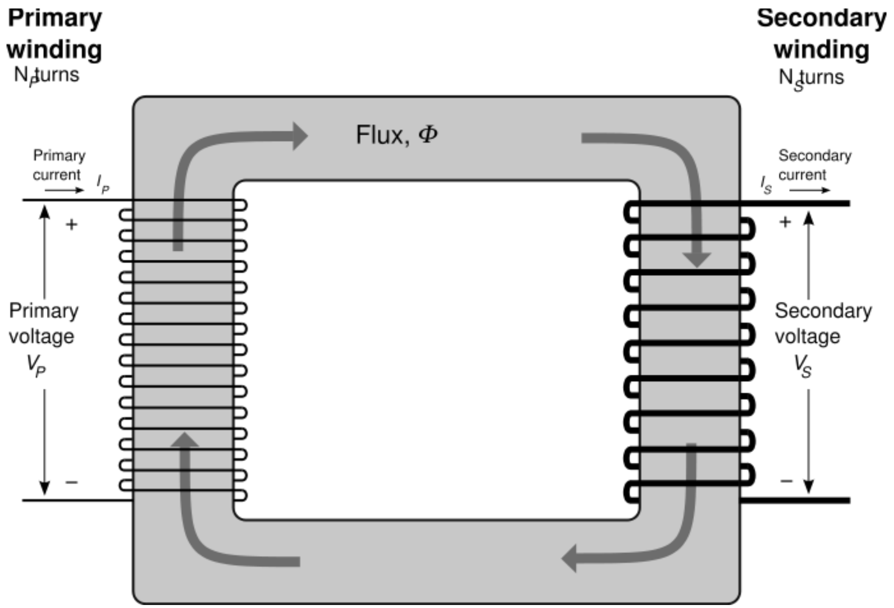
Electrical Machines II

Week 2: Ideal and non-ideal transformer and equivalent circuits

Ideal Transformer Relationships



Note that I_2 and I_2'
are in opposite directions

Primary can be noted by subscript **1**
Secondary can be noted by subscript **2**

An ideal transformer is a lossless device with an input winding and an output winding.

The relationships between the input voltage and the output voltage, and between the input current and the output current, are given by the following equations.

In instantaneous quantities

$$\frac{v_p(t)}{v_s(t)} = \frac{i_s(t)}{i_p(t)} = \boxed{a}$$

Turns Ratio

$$a = {}^{N_1}\!/_{N_2} = {}^{N_p}\!/_{N_s}$$

Ideal Transformer Relationships

Voltage and current relation

$$N_p i_p = N_s i_s$$

$$\frac{i_p}{i_s} = \frac{1}{a}$$

$$\frac{v_p}{v_s} = \boxed{a}$$

Turns Ratio is a scalar quantity that doesn't affect the angle

Power relation

$$P_{in} = V_P I_P \cos\varphi_P$$

$$P_{out} = V_s I_s \cos\varphi_s$$

φ_P → Angle between **primary** voltage and current

φ_s → Angle between **secondary** voltage and current

Thus, *the output power of an ideal transformer is equal to its input power*.

Since in ideal transformer primary and secondary voltage angles are equal, **then the primary and secondary have the same power**

