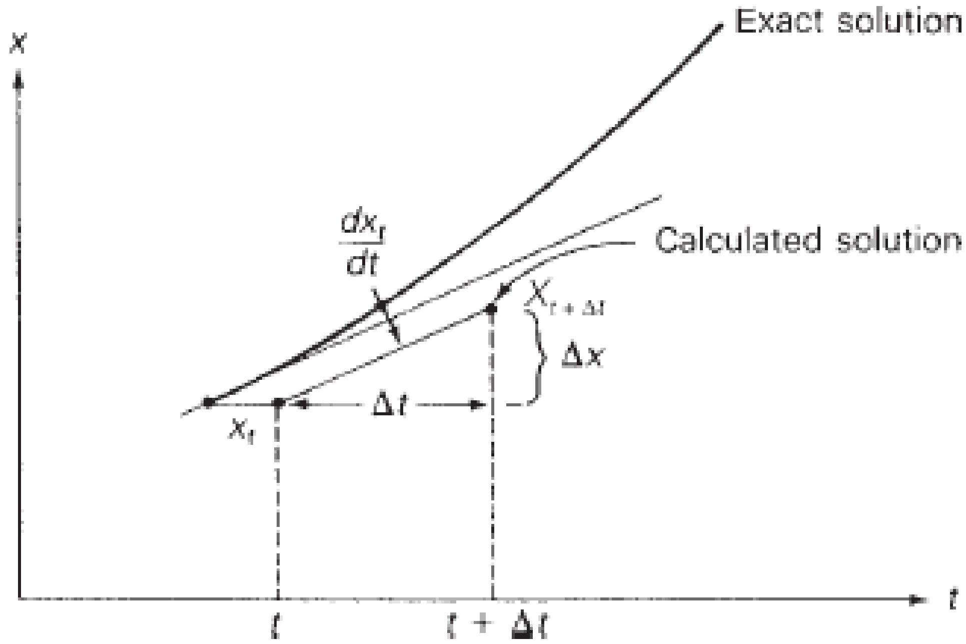
Numerical Integration of the Swing Equation

Euler’s Method

Given the first order differential equation

$$\frac{dx}{dt} = f(x)$$

One simple integration technique is Euler’s method as shown in Figure. The integration step size is Δt.

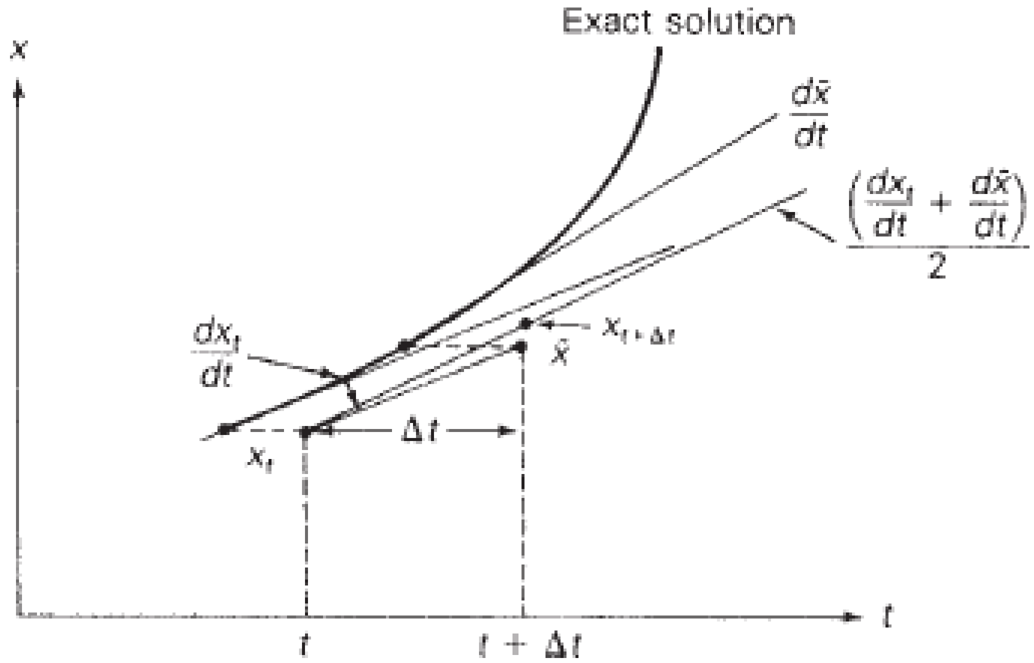


$$\frac{dx_t}{dt} = f(x_t)$$

The new value $x_{t+\Delta t}$ is calculated from the old value x_t by adding the increment Δx,

