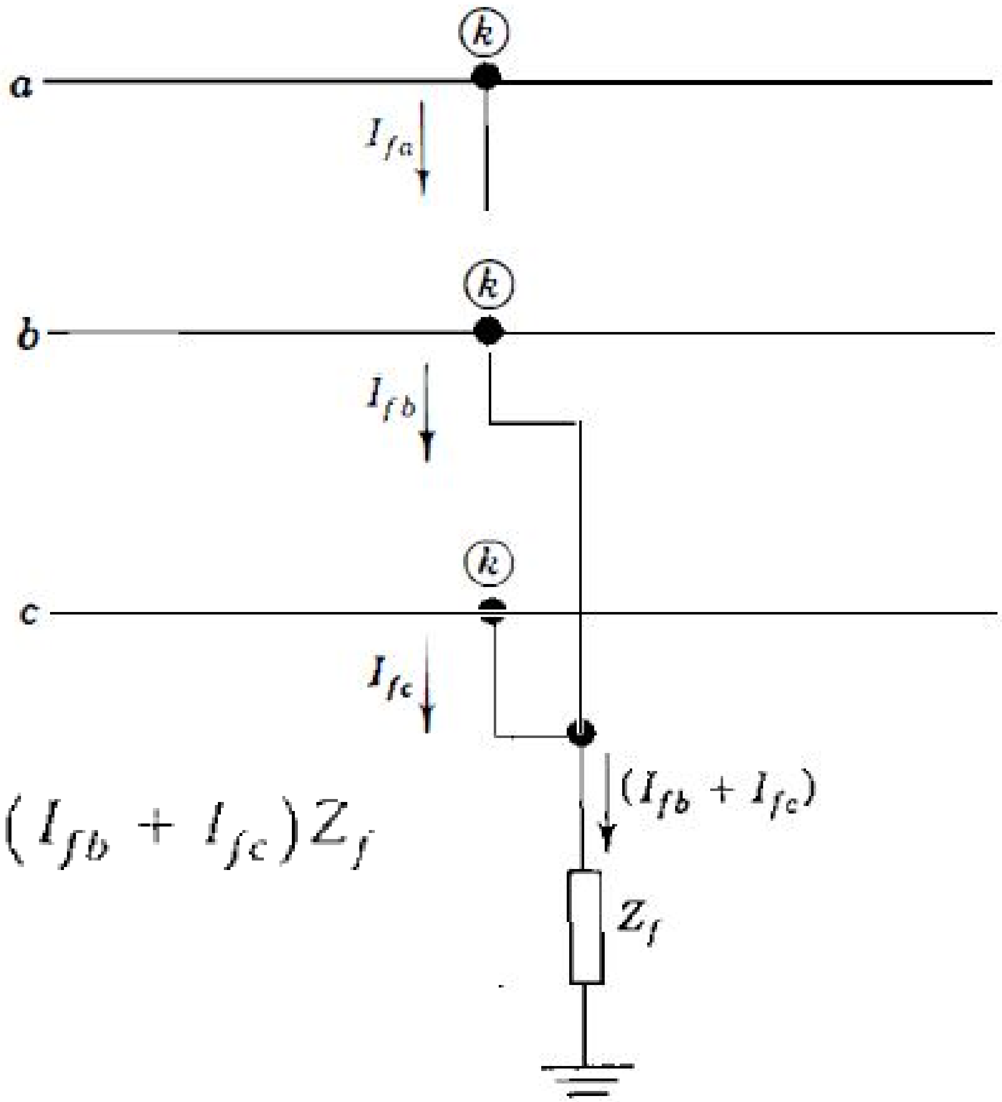
Double Line To Ground Fault



$$I_{fa} = 0 \qquad V_{kb} = V_{kc} = (I_{fb} + I_{fc})Z_f$$

Since I_{fa} is zero, the zero-sequence current is given by $I_{fa}^{(0)} = (I_{fb} + I_{fc})/3$

$$V_{kb} = V_{kc} = 3Z_f I_{fa}^{(0)}$$

$$\begin{bmatrix} V_{ka}^{(0)} \\ V_{ka}^{(1)} \\ V_{ka}^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_{ka} \\ V_{kb} \\ V_{kb} \end{bmatrix}$$