Ideal Transformer Relationships

Impedance Transformation through a Transformer

Impedance of the load

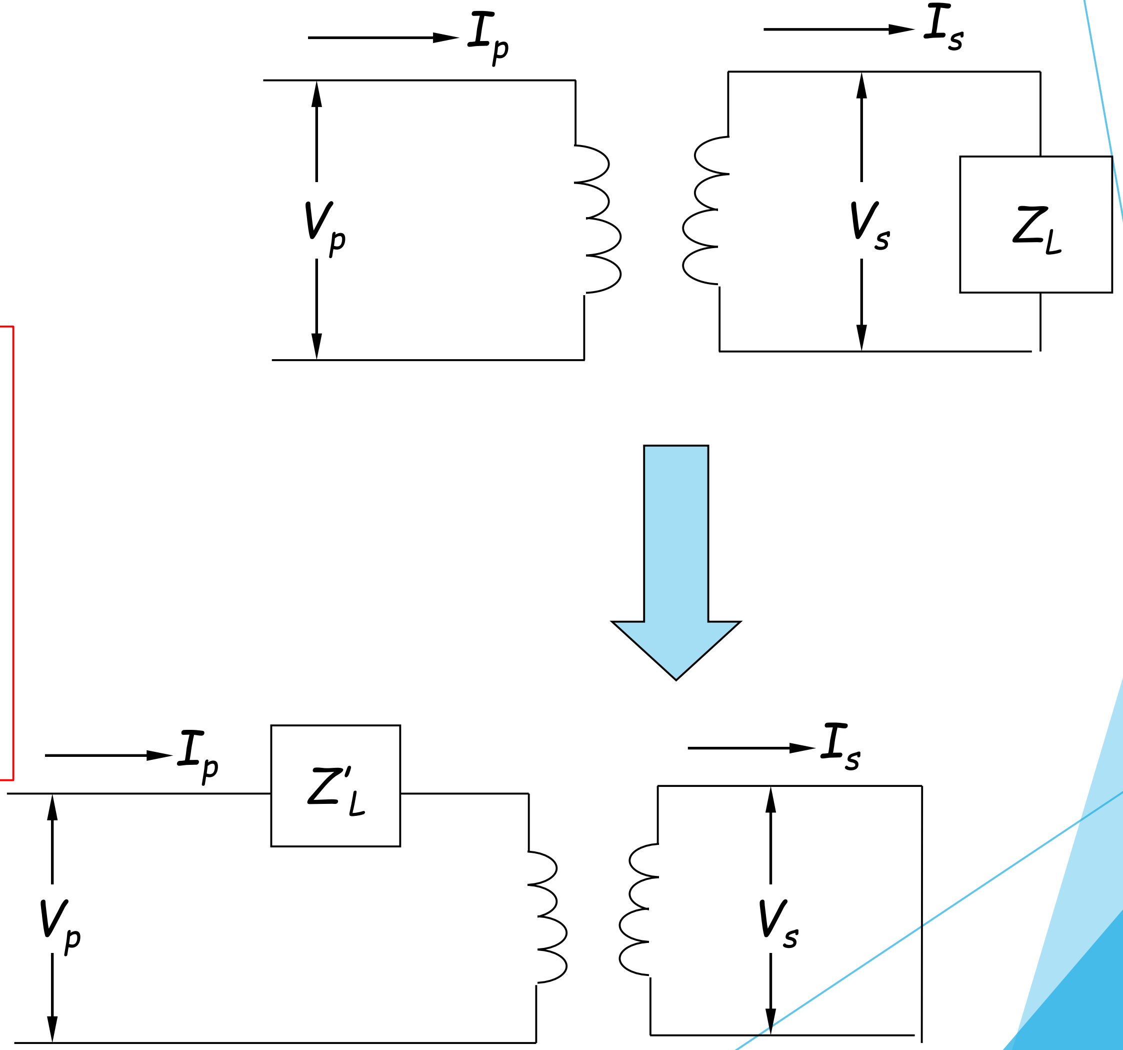
$$\boxed{Z_L = \frac{V_L}{I_L} = \frac{V_s}{I_s}}$$

The impedance of the primary circuit:

$$Z_L' = \frac{V_P}{I_P}$$

$$\because V_P = aV_s, I_P = \frac{I_S}{a}$$

$$\therefore Z_L' = \frac{V_P}{I_P} = \frac{aV_s}{\frac{I_S}{a}} = a^2\frac{V_s}{I_s} = a^2 Z_L$$