$$x_{t+\Delta t} = x_t + \Delta x = x_t + \left(\frac{dx_t}{dt}\right)\Delta t$$

As shown in the figure, Euler's method assumes that the slope is constant over the entire interval Δt. An improvement can be obtained by calculating the slope at both the beginning and end of the interval, and then averaging these slopes. The modified Euler's method is illustrated in Figure



slope at the beginning of the interval is calculated
calculate a preliminary value $\bar{x}$ given by

$$\bar{x} = x_t + \left(\frac{dx_t}{dt}\right)\Delta t$$

Next, the slope at $\bar{x}$ is calculated:

$$\frac{d\bar{x}}{dt} = f(\bar{x})$$

Then, the new value is calculated using the average slope:

$$x_{t+\Delta t} = x_t + \frac{\left(\frac{dx_t}{dt} + \frac{d\bar{x}}{dt}\right)}{2}\Delta t$$

Applying Euler's method to calculate machine frequency Ɯ and power angle ∂

the slopes at the beginning of the interval are

$$\frac{d\delta_t}{dt} = \omega_t - \omega_{\text{syn}}$$

$$\frac{d\omega_t}{dt} = \frac{p_{ap.u.t}\,\omega_{\text{syn}}}{2H\omega_{\text{p.u.}t}}$$

where $p_{ap.u.t}$ is the per-unit accelerating power calculated at $\delta = \delta_t$, and $\omega_{\text{p.u.}t} = \omega_t/\omega_{\text{syn}}$.

$$\tilde{\delta} = \delta_t + \left(\frac{d\delta_t}{dt}\right)\Delta t$$

$$\tilde{\omega} = \omega_t + \left(\frac{d\omega_t}{dt}\right)\Delta t$$

Next, the slopes at $\bar{\delta}$ and $\bar{\omega}$ are calculated,

$$\frac{d\bar{\delta}}{dt} = \bar{\omega} - \omega_{\text{syn}}$$

$$\frac{d\bar{\omega}}{dt} = \frac{\bar{p}_{a\text{p.u.}}\,\omega_{\text{syn}}}{2\text{H}\bar{\omega}_{\text{p.u.}}}$$

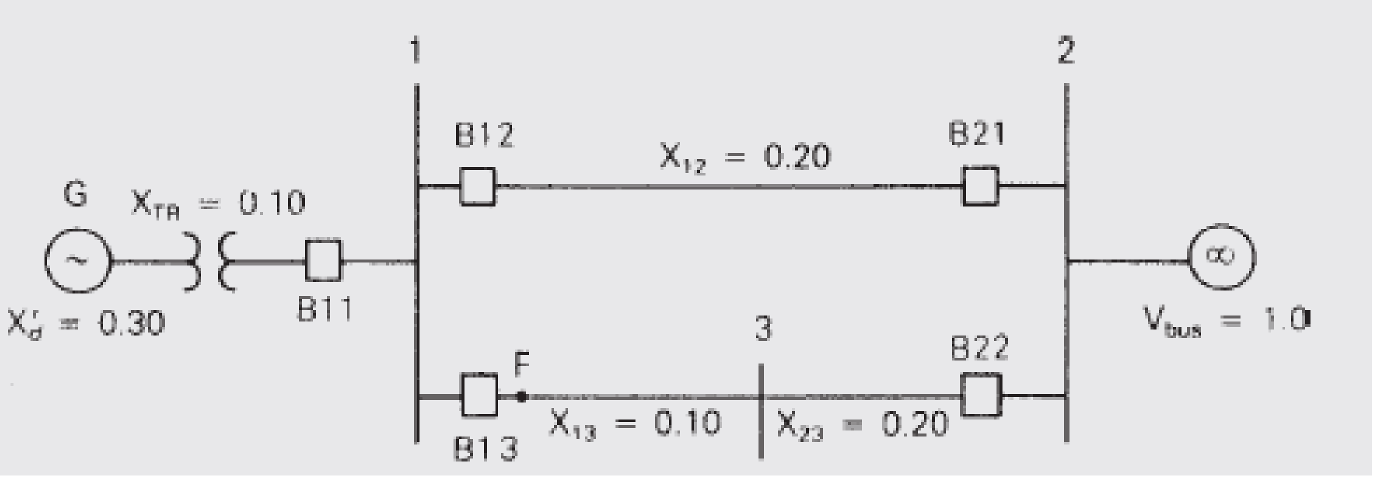
where $\bar{p}_{a\text{p.u.}}$ is the per-unit accelerating power calculated at $\delta = \bar{\delta}$, and $\bar{\omega}_{\text{p.u.}} = \bar{\omega}/\omega_{\text{syn}}$.