The second and third rows of this equation show that

$$V_{ka}^{(1)} = V_{ka}^{(2)}$$

while the first row

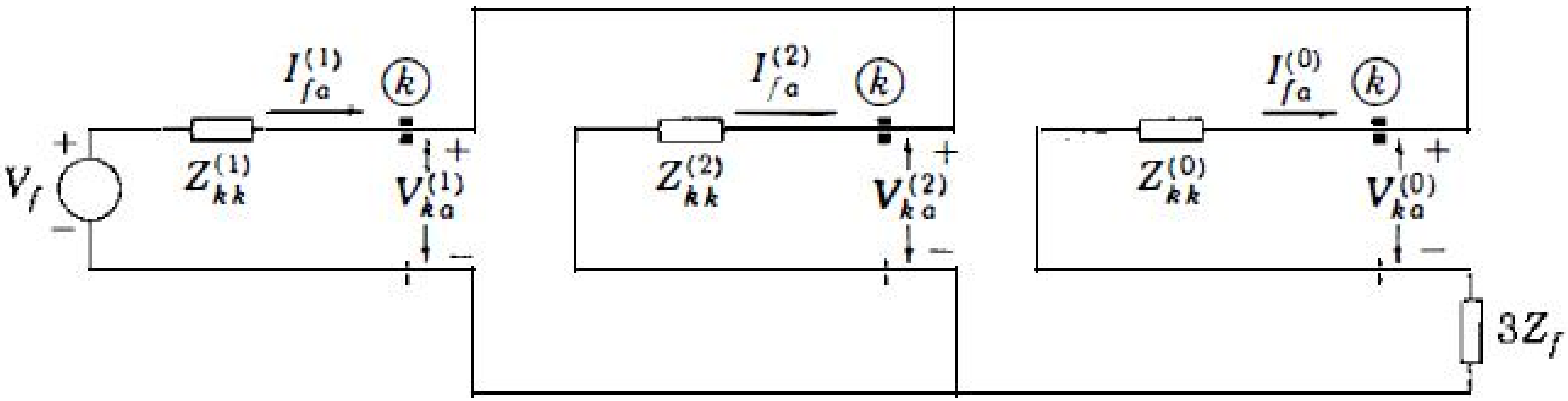
$$3V_{ka}^{(0)} = V_{ka} + 2V_{kb} = \left(V_{ka}^{(0)} + V_{ka}^{(1)} + V_{ka}^{(2)}\right) + 2\left(3Z_f I_{fa}^{(0)}\right)$$

Collecting zero-sequence terms on one side, setting $V_{ka}^{(2)} = V_{ka}^{(1)}$, and solving for $V_{ka}^{(1)}$, we obtain

$$V_{ka}^{(1)} = V_{ka}^{(0)} - 3Z_f I_{fa}^{(0)}$$

$$V_{ka}^{(1)} = V_{ka}^{(2)} = V_{ka}^{(0)} - 3Z_f I_{fa}^{(0)}$$

$$I_{fa}^{(0)} + I_{fa}^{(1)} + I_{fa}^{(2)} = 0$$



$$I_{fa}^{(1)} = \frac{V_f}{Z_{kk}^{(1)} + \left[\dfrac{Z_{kk}^{(2)}\left(Z_{kk}^{(0)} + 3Z_f\right)}{Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f} \right]}$$

The negative- and zero-sequence currents *out* of the system and *into* the fault

$$I_{fa}^{(2)} = -I_{fa}^{(1)}\left[\frac{Z_{kk}^{(0)} + 3Z_f}{Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f}\right]$$

$$I_{fa}^{(0)} = -I_{fa}^{(1)}\left[\frac{Z_{kk}^{(2)}}{Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f}\right].$$