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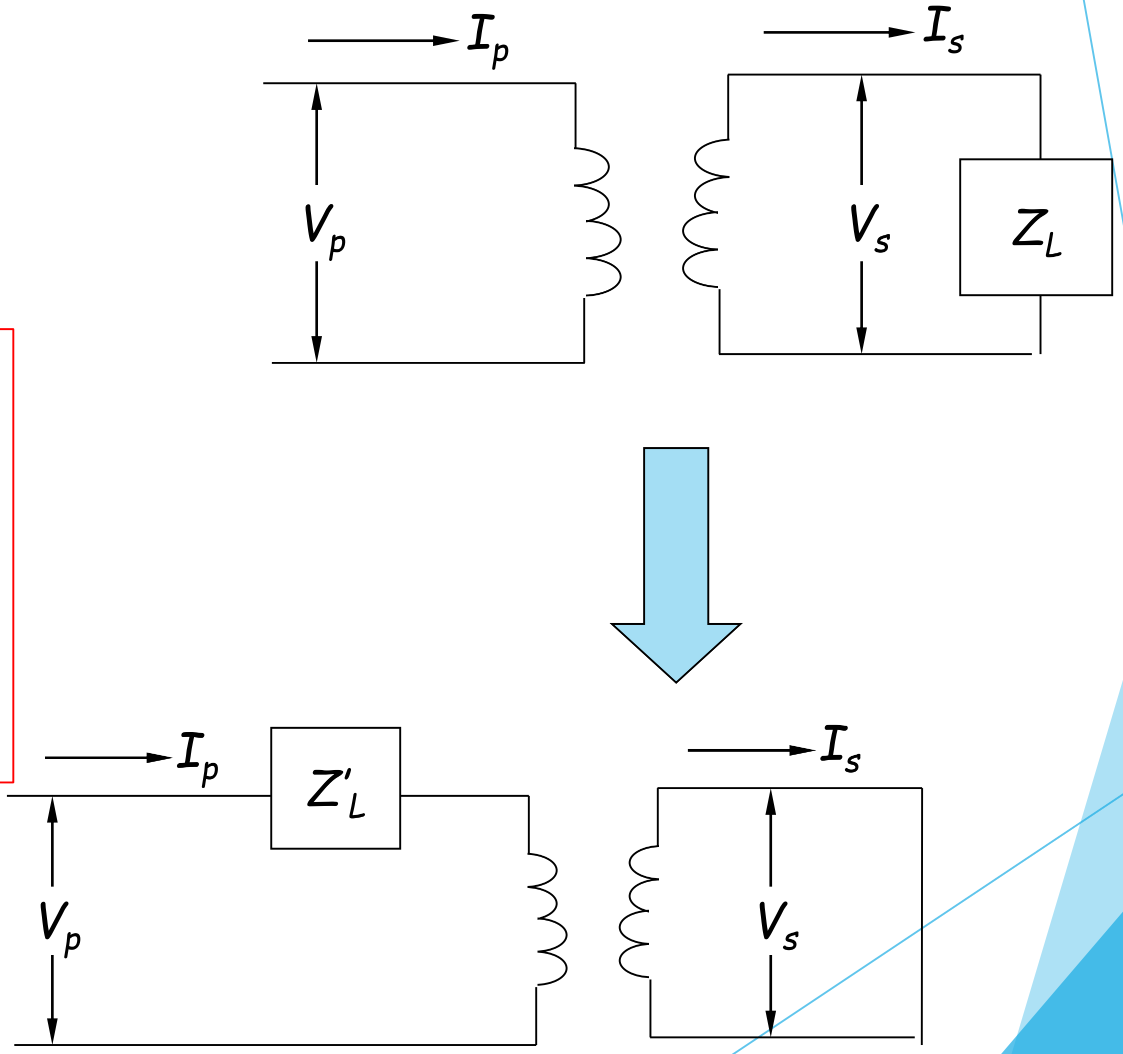
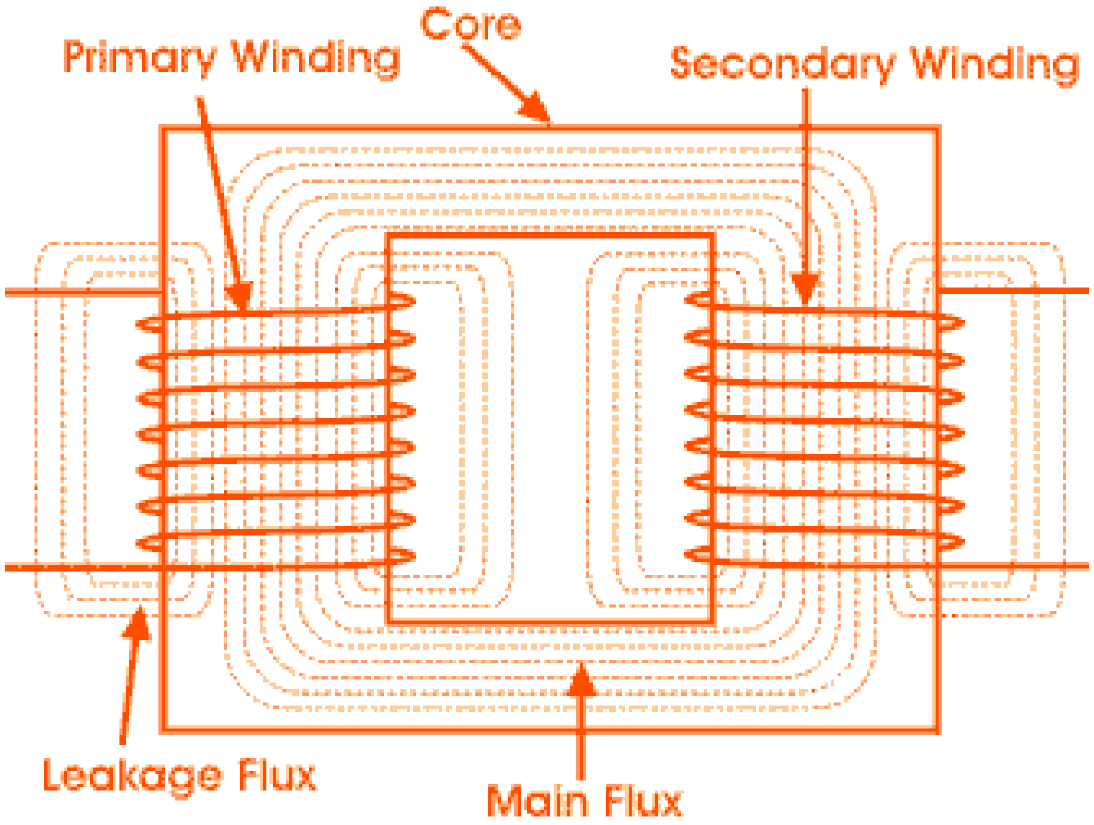