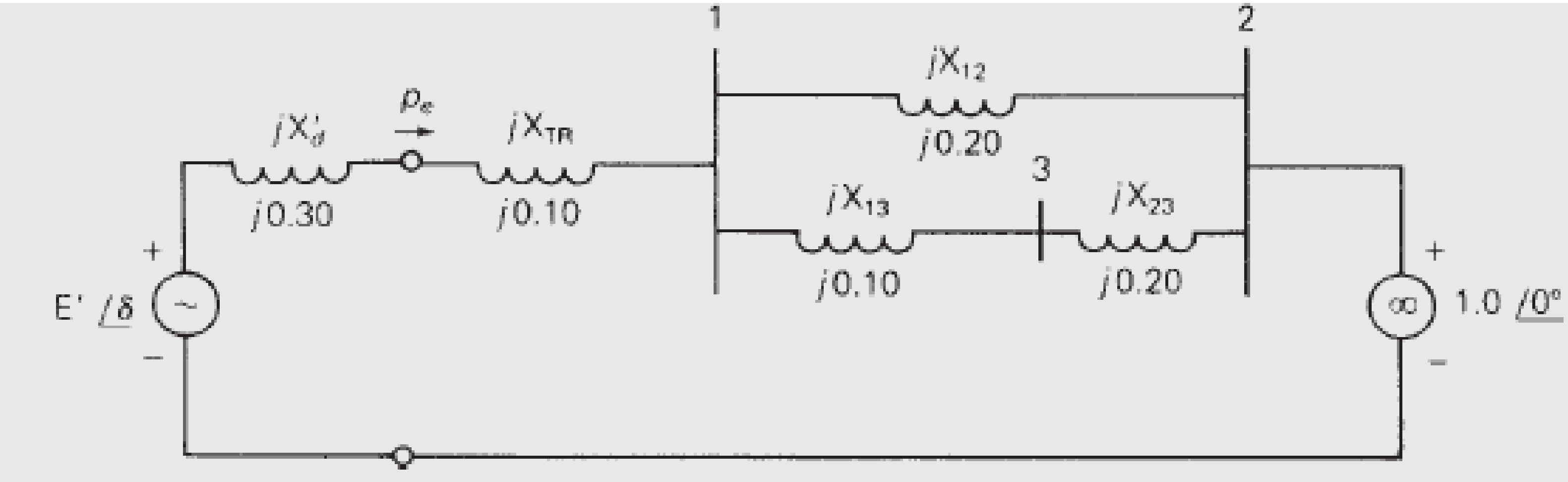
$$\delta_{t+\Delta t} = \delta_t + \frac{\left(\frac{d\delta_t}{dt} + \frac{d\bar{\delta}}{dt}\right)}{2}\Delta t$$

$$\omega_{t+\Delta t} = \omega_t + \frac{\left(\frac{d\omega_t}{dt} + \frac{d\bar{\omega}}{dt}\right)}{2}\Delta t$$

Example:

Figure shows a single-line diagram of a three-phase, 60-Hz synchronous generator, connected through a transformer and parallel transmission lines to an infinite bus. All reactances are given in per-unit on a common system base. If the infinite bus receives 1.0 per unit real power at 0.95 p.f. lagging, determine (a) the internal voltage of the generator and (b) the equation for the electrical power delivered by the generator versus its power angle δ.