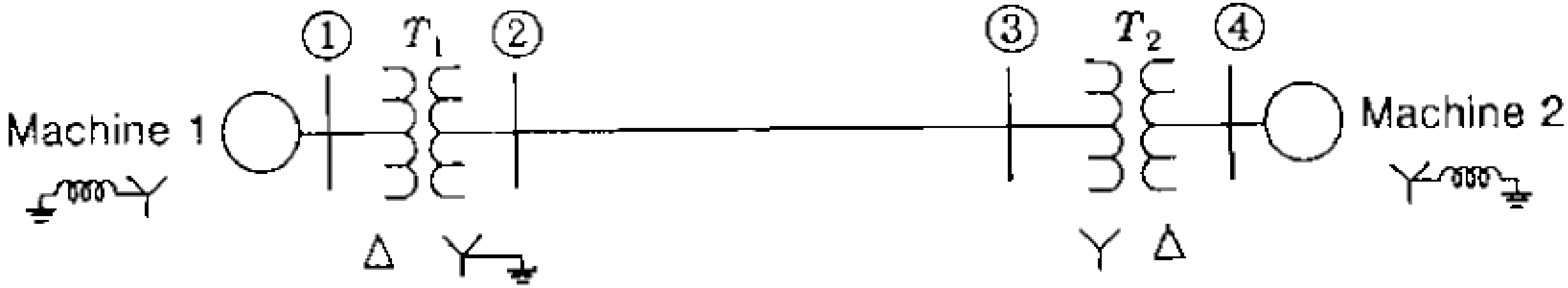
Example 12.4. Find the subtransient currents and the line-to-line voltages at the fault under subtransient conditions when a double line-to-ground fault with $Z_f = 0$ occurs at the terminals of machine 2 in the system of Fig. 12.5. Assume that the system is unloaded and operating at rated voltage when the fault occurs. Use the bus impedance matrices and neglect resistance.

Example 12.1. Two synchronous machines are connected through three-phase transformers to the transmission line shown in Fig. 12.5. The ratings and reactances of the machines and transformers are

Machines 1 and 2: 100 MVA, 20 kV; $X''_d = X_1 = X_2 = 20\%$,

$X_0 = 4\%$, $X_n = 5\%$

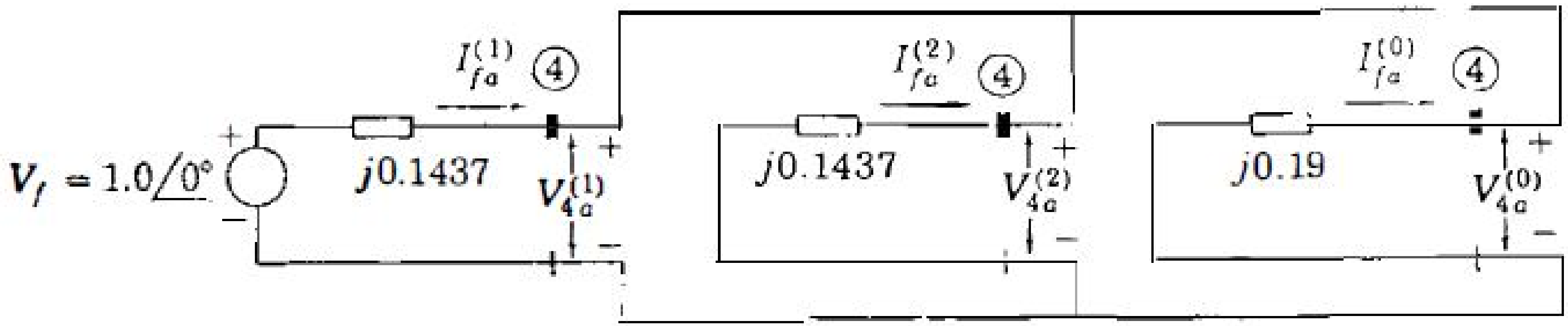
Transformers T_1 and T_2: 100 MVA, 20/345 kV ; $X = 8\%$



$$\mathbf{Z}_{bus}^{(1)} = \mathbf{Z}_{bus}^{(2)} = \begin{array}{c} \\ \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{array} \begin{array}{c} \begin{array}{cccc} \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} \end{array} \\ \begin{bmatrix} j0.1437 & j0.1211 & j0.0789 & j0.0563 \\ j0.1211 & j0.1696 & j0.1104 & j0.0789 \\ j0.0789 & j0.1104 & j0.1696 & j0.1211 \\ j0.0563 & j0.0789 & j0.1211 & j0.1437 \end{bmatrix} \end{array}$$