Real Transformers: Non ideal Transformer

- Real transformers
  - have losses
  - have leakage flux
  - have finite permeability of magnetic core

- Real power losses
  - resistance in windings ($I^2 R$)
  - core losses due to eddy currents and hysteresis

Theory of operation of non-ideal single phase transformer



$$e_{ind} = \frac{d\lambda}{dt}$$

Flux linkage in the coil which the voltage is being induced = all flux from each turn of coil

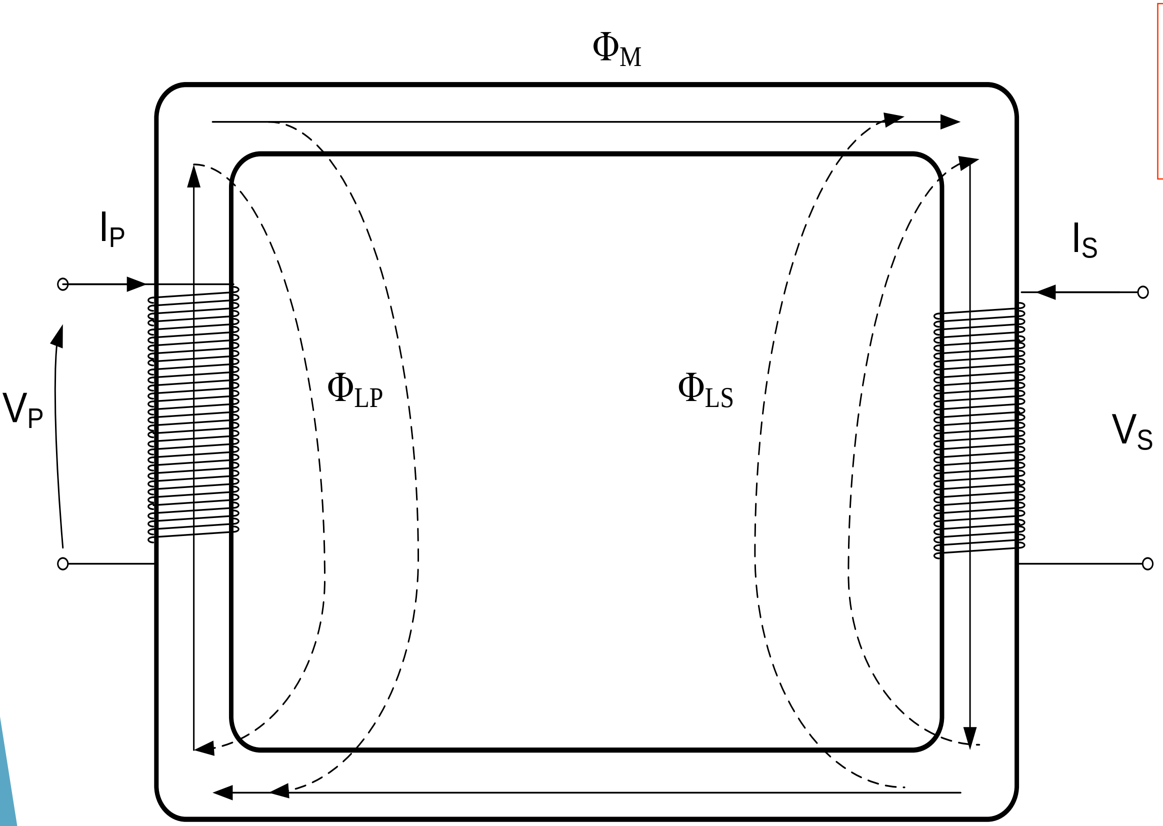
$$\lambda = N\Phi$$

$$\therefore e_{ind} = N\frac{d\Phi}{dt}$$

Leakage flux: flux that goes through one of the transformer windings but not the other one