$$X_{eq} = X'_d + X_{TR} + X_{12} \| (X_{13} + X_{23})$$

$$= 0.30 + 0.10 + 0.20 \| (0.10 + 0.20)$$

$$= 0.520 \quad \text{per unit}$$

The current into the infinite bus is

$$I = \frac{P}{V_{bus}(\text{p.f.})} \underline{/-\cos^{-1}(\text{p.f.})} = \frac{(1.0)}{(1.0)(0.95)} \underline{/-\cos^{-1} 0.95}$$

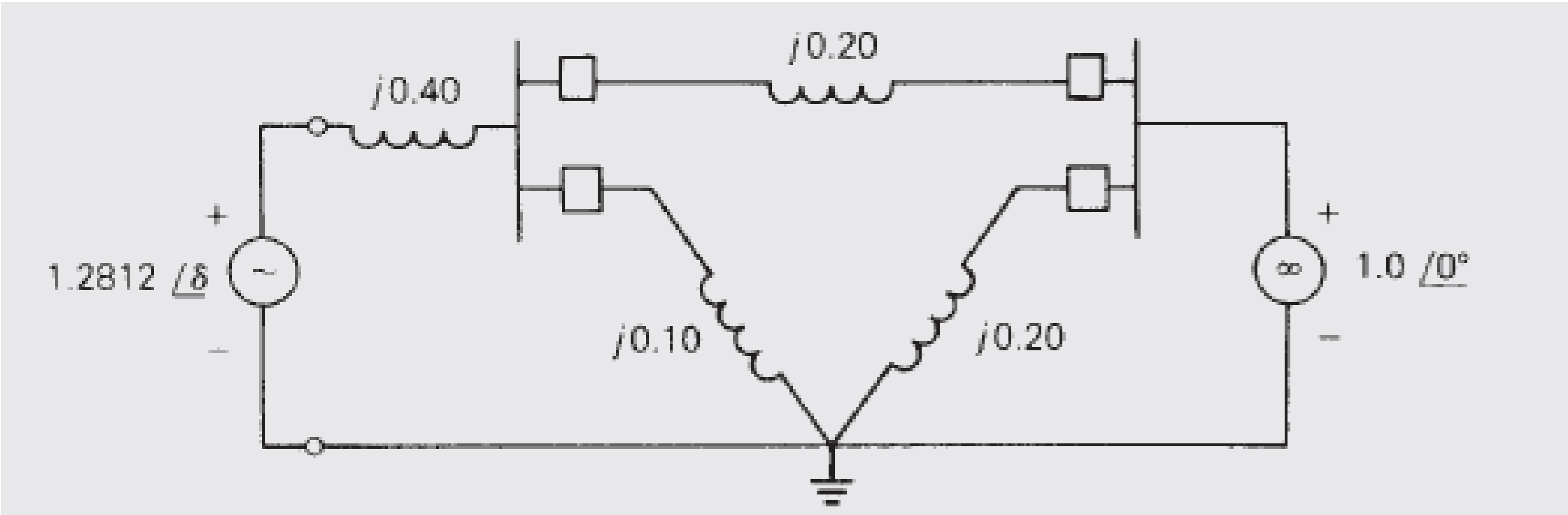
$$= 1.05263 \underline{/-18.195^\circ} \quad \text{per unit}$$

and the machine internal voltage is

$$\begin{aligned}E' = E'\angle\delta &= V_{\text{bus}} + jX_{\text{eq}} I \\ &= 1.0\angle 0^\circ + (j0.520)(1.05263\angle -18.195^\circ) \\ &= 1.0\angle 0^\circ + 0.54737\angle 71.805^\circ \\ &= 1.1709 + j0.5200 \\ &= 1.2812\angle 23.946^\circ \quad \text{per unit}\end{aligned}$$

