Electrical Machines II

Week 4: Transformer Efficiency, Maximum Efficiency Criterion and Phasor diagram

Transformer Losses and Efficiency

Transformer efficiency is defined as

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% \qquad \eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

Types of losses incurred in a transformer:

1- Copper I^2R losses --- Load dependant

2- Core losses (hysteresis losses and eddy current losses), also known as no-load losses. Core losses are load in- dependent.

Transformer Losses and Efficiency

Transformer efficiency may be calculated using the following:

$$\eta = \frac{V_2' I_2' \cos\varphi_2'}{V_2' I_2' \cos\varphi_2' + I_2'^2 R_{eq} + P_c} \times 100\%$$

For maximum efficiency,

Differentiation of efficiency with respect to load current → $$\frac{d\eta}{dI_2'} = 0$$

$$\frac{d\eta}{dI_2'} = \frac{\left[V_2' I_2' \cos\varphi_2' + I_2'^2 R_{eq} + P_c\right]\left(V_2' \cos\varphi_2'\right) - \left[V_2' I_2' \cos\varphi_2'\right]\left[V_2' \cos\varphi_2' + 2 I_2' R_{eq} + 0\right]}{\left[V_2' I_2' \cos\varphi_2' + I_2'^2 R_{eq} + P_c\right]^2} \times 100\%$$

Transformer Losses and Efficiency

$$\frac{d\eta}{dI_2'} = \frac{\left[V_2' I_2' \cos\varphi_2' + I_2'^2 R_{eq} + P_c\right]\left(V_2' \cos\varphi_2'\right) - \left[V_2' I_2' \cos\varphi_2'\right]\left[V_2' \cos\varphi_2' + 2I_2' R_{eq} + 0\right]}{\left[V_2' I_2' \cos\varphi_2' + I_2'^2 R_{eq} + P_c\right]^2} \times 100\%$$

$$0 = \left[V_2' I_2' \cos\varphi_2' + I_2'^2 R_{eq} + P_c\right]\left(V_2' \cos\varphi_2'\right) - \left[V_2' I_2' \cos\varphi_2'\right]\left[V_2' \cos\varphi_2' + 2I_2' R_{eq} + 0\right]$$

$$0 = \left[V_2' I_2' \cos\varphi_2' + I_2'^2 R_{eq} + P_c\right] - I_2'\left[V_2' \cos\varphi_2' + 2I_2' R_{eq}\right]$$

$$-P_c = V_2' I_2' \cos\varphi_2' + I_2'^2 R_{eq} - V_2' I_2' \cos\varphi_2' - 2I_2'^2 R_{eq}$$

$$-P_c = -I_2'^2 R_{eq}$$

$$P_c = I_2'^2 R_{eq}$$

$$\left. I_2' \right|_{\eta \max} = \sqrt{\frac{P_c}{\mathrm{R}_{eq}}}$$

No-load core losses

For maximum efficiency, at a load such that

copper losses = iron losses

Transformer Voltage regulation

Because a real transformer has series impedance within it, the output voltage of a transformer varies with the load even if the input voltage remains constant. The voltage regulation of a transformer is the change in the magnitude of the secondary terminal voltage from no-load to full-load.

$$\%Voltage\ Regulation = \frac{V_s[no-load]-V_s[full-load]}{V_s[full-load]}\times 100$$

$$\approx \frac{V_p[no-load]-V_p[full-load]}{V_p[full-load]}\times 100$$

Referred to the primary side

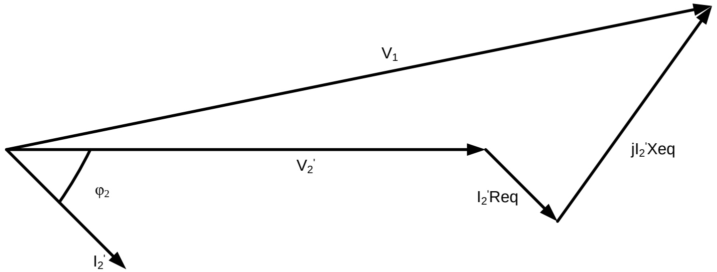
Single phase Transformer Phasor diagram

$$V_1 = V_2' + I_2'\left(R_{eq} + jX_{eq}\right)$$

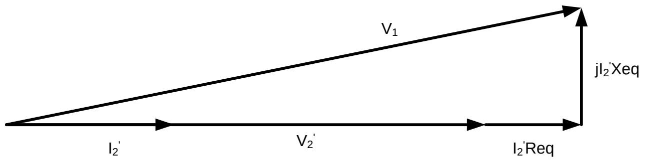
$$V_1 = V_2' + I_2' R_{eq} + jI_2' X_{eq}$$

Very useful in determining the sources of voltage drop and voltage regulation calculation

Lagging p.f



unity p.f



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