Mutual flux: flux that remains in the core and links both windings

Theory of operation of non-ideal single phase transformer



$$\phi_P = \phi_M + \phi_{LP}$$
$$\phi_S = \phi_M + \phi_{LS}$$

$$v_P = N_P \frac{d\Phi_P}{dt} = N_P \frac{d\Phi_M}{dt} + N_P \frac{d\Phi_{LP}}{dt}$$

$$= e_P + e_{LP}$$

$$v_S = N_S \frac{d\Phi_S}{dt} = N_S \frac{d\Phi_M}{dt} + N_S \frac{d\Phi_{LS}}{dt}$$

$$= e_S + e_{LS}$$

ϕ_p: total average primary flux
ϕ_M : flux linking both primary and secondary windings
ϕ_{LP}: primary leakage flux
ϕ_S: total average secondary flux
ϕ_{LS}: secondary leakage flux

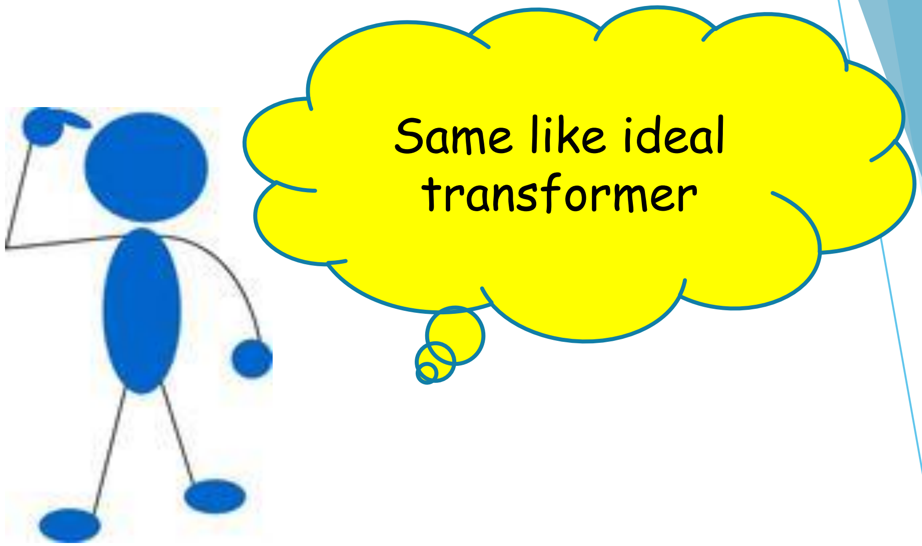
Theory of operation of non-ideal single phase transformer

$$e_P = N_P \frac{d\Phi_M}{dt}$$

$$e_S = N_S \frac{d\Phi_M}{dt}$$

$$\frac{e_P}{N_P} = \frac{d\Phi_M}{dt} = \frac{e_S}{N_S}$$

$$\therefore \frac{e_P}{e_S} = \frac{N_P}{N_S} = a$$



In a well designed transformer,

$$\Phi_M >> \Phi_{LS}$$

$$\Phi_M >> \Phi_{LP}$$

The ratio of the total voltage on primary of transformer to the total voltage on secondary is approximately related by the turns ratio “a”. The smaller the leakage fluxes are, the closer the total transformer voltage ratio approximately as that of an ideal transformer

The Magnetization current in a real transformer

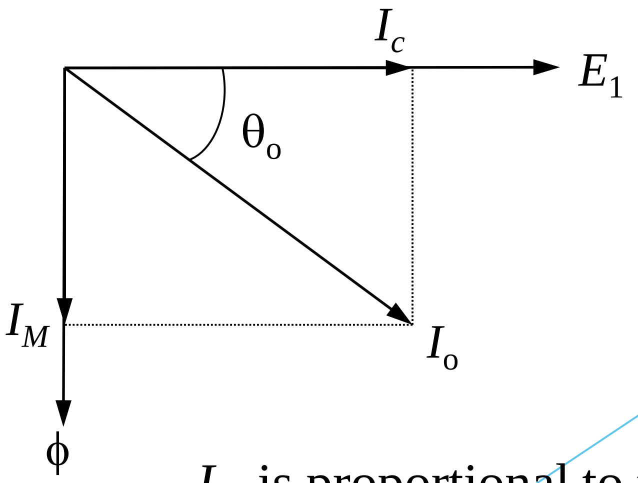
When an ac power source is connected to a transformer, a current flows in its primary circuit, even when the secondary circuit is open circuited. We often call this current as ***no-load current.***

This current is the current required to produce flux in the ferromagnetic core and is called ***excitation current***. It consists of two components:

1. The ***magnetization current*** I_m, which is the current required to produce the flux in the transformer core
2. The ***core-loss current*** I_{h+e}, which is the current required to make up for hysteresis and eddy current losses

Excitation current, I_o

- Magnetization current I_M (current required to produce flux in the core)
- Core-loss current I_{h+e} (current required to make up for hysteresis and eddy current losses)



I_M is proportional to the flux ϕ

$I_c = I_{h+e} = \text{Core loss}/E_1$

The Exact Equivalent Circuit of a Transformer

The losses that occur in transformers have to be accounted for in any accurate model of transformer behavior.