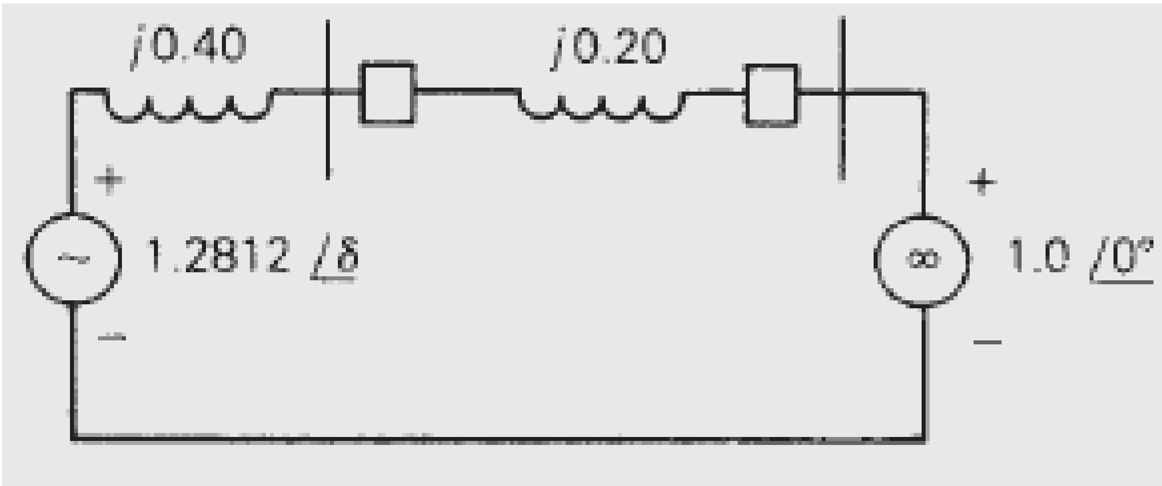
$$p_e = \frac{(1.2812)(1.0)}{0.520} \sin\delta = 2.4638 \sin\delta \quad \text{per unit}$$

If a three phase to ground bolted short circuit occurs on line 1-3 at bus 3m The fault is cleared by opening the circuit breakers at the ends of line 1-3 and line 2-3. These circuit breakers remain open. Calculate the critical clearing angle. H=3.0 pm=1.0p.u.
The faulted network is shown

