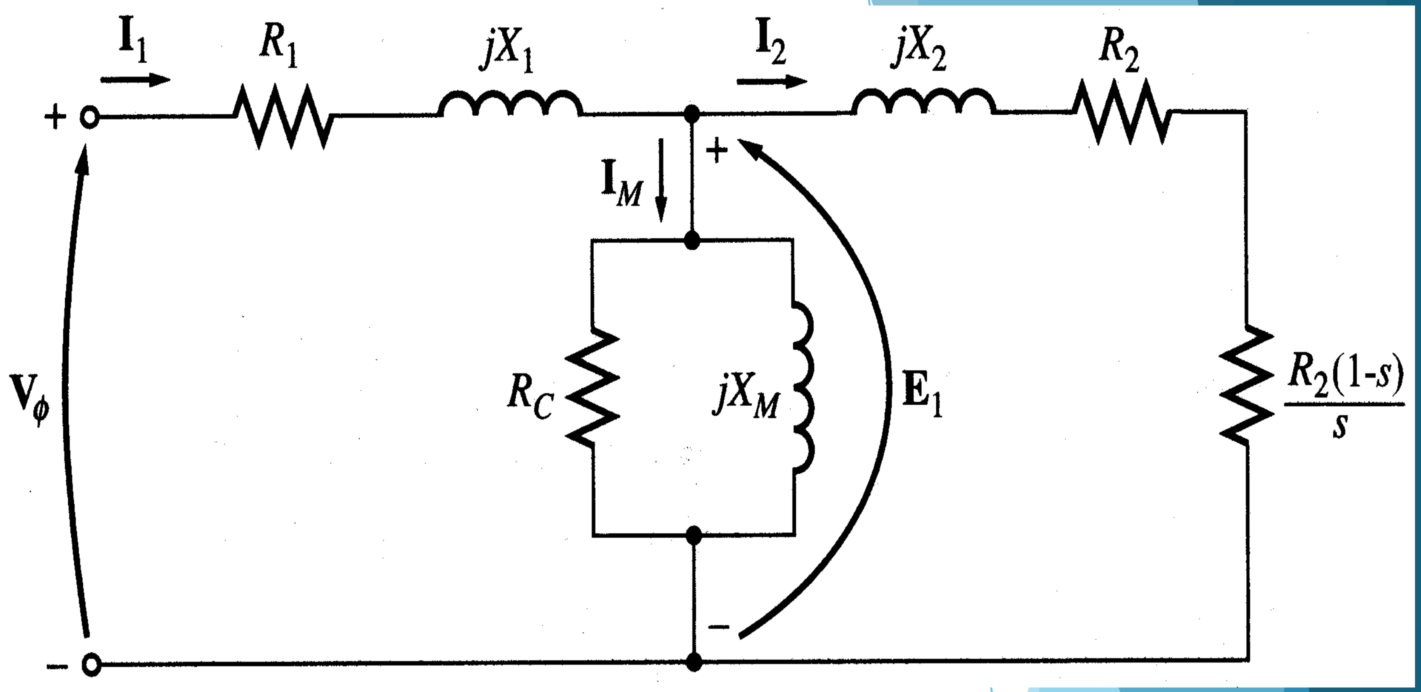
$$= 3 I_r^{'2} \frac{R_r^{'}}{s}$$

$$P_{RCL} = 3 I_r^{'2} R_r^{'}$$

$$P_{dev} = P_{gap} - P_{RCL} = 3 I_r^{'2} R_r^{'} \left(\frac{1-s}{s}\right) = \frac{P_{RCL}(1-s)}{s}$$

$$P_{dev} = (1-s) P_{gap}$$

$$P_{out} = P_{dev} - (P_{mechanical} + P_{stray})$$



per-phase exact equivalent circuit of a balanced three-phase induction motor

$$T = \frac{P_{dev}}{\omega_m} = \frac{(1-s) P_{gap}}{(1-s)\omega_s} = \frac{P_{gap}}{\omega_s}$$

Torque – Speed Characteristics

$$T_L = \frac{P_{out}}{\omega_m} \qquad T_{dev} = \frac{P_{dev}}{\omega_m} = \frac{P_{gap}}{\omega_s} = \frac{3I_r^{'2} R_r^{'}}{s\omega_s}$$

$R_r \Uparrow, I_r \Downarrow$ *at the same rotor emf*, $T_{dev} \Downarrow$

Substituting for *I*

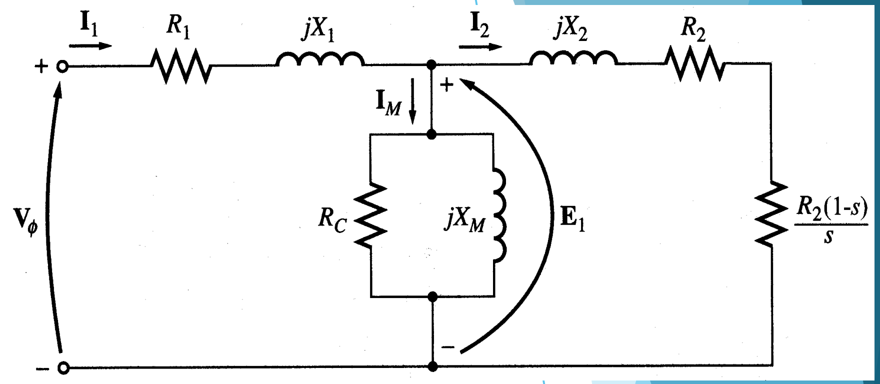
$$T_e = \frac{3R_r^{'}}{s\omega_s} \frac{V_s^{2}}{\left[\left(R_s + \frac{R_r^{'}}{s}\right)^2 + \left(X_{ls} + X_{lr}\right)^2\right]}$$