1. Copper (I^2R) losses. Copper losses are the resistive heating losses in the primary and secondary windings of the transformer. They are proportional to the square of the current in the windings.

2. Eddy current losses. Eddy current losses are resistive heating losses in the core of the transformer. They are proportional to the square of the voltage applied to the transformer.

3. Hysteresis losses. Hysteresis losses are associated with the rearrangement of the magnetic domains in the core during each half-cycle. They are a complex, nonlinear function of the voltage applied to the transformer.

4. Leakage flux. The fluxes which escape the core and pass through only one of the transformer windings are leakage fluxes. These escaped fluxes produce a self-inductance in the primary and secondary coils, and the effects of this inductance must be accounted for.

The Exact Equivalent Circuit of a Transformer

Modeling the copper losses: resistive losses in the primary and secondary windings of the core, represented in the equivalent circuit by R_P and R_S.

Modeling the core excitation: I_m is proportional to the voltage applied to the core and lags the applied voltage by 90°. It is modeled by X_M.

Modeling the core loss current: I_{h+e} is proportional to the voltage applied to the core and in phase with the applied voltage. It is modeled by R_C.

4- Modeling the leakage fluxes: primary leakage flux is proportional to the primary current I_P and secondary leakage flux is proportional to the secondary current I_S, represented in the equivalent circuit by X_P $(=\phi_{LP}/I_P)$ and X_S $(=\phi_{LS}/I_S)$.

$$\Phi_{LP} = \frac{N_P i_P}{\Re} = \mathrm{P} N_P i_P$$

$$\Phi_{LS} = \frac{N_S i_S}{\Re} = \mathrm{P} N_S i_S$$

$$e_{LP} = N_P \frac{d(\mathrm{P} N_P) i_P}{dt} = N_P^2 \mathrm{P} \frac{di_P}{dt}$$

$$e_{LS} = N_S \frac{d(\mathrm{P} N_S) i_S}{dt} = N_S^2 \mathrm{P} \frac{di_S}{dt}$$