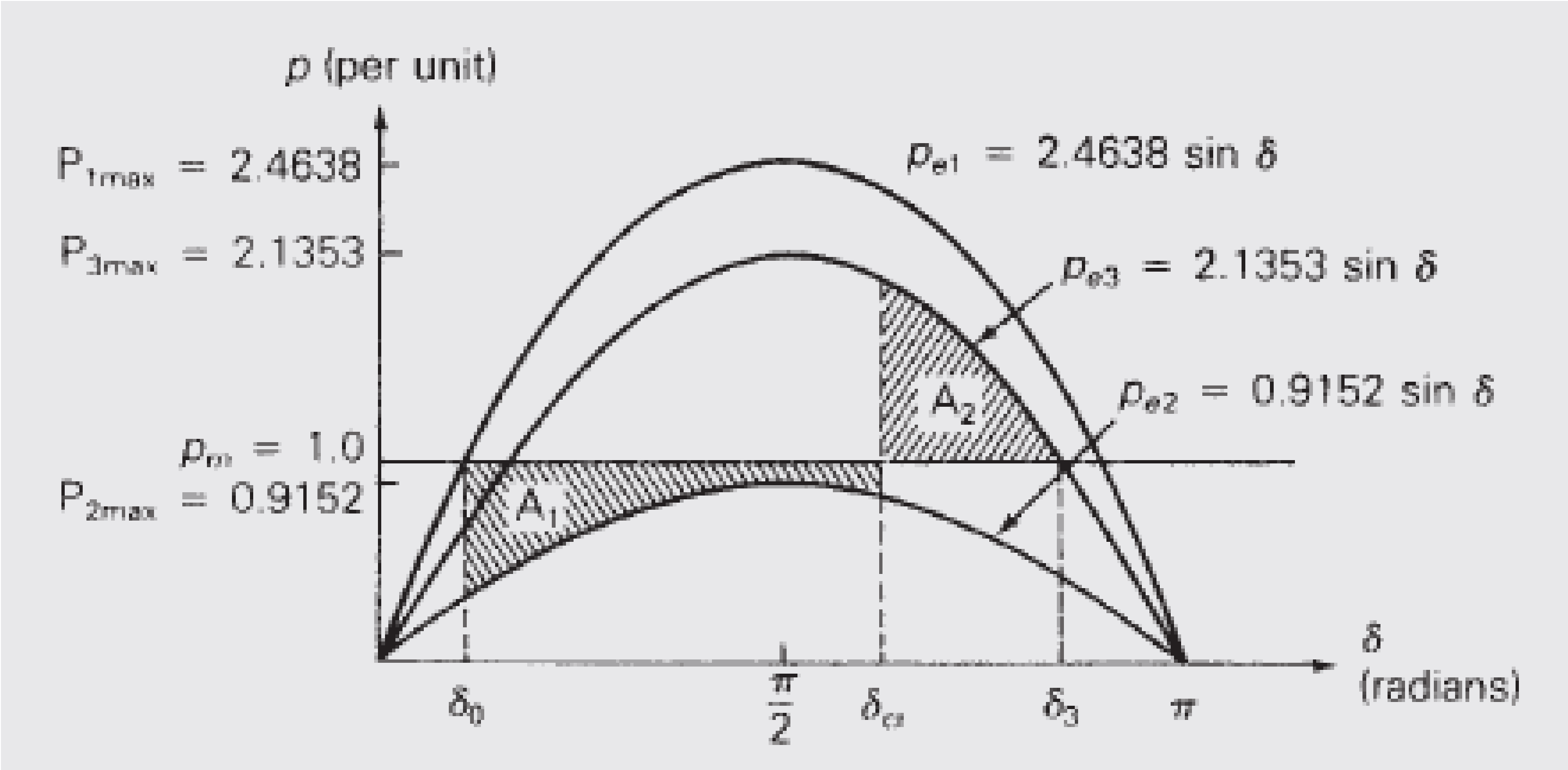


$$P_{e2} = 0.9152 \sin \partial$$

After fault clearance



$$P_{e3} = 2.1353 \sin \partial$$



To obtain critical clearing angle A1=A2

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (p_m - P_{2max} \sin\delta)\, d\delta = A_2 = \int_{\delta_{cr}}^{\delta_3} (P_{3max} \sin\delta - p_m)\, d\delta$$

$$\int_{0.4179}^{\delta_{cr}} (1.0 - 0.9152 \sin\delta)\, d\delta = \int_{\delta_{cr}}^{2.6542} (2.1353 \sin\delta - 1.0)\, d\delta$$

$$\delta_{cr} = 1.9812 \text{ radians} = 113.5$$

Verify the critical clearing angle obtained, and calculate the critical clearing time by applying the modified Euler's method to solve the swing equation for the following two cases:

Case 1 The fault is cleared at $\delta = 1.95$ radians $= 112^\circ$ (which is less than δ_{cr})

Case 2 The fault is cleared at $\delta = 2.09$ radians $= 120^\circ$ (which is greater than δ_{cr})

For calculations, use a step size $\Delta t = 0.01$ s, and solve the swing equation from $t = 0$ to $t = T = 0.85$ s.