$$
\mathbf{Z}_{\text{bus}}^{(0)} = \begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \end{array} \begin{array}{c} \begin{array}{cccc} ① & ② & ③ & ④ \end{array} \\ \begin{bmatrix} j0.19 & 0 & 0 & 0 \\ 0 & j0.08 & j0.08 & 0 \\ 0 & j0.08 & j0.58 & 0 \\ 0 & 0 & 0 & j0.19 \end{bmatrix} \end{array}
$$

Solution. The bus impedance matrices $\mathbf{Z}_{\text{bus}}^{(1)}$, $\mathbf{Z}_{\text{bus}}^{(2)}$, and $\mathbf{Z}_{\text{bus}}^{(0)}$ are the same as in Example 12.1, and so the Thévenin impedances at fault bus ④ are equal in per unit to the diagonal elements $Z_{44}^{(0)} = j0.19$ and $Z_{44}^{(1)} = Z_{44}^{(2)} = j0.1437$. To simulate the double line-to-ground fault at bus ④, we connect the Thévenin equivalents of all three sequence networks in parallel, as shown in Fig.



$$I_{fa}^{(1)} = \frac{V_f}{Z_{44}^{(1)} + \left[\dfrac{Z_{44}^{(2)} Z_{44}^{(0)}}{Z_{44}^{(2)} + Z_{44}^{(0)}}\right]} = \frac{1 + j0}{j0.1437 + \left[\dfrac{(j0.1437)(j0.19)}{(j0.1437 + j0.19)}\right]}$$

$$= -j4.4342 \text{ per unit}$$

$$V_{4a}^{(1)} = V_{4a}^{(2)} = V_{4a}^{(0)} = V_f - I_{fa}^{(1)} Z_{44}^{(1)} = 1 - (-j4.4342)(j0.1437) = 0.3628 \text{ per unit}$$

$$I_{fa}^{(2)} = -I_{fa}^{(1)}\left[\frac{Z_{44}^{(0)}}{Z_{44}^{(2)} + Z_{44}^{(0)}}\right] = j4.4342\left[\frac{j0.19}{j(0.1437 + 0.19)}\right] = j2.5247 \text{ per unit}$$

$$I_{fa}^{(0)} = -I_{fa}^{(1)}\left[\frac{Z_{44}^{(2)}}{Z_{44}^{(2)} + Z_{44}^{(0)}}\right] = j4.4342\left[\frac{j0.1437}{j(0.1437 + 0.19)}\right] = j1.9095 \text{ per unit}$$

$$I_{fa} = I_{fa}^{(0)} + I_{fa}^{(1)} + I_{fa}^{(2)} = j1.9095 - j4.4342 + j2.5247 = 0$$

$$I_{fb} = I_{fa}^{(0)} + a^2 I_{fa}^{(1)} + a I_{fa}^{(2)}$$

$$= j1.9095 + (1\angle 240^\circ)(4.4342\angle -90^\circ) + (1\angle 120^\circ)(2.5247\angle 90^\circ)$$

$$= -6.0266 + j2.8642 = 6.6726\angle 154.6^\circ \text{ per unit}$$

$$I_{fc} = I_{fa}^{(0)} + a I_{fa}^{(1)} + a^2 I_{fa}^{(2)}$$

$$= j1.9095 + (1\angle 120^\circ)(4.4342\angle -90^\circ) + (1\angle 240^\circ)(2.5247\angle 90^\circ)$$

$$= 6.0266 + j2.8642 = 6.6726\angle 25.4^\circ \text{ per unit}$$

and the current I_f into the ground is

$$I_f = I_{fb} + I_{fc} = 3I_{fa}^{(0)} = j5.7285 \text{ per unit}$$

Calculating *a-b-c* voltages at the fault bus, we find that

$$V_{4a} = V_{4a}^{(0)} + V_{4a}^{(1)} + V_{4a}^{(2)} = 3V_{4a}^{(1)} = 3(0.3628) = 1.0884 \text{ per unit}$$