$$\because L = \mathrm{P} N^2$$

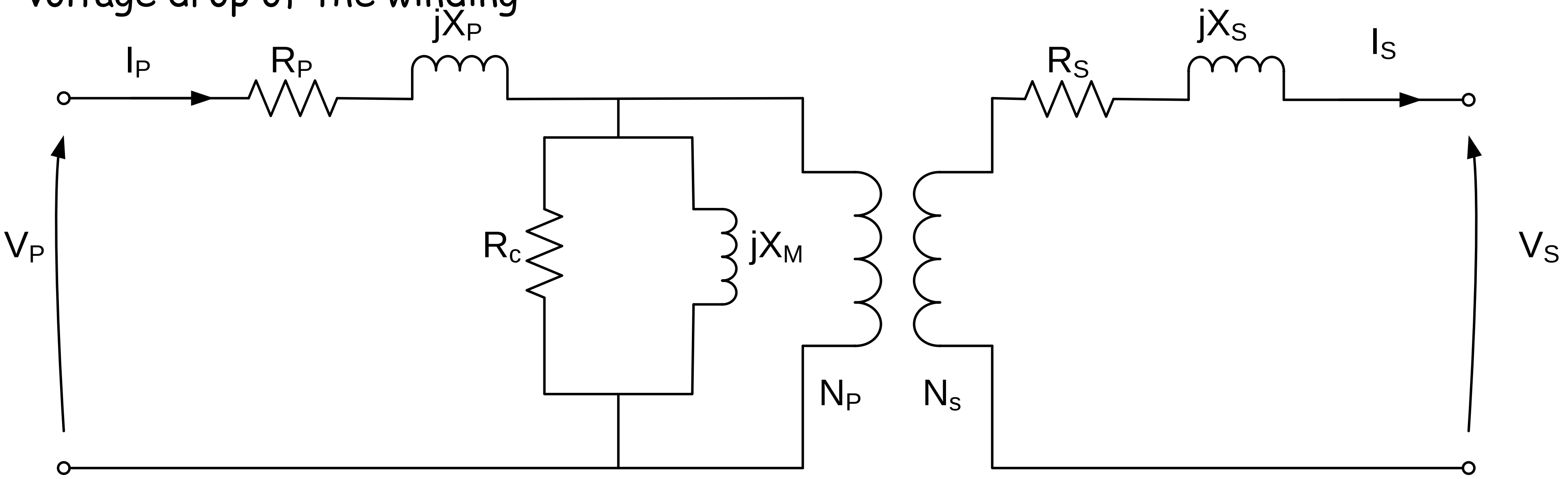
$$e_{LP} = L_P \frac{di_P}{dt}$$

$$e_{LS} = L_S \frac{di_S}{dt}$$

P= permeance =inverse of reluctance

The Exact Equivalent Circuit of a Transformer

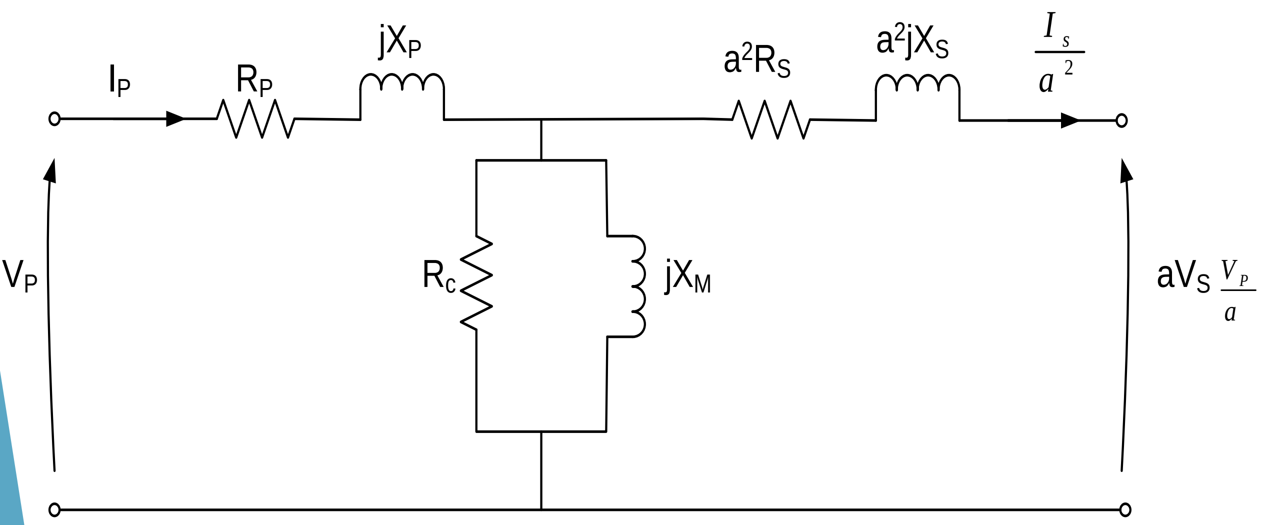
- The excitation branch is modeled and placed inside the primary winding as the voltage actually applied to the cores is really the input voltage less the internal voltage drop of the winding



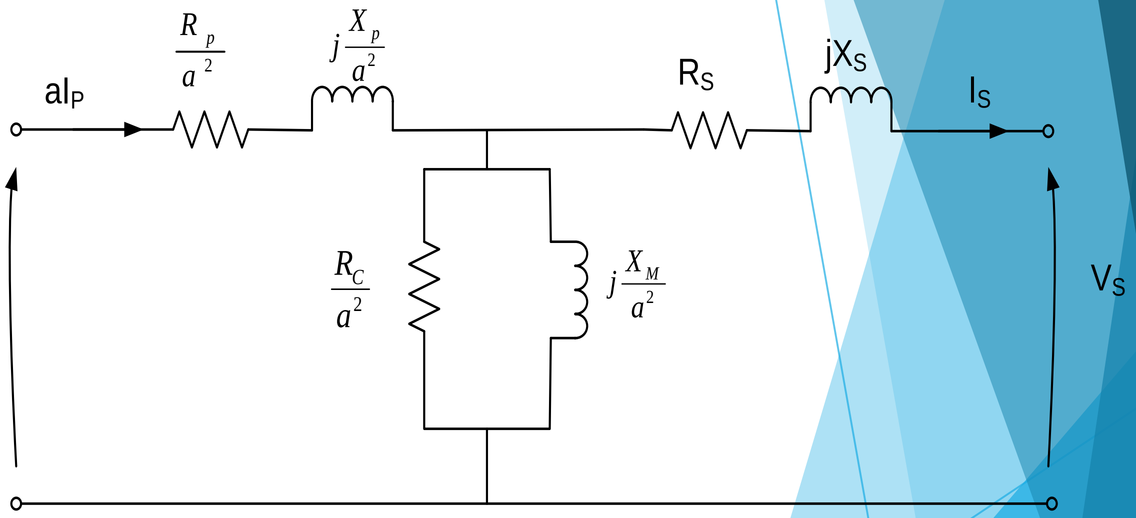
Model of a real transformer

The Exact Equivalent Circuit of a Transformer

Although the previous equivalent circuit is an accurate model of a transformer, it is not a very useful one. To analyze practical circuits containing transformers, it is normally necessary to convert the entire circuit to an equivalent circuit at a single voltage level. Therefore, the equivalent circuit must be referred either to its primary side or to its secondary side in problem solutions.