$t = 0$ are

$$\delta_0 = 0.4179 \quad \text{rad}$$

$$\omega_0 = \omega_{\text{syn}} = 2\pi 60 \quad \text{rad/s}$$

Also, the H constant is 3.0 p.u.-s, and the faulted accelerating power is

$$p_{a\text{p.u.}} = 1.0 - 0.9152 \sin \delta$$

The postfault accelerating power is

$$p_{a\text{p.u.}} = 1.0 - 2.1353 \sin \delta \quad \text{per unit}$$

Case 1 Stable			Case 2 Unstable		
Time s	Delta rad	Omega rad/s	Time s	Delta rad	Omega rad/s
0.000	0.418	376.991	0.000	0.418	376.991
0.020	0.426	377.778	0.020	0.426	377.778
0.040	0.449	378.547	0.040	0.449	378.547
0.060	0.488	379.283	0.060	0.488	379.283
0.080	0.541	379.970	0.080	0.541	379.970
0.100	0.607	380.599	0.100	0.607	380.599
0.120	0.685	381.159	0.120	0.685	381.159
0.140	0.773	381.646	0.140	0.773	381.646
0.160	0.870	382.056	0.160	0.870	382.056
0.180	0.975	382.392	0.180	0.975	382.392
0.200	1.086	382.660	0.200	1.086	382.660
0.220	1.202	382.868	0.220	1.202	382.868
0.240	1.321	383.027	0.240	1.321	383.027
0.260	1.443	383.153	0.260	1.443	383.153
0.280	1.567	383.262	0.280	1.567	383.262
0.300	1.694	383.370	0.300	1.694	383.370
0.320	1.823	383.495	0.320	1.823	383.495
0.340	1.954	383.658	0.340	1.954	383.658
	Fault Cleared		0.360	2.090	383.876
0.360	2.076	382.516		Fault Cleared	
0.380	2.176	381.510	0.380	2.217	382.915
0.400	2.257	380.638	0.400	2.327	382.138
0.420	2.322	379.886	0.420	2.424	381.546
0.440	2.373	379.237	0.440	2.511	381.135
0.460	2.413	378.674	0.460	2.591	380.902
0.480	2.441	378.176	0.480	2.668	380.844
0.500	2.460	377.726	0.500	2.746	380.969
0.520	2.471	377.307	0.520	2.828	381.288
0.540	2.473	376.900	0.540	2.919	381.824
0.560	2.467	376.488	0.560	3.022	382.609
0.580	2.453	376.056	0.580	3.145	383.686
0.600	2.429	375.583	0.600	3.292	385.111
0.620	2.396	375.053	0.620	3.472	386.949
0.640	2.351	374.446	0.640	3.693	389.265
0.660	2.294	373.740	0.660	3.965	392.099
0.680	2.221	372.917	0.680	4.300	395.426
0.700	2.130	371.960	0.700	4.704	399.079
0.720	2.019	370.855	0.720	5.183	402.689
0.740	1.884	369.604	0.740	5.729	405.683
0.760	1.723	368.226	0.760	6.325	407.477
0.780	1.533	366.773	0.780	6.941	407.812
0.800	1.314	365.341	0.800	7.551	406.981

```
10 REM EXAMPLE 13.7
20 REM SOLUTION TO SWING EQUATION
30 REM THE STEP SIZE IS DELTA
40 REM THE CLEARING ANGLE IS DLTCLR
50 DELTA + .01
60 DLTCLR = 1.95
70 J = 1
80 PMAX = .9152
90 PI = 3.1415927 #
100 T = 0
110 X1 = .4179
120 X2 = 2 * PI * 60
130 LPRINT "TIME DELTA OMEGA"
140 LPRINT "s rad rad/s"
150 LPRINT USING "#####.###"; T;X1;X2
160 FOR K = 1 TO 86
170 REM LINE 180 IS EQ(13.4.7)
180 X3 = X2 − (2 * PI * 60)
190 IF J=2 THEN GOTO 240
200 IF X1 > DLTCLR OR X1 = DLTCLR THEN
   PMAX=2.1353
210 IF X1 > DLTCLR OR X1 = DLTCLR THEN
   LPRINT "FAULT CLEARED"
220 IF X1 > DLTCLR OR X1 = DLTCLR THEN
   J=2
230 REM LINES 240 AND 250 ARE EQ(13.4.8)
240 X4 = 1 − PMAX * SIN(X1)
250 X5 = X4 * (2 * PI * 60) * (2 * PI * 60)/(6 * X2)
260 REM LINE 270 IS EQ(13.4.9)
270 X6 = X1 + X3 * DELTA
280 REM LINE 290 IS EQ(13.4.10)
290 X7 = X2 + X5 * DELTA
300 REM LINE 310 IS EQ(13.4.11)
310 X8 = X7 − 2 * PI * 60
320 REM LINES 330 AND 340 ARE EQ(13.4.12)
330 X9 = 1 − PMAX * SIN(X6)
340 X10 = X9 * (2 * PI * 60) * (2 * PI * 60)/(6 * X7)
350 REM LINE 360 IS EQ(13.4.13)
360 X1 = X1 + (X3 + X8) * (DELTA/2)
370 REM LINE 380 IS EQ(13.4.14)
380 X2 = X2 + (X5 + X10) * (DELTA/2)
390 T = K * DELTA
400 Z = K/2
410 M = INT(Z)
420 IF M = Z THEN LPRINT USING
   "#####.###"; T;X1;X2
430 NEXT K
440 END
```

This program is taken from "Power System Analysis and Design" Glover & Sarma