$$V_{4b} = V_{4c} = 0$$

$$V_{4,ab} = V_{4a} - V_{4b} = 1.0884 \text{ per unit}$$

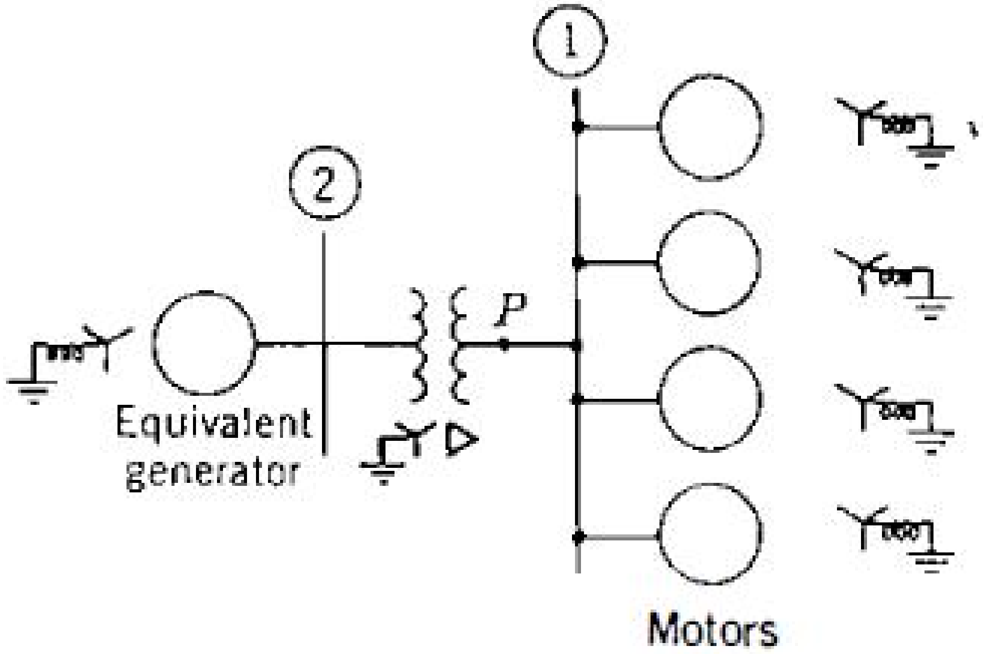
$$V_{4,bc} = V_{4b} - V_{4c} = 0$$

$$V_{4,ca} = V_{4c} - V_{4a} = -1.0884 \text{ per unit}$$

Example 12.6. A group of identical synchronous motors is connected through a transformer to a 4.16-kV bus at a location remote from the generating plants of a power system. The motors are rated 600 V and operate at 89.5% efficiency when carrying a full load at unity power factor and rated voltage. The sum of their output ratings is 4476 kW (6000 hp). The reactances in per unit of each motor based on its own input kilovoltampere rating are $X_d'' = X_1 = 0.20$, $X_2 = 0.20$, $X_0 = 0.04$, and each is grounded through a reactance of 0.02 per unit. The motors are connected to the 4.16-kV bus through a transformer bank composed of three single-phase units, each of which is rated 2400/600 V, 2500 kVA. The 600-V windings are connected in Δ to the motors and the 2400-V windings are connected in Y. The leakage reactance of each transformer is 10%.

The power system which supplies the 4.16-kV bus is represented by a Thévenin equivalent generator rated 7500 kVA, 4.16 kV with reactances of $X''_d = X_2 = 0.10$ per unit, $X_0 = 0.05$ per unit, and X_n from neutral to ground equal to 0.05 per unit.

Each of the identical motors is supplying an equal share of a total load of 3730 kW (5000 hp) and is operating at rated voltage, 85% power-factor lag, and 88% efficiency when a single line-to-ground fault occurs on the low-voltage side of the transformer bank. Treat the group of motors as a single equivalent motor. Draw the sequence networks showing values of the impedances. Determine the subtransient line currents in all parts of the system with prefault current neglected.



1
2
P
Equivalent generator
Motors