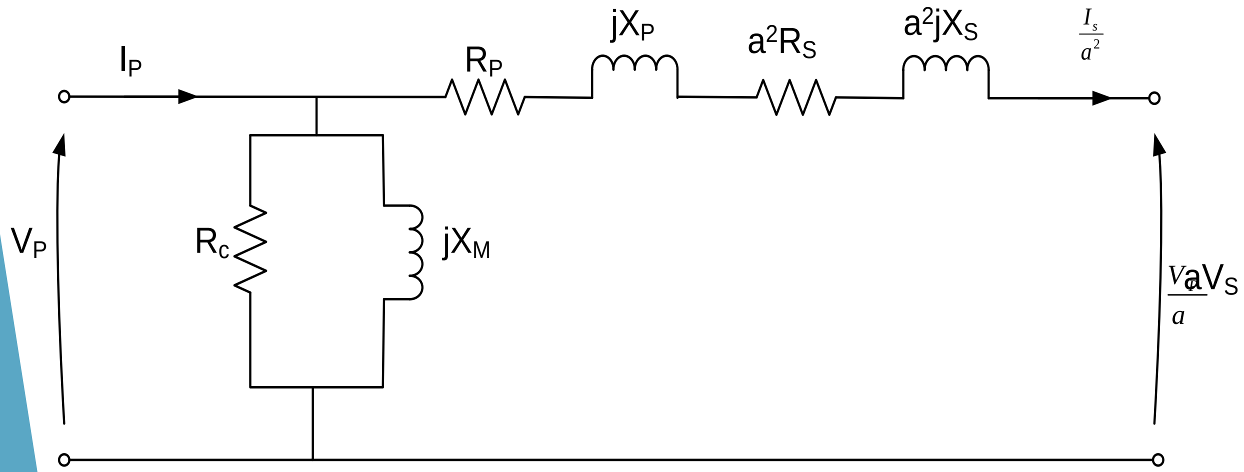
equivalent circuit of the transformer referred to its primary side.



equivalent circuit referred to its secondary side.

The Exact Equivalent Circuit of a Transformer

- The transformer models shown before are often complex than necessary to get good results in practical engineering applications.
- One principle complain is the excitation branch which adds an extra node to the circuit.
- Excitation branch has very small current compared to the load current. It could be 2-3% of full load current for a typical transformer
- A simplified can be produced that reduces model complexity and at the same time is acceptable.



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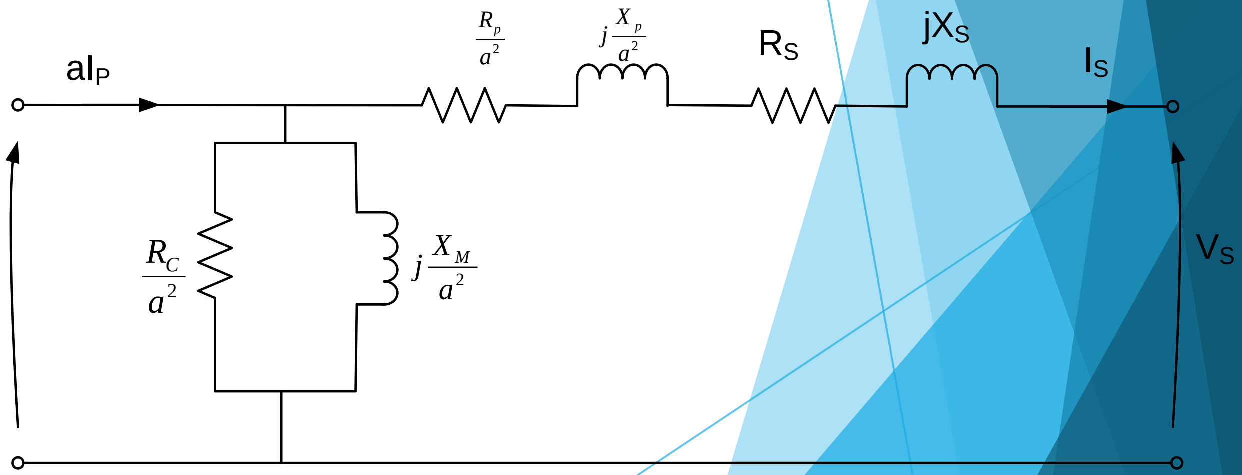

equivalent circuit referred to its secondary side.