- The equation reveals that the torque developed by an induction motor is directly proportional to the **square of the current** in the rotor circuit and **the equivalent hypothetical resistance** of the rotor.
- However, the two quantities, the rotor current and the hypothetical rotor resistance, **are inversely related** to each other.
- For instance, if the rotor resistance is increased, we expect the torque developed by the motor to increase linearly. But any increase in the rotor resistance is accompanied by a decrease in the rotor current for the same induced emf in the rotor.
- A decrease in the rotor current causes a reduction in the torque developed. Whether the overall torque developed increases or decreases depends upon which parameter plays a dominant role.

Torque – Speed Characteristics

At standstill, the rotor slip is unity (s=1) and the effective rotor resistance is R'_r,. The magnitude of the rotor current is:

$$\left|I_r^{'}\right| = \frac{E_1}{\sqrt{R_r^{'2} + X_r^{'2}}}$$



per-phase exact equivalent circuit of a balanced three-phase induction motor

Note that the rotor winding resistance R'_r, is usually very small compared with its leakage reactance X'_r,. That is, R'_r, $<< X'_r$

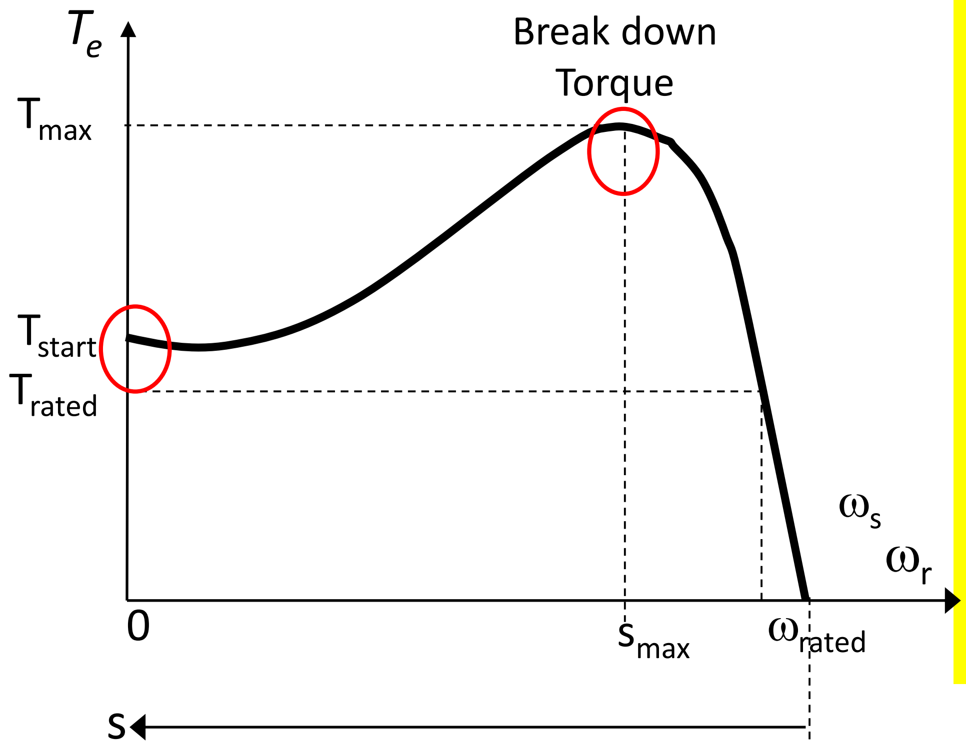
The starting torque developed by the motor is:

$$T_{start} = \frac{3 I_r^{'2} R_r^{'}}{\omega_s}$$

As the rotor starts rotating, an increase in its speed is accompanied by a decrease in its slip. As *s* decreases, increases. As long as R'_r/s is smaller than X'_r, the reduction in the rotor current is minimal. Thus, in this speed range, the rotor current may be approximated as

$$I_r^{'} \approx \frac{E_1}{X_r^{'}}$$

Torque – Speed Characteristics



- Since the rotor current is almost constant, the torque developed by the motor increases with the increase in the effective resistance R'_r/s.

Thus, the torque developed by the motor keeps increasing with the decrease in the slip as long as the rotor resistance has little influence on the rotor current.

- When the slip falls below a certain value called the breakdown slip s_b, the hypothetical resistance becomes the dominating factor. In this range, $R'_r/s >> X'_r$, and the rotor current can be approximated as

$$I'_r \approx \frac{sE_1}{R'_r}$$

- The **torque developed by the motor is now proportional to the slip s**. As the slip decreases, so does the torque developed.
- At no load, the slip is almost zero, the hypothetical rotor resistance is nearly infinite, **the rotor current** is approximately **zero**, and the **torque developed is virtually zero**. With this understanding, we are able to sketch the speed-torque curve of an induction motor. Such a curve is depicted in Figure